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Effect of Water Scarcity in the Society: A Standard Incidence Model

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ABSTRACT

This paper presents the formulation of a water scarcity model and its analysis using the theory of differential equations. Equilibrium point of the model is found and analyzed its local stability and global stability analytically. Numerical simulation for the deterministic model is exhibited to validate our analytical findings. Our results show the better ways for water recovery through the compartments of the model

Keywords
Mathematical model; Water Scarcity; Local Stability; Global stability; Simulation

1.Introduction

Water is one of the most important natural renewable resources and no one can survive without it, either humans or animals. Water comes from various sources including precipitation, surface water, and ground water. India is abundant in the various natural resources, and water is one of them. That plays a significant role in water supply for India. Every three to four months, India receives 70 per cent of surface water in the form of rain (monsoon).

In this paper [1], Water is one of the most essential natural renewable resources, and no one, either humans or animals can live without it. Water comes from numerous sources, including runoff, groundwater, and surface water. The main contributor to the world growth and development are water supplies.

The paper concludes 70 percent of the Earth's surface is filled by 1400 million cubic kilometers of water (m km3). 2.5% freshwater and 97.5% saltwater. 2.5 percent is groundwater, 0.3 percent is lakes and rivers, 68.9 percent is frozen in ice caps, 30.8 percent. One-third of the population of the world currently resides in countries where the quality of the water is not adequately compromised, but by 2025 it is projected to increase by two-thirds [2].

The water scarcity situation has been investigated in cities with a population of more than one million. This was done by using the methodology of the Composite Index to make water-related statistics more intelligible. A forecast was created for the years 2020 to 2030 to show potential improvements in the supply and demand for water in selected Middle East countries. With rising urbanization, there is a moderate to high water risk for all countries at present [3]. [4] Water shortage is a common issue in many parts of the world in this paper chat. Many previous water shortage evaluation strategies only considered the volume of water, and overlooked the quantity of water.

The formulation of a corruption control model and its analysis using the theory of differential equations are presented in this paper, [5]. The equilibria of the model and the stability of these

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equilibria are discussed in detail. They propose and evaluate mathematical models to research the dynamics of smoking activity under the influence of educational programs and also the willingness of the person to quit smoking [6]. A nonlinear mathematical model is formulated and analyzed in this paper [7] to research the relationship between the criminal population and non-criminal population by taking into account the rate of non-monotone incidence. See [8],[9].

[10] suggested and analyzed a mathematical model using oncolytic virotherapy for cancer care. The growth of tumor cells is presumed to obey logistic growth and the interaction between tumor cells and viruses is of type saturation. Several nonlinear mathematical models are proposed and analyzed in this paper [11] to study the spread of asthma due to inhaled industry pollutants. [12],[13], [14], [15] are also referenced. This paper aims to illustrate the requirements and availability of water. As a result of growing populations, rising urbanization, and rapid industrialization, combined with the need to increase agricultural production, water demand has been found to increase significantly. Water per capital supply is also slowly declining. More than 2.2 million people are expected to die every year from diseases related to polluted drinking water and poor sanitation.

The aim of this paper is to highlight water demands and supply. We are here giving a new try to prove the same by using the Mathematical model. Using the principle of an ordinary differential equation, we analyze our model and record comprehensive results of numerical simulations to support the analytical results. The remainder of this article is structured as follows: Section 2 explains the model and the presence of equilibria and illustrates local stability, Global equilibrium stability. Section 3 displays the effects of simulation for deterministic model. Our results are summarized in Section 4 as a conclusion

2. The Model and Analysis

We proposed and analyzed a nonlinear model for Water Scarcity by dividing into four different compartments, namely total usage of Water (W), Human (H), Water scarcity (W_s), Water recover (W_r). All variables are Time t functions. The transfer diagram of the model is described in Figure 1.

$$\begin{aligned}
\frac{dW}{dt} &= \Lambda - \alpha_1 W - \alpha_2 W\left(\frac{H}{N}\right) + \delta_2 W_r \\
\frac{dH}{dt} &= \alpha_2 W\left(\frac{H}{N}\right) - \beta H - \mu H - \mu_1 H \\
\frac{dW_s}{dt} &= \alpha_1 W + \beta H - \delta_1 W_s \\
\frac{dW_r}{dt} &= \delta_1 W_s - \delta_2 W_r
\end{aligned} \tag{1}$$

