Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 1944 - 1953
Received 25 April 2021; Accepted 08 May 2021.

# Dynamics of SEIR Model for Second Wave Indian COVID-19 Pandemic

E.Muralikrishnan(1)
*Department of Mathematics,*
*College of Engineering and Technology,*
*SRM Institute of Science and Technology,*
*Ramapuram - 600 089, Kancheepuram District,*
*Tamil Nadu, INDIA.*
*Email: muralike@srmist.edu.in*

S.Balamuralitharan(2)
*Department of Mathematics,*
*College of Engineering and Technology,*
*SRM Institute of Science and Technology,*
*Kattankulathur - 603 203, Chengalpattu District,*
*Tamil Nadu, INDIA.*
*Email: balamurs@srmist.edu.in*

Corresponding Author: *balamurs@srmist.edu.in*

**ABSTRACT**

The aim of this study is to build an SEIR model for Corona Virus Diseases-2019(COVID-19) second wave pandemic in India. We discussed stability analysis of SEIR model with control strategy for global asymptotically stable (or) not. This method used to find the control strategy of government policy such as Indian government and other government organized bodies for high affected states. We analysis the equation using reproduction number from next generation matrix method. The Lyapunov technique to find the given system of equation is globally asymptotically stable (or) not from reproduction number.

**Keywords:**

Indian second wave COVID-19 pandemic, SEIR model, Stability Analysis, Reproduction Number, Lyapunov function.

## Introduction

Already we know that, the history of Indian COVID-19 pandemic on first wave. Recently highly affected by the second wave, globally 148,472,884 as on date April 27, 2021. Similarly 17,625,735 by Indian second wave pandemic as on dateApril 27, 2021. The first wave lies between from April 2020 to February 2021. Then the second wave lies between March 2021 and till date. Now highly affected people as rank wise such as Maharashtra, Kerala, Karnataka, Uttar Pradesh, Tamil Nadu.

We obtain SEIR model for common second Indian pandemic for the study of equation analysis. In this regard, the equilibrium analysis, endemic equilibrium, Reproduction number and

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 1944 - 1953
Received 25 April 2021; Accepted 08 May 2021.

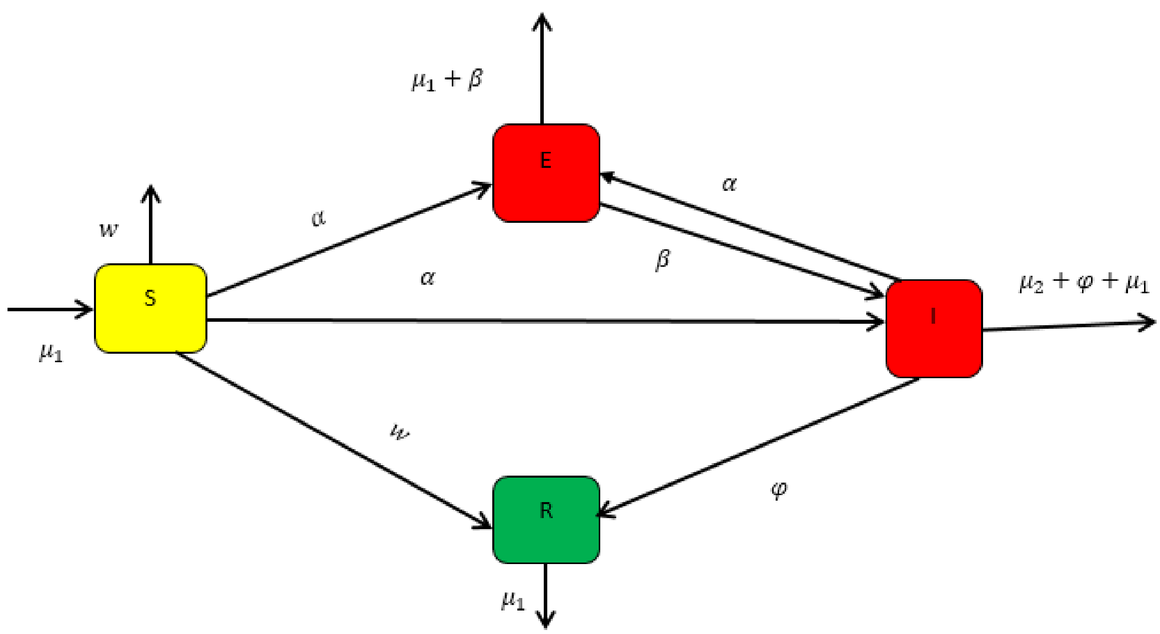
Lyapunov technique by using $R_0$ in terms of globally asymptotically stable (or) not. Based on this analytic problem is used to identify the cure rate and infected rate from the real life data. This paper is useful for researcher in Mathematical Biology and Covid-19 researcher also. The several authors have been prepared lot of papers in COVID-19

## Method

A computational investigation is the SEIR mathematical modelling of COVID-19 distribution.The model was built using the SEIR model [12], which included vaccination and isolation variables as model parameters.The basic reproduction number and global stability for COVID-19 spread were calculated using the generation matrix method [9].

## COVID-19 SEIR type formulation

(S) -Suspected, (E) -Exposed, (I) - Infected, and (R)-Recovered are the four compartments of SEIR model for COVID-19 spread.Infected members of a class have the ability to infect others.For the SEIR model, the following figure illustrates the compartmental model.




Parameters and variables definition

| Variable/Parameter | Definition |
|---|---|
| S | Population under suspicion |
| E | People that have been exposed |
| I | Community infected with the |
| R | virus |
| $\mu_1$ | The population has recovered. |
| $\alpha$ | Birth-to-death ratio in the |
| $\beta$ | population |
| $\mu_2$ | Changeover time from S to E |

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 1944 - 1953
Received 25 April 2021; Accepted 08 May 2021.

| | |
|---|---|
| $\varphi$<br>$w$ | Changeover time from E to I<br>COVID-19 population death rate<br>Changeover time from I to R<br>Suspected population vaccination |

Using the compartmental modelin Fig. 1, the rate of change in the number of people Suspected, Exposed, Infected, and Recovered over time in the SEIR statistical model of Covid- 19 spread can be interpreted as follows.

$$\frac{dS}{dt} = \mu_1 - (\alpha I + w)S \quad (1)$$

$$\frac{dE}{dt} = \alpha SI - (\beta + \mu_1)E \quad (2)$$

$$\frac{dI}{dt} = \beta E - (\mu_2 + \varphi + \mu_1)I \quad (3)$$

$$\frac{dR}{dt} = \varphi I + \omega S - \mu R \quad (4)$$

**Analysis of the COVID-19 SEIR model**

**Analysis of equilibrium**

Eqs. (1)–(4) was used to determine the equilibrium point for disease free and the equilibrium point for endemic using stability analysis .To determine the two equilibrium points, each equation in Eqs. (1) – (4) must be equal to zero, or, or $\frac{dS}{dt} = 0$, $\frac{dE}{dt} = 0$, $\frac{dI}{dt} = 0$, $\frac{dR}{dt} = 0$

$$\mu_1 - (\alpha I + w)S = 0$$

$$\alpha SI - (\beta + \mu_1)E = 0$$

$$\beta E - (\mu_2 + \varphi + \mu_1)I = 0$$

$$\varphi I + \omega S - \mu R = 0$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 1944 - 1953
Received 25 April 2021; Accepted 08 May 2021.

**Covid-19 is in a state of disease-free equilibrium**.

When there is no distribution of COVID-19, the equilibrium point for disease-free conditions is E = I = 0.

From Eq (5)

$$S = \frac{\mu}{w}$$

From Eq (8)

$$R = \frac{\mu_1}{\mu_2}$$

As a result, the disease-free covid 19 equilibrium point is

$$(S, E, I, R) = \left(\frac{\mu_1}{w}, 0, 0, \frac{\mu_1}{\mu_2}\right)$$

**Endemic Equilibrium**

The probability of disease transmission is indicated by endemic equilibrium points. Since the population S ≠ 0, E ≠ 0, I ≠ 0, and R ≠ 0 in endemic conditions and disease spread.