In the table, the parameters used in the (1) model are defined. (1)

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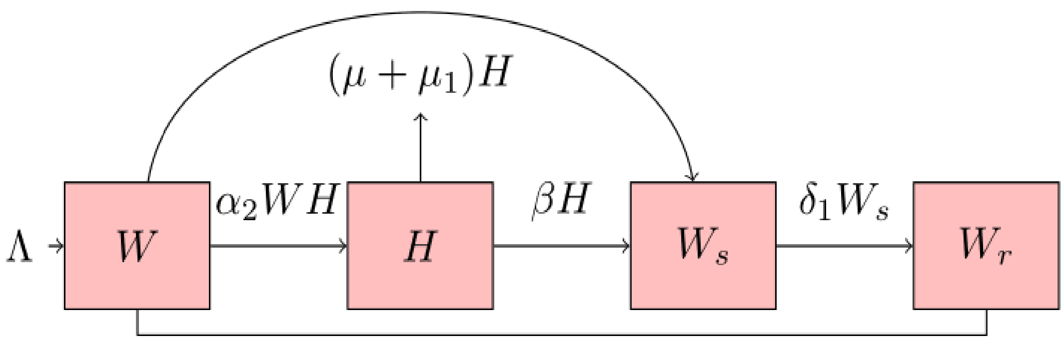
Table 1: Description of parameters

Parameter	Description
Λ	Recruitment rate
α_1	Water draining rate
α_2	The rate of human consumption of water
δ_1	The rate of water Recovery
δ_2	The rate of water going to Normal water
β	Rate of human population affected water scarcity
μ	Natural death
μ_1	Rate of due to Scarcity death



2.1 Existence of Equilibria

As $N(t) = W(t) + H(t) + W_s(t) + W_r(t)$, for the analysis purpose we consider the following system:

$$\frac{dW}{dt} = \Lambda - (\mu + \mu_1)H$$

$$\frac{dH}{dt} = \alpha_2(W + H + W_s + W_r)\left(\frac{H}{N}\right) - k_1H \qquad (2)$$

$$\frac{dW_s}{dt} = \alpha_1(W + H + W_s + W_r) + \beta H - \delta_1W_s$$

$$\frac{dW_r}{dt} = \delta_1W_s - \delta_2W_r$$

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Our model's equilibrium is calculated by setting the right-hand side of the model to zero. The system has following equilibria namely Endemic Equilibrium (EE)
$E^*(N^*, H^*, W_s^*, W_r^*)$

$$N^* = -\frac{\Lambda\alpha_2(\beta\delta_1 + \beta\delta_2 + \delta_1\delta_2)}{(\mu + \mu_1)(\alpha_1\delta_1k_1 + \alpha_1\delta_2k_1 - \alpha_2\delta_1\delta_2 + \delta_1\delta_2k_1)}$$

$$H^* = \frac{\Lambda}{\mu + \mu_1}$$

$$W_s^* = -\frac{\Lambda\delta_2(\alpha_1k_1 + \alpha_2\beta - k_1\beta)}{(\mu + \mu_1)(\alpha_1\delta_1k_1 + \alpha_1\delta_2k_1 - \alpha_2\delta_1\delta_2 + \delta_1\delta_2k_1)}$$

$$W_r^* = -\frac{\Lambda\delta_1(\alpha_1k_1 + \alpha_2\beta - k_1\beta)}{(\mu + \mu_1)(\alpha_1\delta_1k_1 + \alpha_1\delta_2k_1 - \alpha_2\delta_1\delta_2 + \delta_1\delta_2k_1)}$$

Where $k_1 = \beta + \mu + \mu_1$

2.2 Stability Analysis

The variational matrix for the system is given by

$$M =$$

$$\begin{pmatrix} 0 & -(\mu+\mu_1) & 0 & 0 \\ \alpha_2\left(\frac{H}{N}\right) - \alpha_2(N - H - W_s - W_r)\left(\frac{1}{N^2}\right) & \alpha_2\left(\frac{H}{N}\right) - \alpha_2(N - H - W_s - W_r)\left(\frac{1}{N^2}\right) & -\alpha_2\left(\frac{H}{N}\right) & -\alpha_2\left(\frac{H}{N}\right) \\ \alpha_1 & -\alpha_1 + \beta & -(\alpha_1 + \delta_1) & -\alpha_1 \\ 0 & 0 & \delta_1 & -\delta_2 \end{pmatrix}$$

2.2.1 Stability analysis of EE point

The variation matrix, M* corresponding to the point E^* of the Endemic Equilibrium, is given by

$$M^* = \begin{pmatrix} 0 & n_{12} & 0 & 0 \\ n_{21} & n_{22} & n_{23} & n_{24} \\ n_{31} & n_{32} & n_{33} & 0 \\ 0 & 0 & n_{43} & n_{44} \end{pmatrix}$$

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Where

$n_{12} = -(\mu + \mu_1)$, $n_{21} = \alpha_2\left(\frac{H}{N}\right) - \alpha_2(N - H - W_s - W_r)\left(\frac{1}{N^2}\right)$,
$n_{22} = \alpha_2\left(\frac{H}{N}\right) - \alpha_2(N - H - W_s - W_r)\left(\frac{1}{N^2}\right)$, $n_{23} = -\alpha_2\left(\frac{H}{N}\right)$, $n_{24} = -\alpha_2\left(\frac{H}{N}\right)$
$n_{31} = \alpha_1$, $n_{32} = -\alpha_1 + \beta$, $n_{33} = -(\alpha_1 + \delta_1)$, $n_{34} = -\alpha_1$
$n_{43} = \delta_1$, $n_{44} = -\delta_2$

The bi-quadratic equation

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$$

Where

$a_1 = -(n_{22} + n_{33} + n_{44})$

$a_2 = n_{22}n_{33} + n_{33}n_{44} + n_{22}n_{44} - n_{12}n_{21} - n_{23}n_{32}$

$a_3 = n_{12}n_{21}n_{33} + n_{12}n_{21}n_{44} + n_{23}n_{32}n_{44} - n_{12}n_{23}n_{31} - n_{24}n_{32}n_{43} - n_{22}n_{33}n_{44}$

$a_4 = n_{12}n_{23}n_{31}n_{44} - n_{12}n_{24}n_{31}n_{43} - n_{12}n_{21}n_{33}n_{44}$

E^* will be locally asymptotically stable by using Routh-Hurwitz criteria if the following conditions are satisfied:

$\mathbf{a_1 > 0}$, $\mathbf{a_3 > 0}$, $\mathbf{a_1a_2a_3 - a_3^2 - a_1^2a_4 > 0}$, $\mathbf{a_3 > 0}$. If two other inequalities referred to above are satisfied, $\boldsymbol{E^*}$ is locally asymptotically stable

2.2.2 Global Stability of Endemic Equilibrium

In order to analyze the global stability of the endemic equilibrium E^*, We adopt the approach developed by [8] Korobeinikov (2006) and it is successfully applied in [9]. E^* exists for all x, y, z, w > ϵ , for some $\epsilon > 0$.

Let $k_1y = [\beta + \mu + \mu_1]y = g(x, y, z, w)$ be positive and monotonic functions in R_+^4 (for more details, see [8, 9]).

$$V(x,y,z,w) = x - \int_\epsilon^x \frac{g(x^*,y^*,z^*,w^*)}{g(\eta^*,y^*,z^*,w^*)}d\eta + y - \int_\epsilon^y \frac{h(x^*,y^*,z^*,w^*)}{h(x^*,\eta^*,z^*,w^*)}d\eta$$
$$+z - \int_\epsilon^z \frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y^*,\eta^*,w^*)}d\eta + w - \int_\epsilon^w \frac{g(x^*,y^*,z^*,w^*)}{g(x^*,y^*,z^*,\eta^*)}d\eta$$