Endemic equilibrium points obtained from Eqs(5) to (8) are:

$$S = \frac{(\mu_1 + \beta)(\mu_2 + \varphi + \mu_1)}{\alpha\beta}$$

$$E = \frac{\alpha\beta\gamma - (\mu_1 + \beta)(\mu_2 + \varphi + \mu_1)}{\alpha\beta(\mu_1 + \beta)}$$

$$I = \frac{\alpha\beta\mu_1 - \omega(\mu_1 + \beta)(\mu_2 + \varphi + \mu_1)}{\alpha(\mu_1 + \beta)(\mu_2 + \varphi + \mu_1)}$$

$$R = \frac{\varphi\alpha\beta^2\mu_1 - \varphi\omega\beta(\mu_1 + \beta)(\mu_2 + \varphi + \mu) + w(\mu_1 + \beta)^2(\mu_2 + \varphi + \mu_1)}{\alpha\beta\mu_2(\mu_1 + \beta)(\mu_2 + \varphi + \mu_1)}$$

Then, for COVID-19, the endemic equilibrium points are:

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 1944 - 1953
Received 25 April 2021; Accepted 08 May 2021.

$$(S,E,I,R) = \begin{pmatrix} \frac{(\mu_1+\beta)(\mu_2+\varphi+\mu_1)}{\alpha\beta}, \; \frac{\alpha\beta\gamma-(\mu_1+\beta)(\mu_2+\varphi+\mu_1)}{\alpha\beta(\mu_1+\beta)}, \\ \frac{\alpha\beta\mu_1-\omega(\mu_1+\beta)(\mu_2+\varphi+\mu_1)}{\alpha(\mu_1+\beta)(\mu_2+\varphi+\mu_1)}, \\ \frac{\varphi\alpha\beta^2\mu_1-\varphi\omega\beta(\mu_1+\beta)(\mu_2+\varphi+\mu)+w(\mu_1+\beta)^2(\mu_2+\varphi+\mu_1)}{\alpha\beta\mu_2(\mu_1+\beta)(\mu_2+\varphi+\mu_1)} \end{pmatrix}$$

Using Eqs (1)- (4), the Jacobian matrices is given by

$$J = \begin{pmatrix} -(\alpha I+w) & 0 & \alpha S & 0 \\ \alpha I & -(\mu_1+\beta) & \alpha S & 0 \\ 0 & \beta & -(\mu_2+\varphi+\mu_1) & 0 \\ w & 0 & \varphi & -\mu_1 \end{pmatrix}$$

Then

$$|\lambda I - J| = \begin{pmatrix} \lambda+(\alpha I+w) & 0 & -\alpha S & 0 \\ -\alpha I & \lambda+(\mu_1+\beta) & -\alpha S & 0 \\ 0 & -\beta & \lambda+(\mu_2+\varphi+\mu_1) & 0 \\ -w & 0 & -\varphi & \lambda+\mu_1 \end{pmatrix}$$

Substitute

$$S = \frac{\mu}{w} \; and \; I = 0$$

$$\begin{vmatrix} \lambda+w & 0 & -\alpha\mu_1 w^{-1} & 0 \\ 0 & \lambda+(\mu_1+\beta) & -\alpha\mu_1 w^{-1} & 0 \\ 0 & -\beta & \lambda+(\mu_2+\varphi+\mu_1) & 0 \\ -w & 0 & -\varphi & \lambda+\mu_1 \end{vmatrix}$$

$$(\lambda+\mu_1)\big[(\lambda+w)[\lambda+(\mu_1+\beta)(\lambda+(\mu_2+\varphi+\mu_1))-\alpha\beta\mu_1 w^{-1}]\big] = 0$$

$$(\lambda+a)(\lambda+b)[(\lambda+c)(\lambda+d)-e] = 0$$

With

$$a = \mu_1 \; ; b = w \; ; c = \mu_1+\beta \; ; d = (\mu_2+\varphi+\mu_1) \; ; e = (\alpha\beta\gamma)/w$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 1944 - 1953
Received 25 April 2021; Accepted 08 May 2021.