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If g (x, y, z, w) is monotonic with respect to its variables, then the state E is the only extreme and the global minimum of this function. So obviously

$$\frac{\partial V}{\partial x} = 1 - \frac{g(x^*, y^*, z^*, w^*)}{g(x^*, y^*, z^*, w^*)}, \qquad \frac{\partial V}{\partial y} = 1 - \frac{h(x^*, y^*, z^*, w^*)}{h(x^*, y^*, z^*, w^*)}$$

$$\frac{\partial V}{\partial z} = 1 - \frac{h(x^*, y^*, z^*, w^*)}{h(x^*, y^*, z^*, w^*)}, \qquad \frac{\partial V}{\partial w} = 1 - \frac{g(x^*, y^*, z^*, w^*)}{g(x^*, y^*, z^*, w^*)}$$

The g (x, y, z, w) and h (x, y, z, w) functions grow monotonically, then have only one stationary point. Further, since

$$\frac{\partial^2 V}{\partial x^2} = \frac{g(x^*, y^*, z^*, w^*)}{[g(x, y^*, z^*, w^*)]^2} \cdot \frac{g(x, y^*, z^*, w^*)}{\partial x},$$

$$\frac{\partial^2 V}{\partial y^2} = \frac{g(x^*, y^*, z^*, w^*)}{[g(x^*, y, z^*, w^*)]^2} \cdot \frac{g(x^*, y, z^*, w^*)}{\partial y},$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{g(x^*, y^*, z^*, w^*)}{[g(x^*, y^*, z, w^*)]^2} \cdot \frac{g(x^*, y^*, z, w^*)}{\partial z},$$

$$\frac{\partial^2 V}{\partial w^2} = \frac{g(x^*, y^*, z^*, w^*)}{[g(x^*, y^*, z^*, w)]^2} \cdot \frac{g(x^*, y^*, z^*, w)}{\partial w}$$

are non-negative, then g(x, y, z, w) and h(x, y, z, w) have minimum. That is,

$$V(x, y, z, w) \geq V(x^*, y^*, z^*, w^*)$$

and hence, V is a Lyapunov function, and its derivative is given by

$$\frac{\partial V}{\partial t} = \dot{x} - \dot{x}\frac{g(x^*, y^*, z^*, w^*)}{g(x, y^*, z^*, w^*)} + \dot{y} - \dot{y}\frac{g(x^*, y^*, z^*, w^*)}{g(x^*, y, z^*, w^*)}$$

$$+\dot{z} - \dot{z}\frac{g(x^*, y^*, z^*, w^*)}{g(x^*, y^*, z, w^*)} + \dot{w} - \dot{w}\frac{g(x^*, y^*, z^*, w^*)}{g(x^*, y^*, z^*, w)}$$

$$= -\alpha_1 x^* \left(1 - \frac{x}{x^*}\right) \left[1 - \frac{h(x^*, y^*, z^*, w^*)}{h(x^*, y^*, z, w^*)}\right] + \alpha_1 y^* \left(1 - \frac{y}{y^*}\right) \left[1 - \frac{h(x^*, y^*, z^*, w^*)}{h(x^*, y^*, z, w^*)}\right]$$

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$$+\alpha_1 z^*\left(1-\frac{z}{z^*}\right)\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y^*,z,w^*)}\right]+\alpha_1 w^*\left(1-\frac{w}{w^*}\right)\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y^*,z,w^*)}\right]$$

$$-\beta y^*\left(1-\frac{y}{y^*}\right)\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y^*,z,w^*)}\right]+\delta_1 z^*\left(1-\frac{z}{z^*}\right)\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y^*,z,w^*)}\right]$$

$$-\delta_1 z^*\left(1-\frac{z}{z^*}\right)\left[1-\frac{g(x^*,y^*,z^*,w^*)}{g(x^*,y^*,z^*,w)}\right]+\delta_2 w^*\left(1-\frac{w}{w^*}\right)\left[1-\frac{g(x^*,y^*,z^*,w^*)}{g(x^*,y^*,z^*,w)}\right]$$

$$-\alpha_2 y^*\left(1-\frac{y}{y^*}\right)\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y,z^*,w^*)}\right]+(\mu+\mu_1)y^*\left(1-\frac{y}{y^*}\right)\left[1-\frac{g(x^*,y^*,z^*,w^*)}{g(x,y^*,z^*,w^*)}\right]$$

$$+g(x,y^*,z^*,w^*)\left[1-\frac{g(x,y,z,w)}{g(x,y^*,z^*,w)}\right]\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y,z^*,w^*)}\right]$$

$$+g(x,y^*,z,w^*)\left[1-\frac{g(x,y,z,w)}{g(x,y^*,z,w^*)}\right]\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y,z^*,w^*)}\right]$$

It is noted here that $g(x^*,y^*,z^*,w^*)=h(x^*,y^*,z,w)$ is explicitly given as g and h in terms of x, y, z and w.

Since E > 0, the functions g (x, y, z, w) is concave with respect to y, z & w and

$$\frac{\partial^2 g(x,y,z,w)}{\partial y^2}\leq 0,\frac{\partial^2 g(x,y,z,w)}{\partial z^2}\leq 0$$

Then $\frac{dV}{dt}\leq 0$ for all x, y, z, w > 0. Also, the monotonicity of g (x, y, z, w) with respect to x, y, z & w ensures that

$$\left(1-\frac{x}{x^*}\right)\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x,y^*,z^*,w^*)}\right]\leq 0,\quad \left(1-\frac{y}{y^*}\right)\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y,z^*,w^*)}\right]$$

$$\left(1-\frac{z}{z^*}\right)\left[1-\frac{h(x^*,y^*,z^*,w^*)}{h(x^*,y^*,z,w^*)}\right]\leq 0,\quad \left(1-\frac{w}{w^*}\right)\left[1-\frac{g(x^*,y^*,z^*,w^*)}{g(x^*,y^*,z^*,w)}\right]$$

holds for all x, y, z, w > 0. Thus, we establish the following result.

The endemic equilibrium E^* of model (1) is globally asymptotically stable whenever conditions outlined in Eq. (9) are satisfied.

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3. Numerical simulation

The system (1) is simulated for various set of parameters satisfying the condition of local and globally asymptotic stability of equilibrium E^*. Here, the results of deterministic model as the curve corresponding to Scarcity lies below the one that corresponds to the deterministic model.

Λ = 100, α_1 = 0.09, α_2= 0.35, μ= 0.0143, μ_1 = 0.0167, β = 0.0005, δ_1 = 0.06, δ_2 = 0.3

The system (1) is simulated for different set of parameters satisfying the condition of local and globally asymptotic stability of equilibrium- E^* (see Fig.2). Figs 3 - 6 demonstrate the impact of various parameters on the equilibrium level of Water scarcity and recovery.

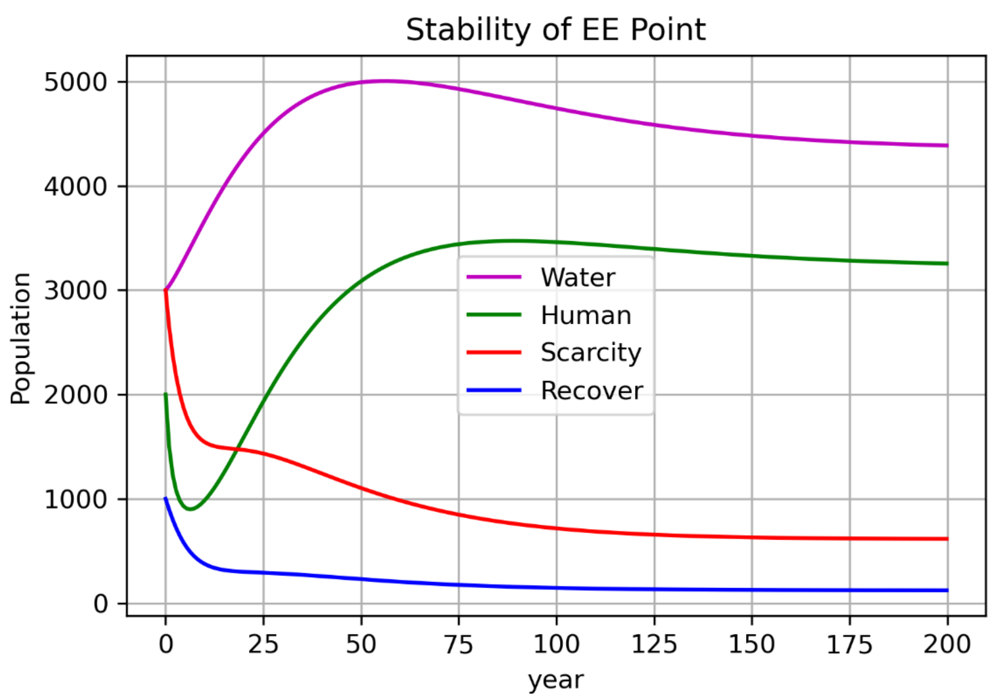

Figure 2: Variation of all compartments of the model showing the stability

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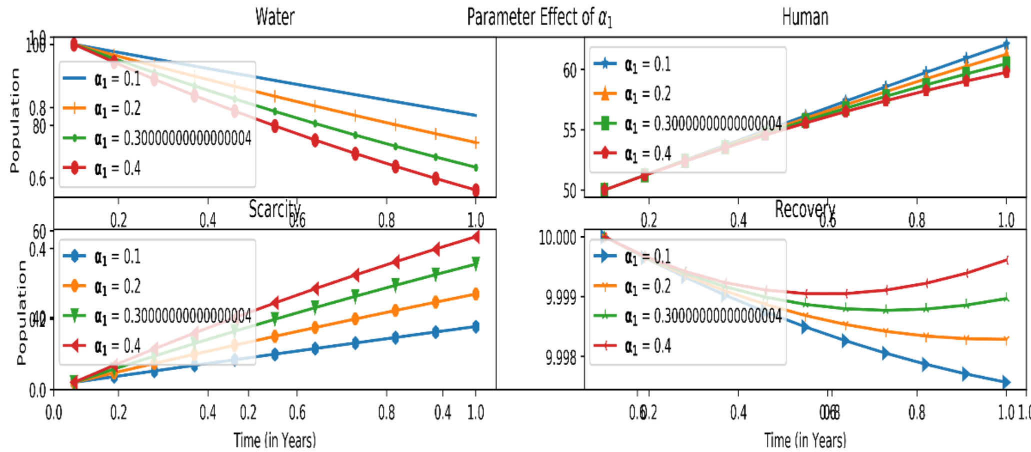

Figure 3: Effect of α_1 on the variation of all compartments of the model

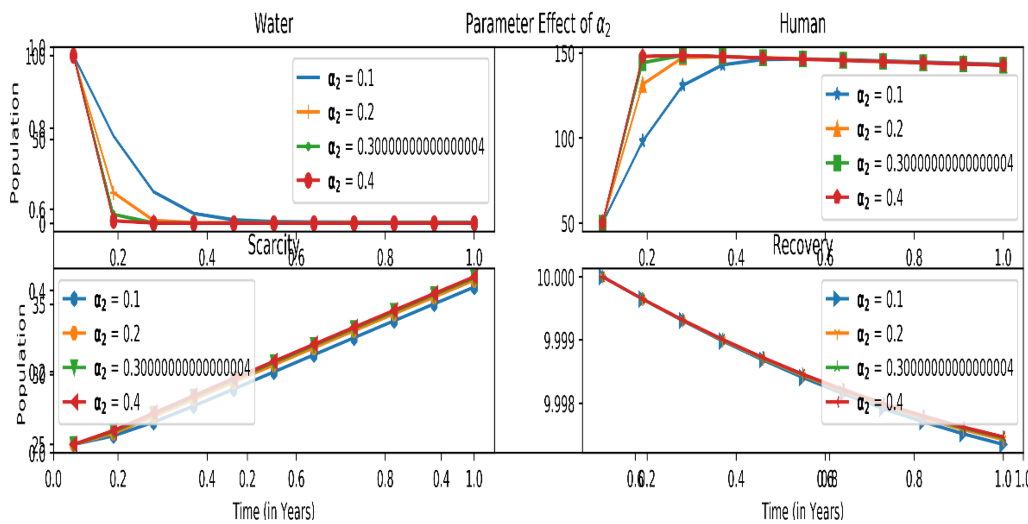

Figure 4: Effect of α_2 on the variation of all compartments of the model

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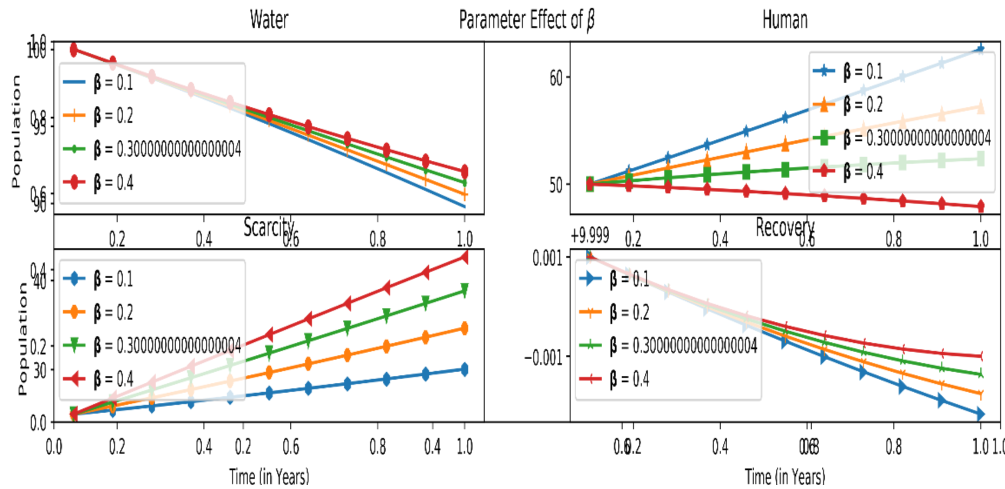

Figure 5: Effect of β on the variation of all compartments of the model

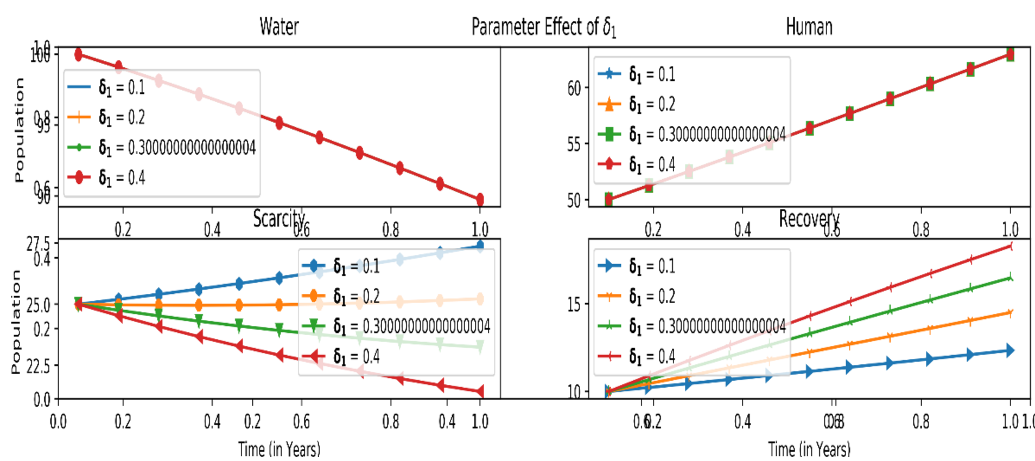

Figure 6: Effect of δ_1 on the variation of all compartments of the model

4. Result Discussion and Conclusion

In this paper, a deterministic mathematical model on water resource-related water scarcity problems were proposed and analyzed. We calculate the equilibrium of the proposed model and analyze in detail the local stability and global stability of endemic equilibria. The impact of various parameters on the equilibrium points of water scarcity a recovery is demonstrated. All of us have social responsible to become stronger of the reduction of water scarcity to the society, in that aspect we have taken one kind of initiative a model to predict to show the better result using possible strategies. Through this model we found the effectiveness of the population progress from Human to Water scarcity by simulation.

When β (The rate of human population affected water scarcity) value increases at the time stable point is differed in all compartment (see Fig. 5). Fig. 3 and 4 depicts if α_1 and α_2 value increase or decrease there is no major different in all compartment. Fig.6 depicts the parameter δ_1 (The rate of water Recovery) value increasing time the water scarcity is decreased and the recovery is increased.

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