$$\lambda^4 + (a+b+c+d)\lambda^3 + (ab + (a+b)(c+d) + cd - e)\lambda^2$$
$$+\big((a+b)(cd-e) + ab(c+d)\big)\lambda + abcd - abe = 0$$

$$K = M_1\lambda^4 - M_2\lambda^3 + M_3\lambda^2 - M_4\lambda + M_5$$

With

$M_1 = 1$

$M_2 = a + b + c + d$

$M_3 = (ab + (a+b)(c+d) + cd - e)$

$M_4 = (a+b)(cd-e) + ab(c+d)$

$M_5 = abcd - abe$

With condition $M_1, M_2,\ M_3,\ M_4,\ M_5 > 0$

This brings us to the conclusion that if $cd > e$and $M_1, M_2,\ M_3,\ M_4,\ M_5 > 0$, then $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4 < 0$

The equilibrium point in the COVID-19 model is a stable global asymptotic equilibrium point since the characteristic values of the equation system are negative.

COVID-19 Basic reproduction number $R_0$ for SEIR model

From Eqs (1) –(4), $R_0$ is calculated by applying generation Matrices method.

Let $F = \begin{pmatrix} 0 & \alpha S \\ \beta & 0 \end{pmatrix}$

$$V = \begin{pmatrix} (\mu_1 + \beta) & 0 \\ 0 & (\mu_2 + \varphi + \mu_1) \end{pmatrix}$$

Then

$$V^{-1} = \frac{1}{(\mu_1 + \beta)(\mu_2 + \varphi + \mu_1)} \begin{pmatrix} \mu_2 + \varphi + \mu_1 & 0 \\ 0 & \mu_1 + \beta \end{pmatrix}$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 1944 - 1953
Received 25 April 2021; Accepted 08 May 2021.

$$V^{-1} = \begin{pmatrix} \frac{1}{\mu_1 + \beta} & 0 \\ 0 & \frac{1}{\mu_2 + \varphi + \mu_1} \end{pmatrix}$$

$$FV^{-1} = \begin{pmatrix} 0 & \alpha S \\ \beta & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\mu_1 + \beta} & 0 \\ 0 & \frac{1}{\mu_2 + \varphi + \mu_1} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{\alpha S}{\mu_2 + \varphi + \mu_1} \\ \frac{\beta}{\mu_1 + \beta} & 0 \end{pmatrix}$$

$$R_0 = \frac{\alpha\beta\mu_1}{(\mu_1 + \beta)(\mu_1 + w)\,(\mu_2 + \varphi + \mu_1)} \dots\dots\dots\dots (3)$$

**COVID-19 Stability analysis for SEIR model**

Theorem

1 The system of Eqs. (5)–(8) is global asymptotic stable for $R_0 \leq 1$

2 The system of Eqs. (5)–(8)is unstable in $D = \{ (S, E, I, R) \in R_+^4,\ S + E + I + R \leq S_0 \}$for$R_0 > 1$

Let $Y = (E, I)^T$,be the case that $\frac{dY}{dt} \leq (F - V)Y$, with F and V are matrices used in $R_{0,}$ and u being an Eigen vector of F

$$R_0 = \rho(FV^{-1}) = \rho(V^{-1}F)$$

If $V^{-1}F = R_0$, then $uV^{-1}F = R_0 u$

Let Lyapunov function

$L_0 = uV^{-1}Y$

$\frac{dL_0}{dt} = uV^{-1}\frac{dY}{dt}$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 1944 - 1953
Received 25 April 2021; Accepted 08 May 2021.

$$\frac{dL_0}{dt} = uV^{-1}(F - V)Y$$

$$\frac{dL_0}{dt} = (uV^{-1}F - u)Y$$

$$\frac{dL_0}{dt} = u(R_0 - 1)Y$$

If $R_0 \leq 1$, then $\frac{dL_0}{dt} = 0$, resulting in $uY = 0$, and $E = I = 0$ with $u > 0$

If $R_0 < 1$then $\frac{dE}{dt} + \frac{dI}{dt} = 0$ and we realized

$$\alpha SI - (\beta + \mu_1)E + \beta E - (\mu_2 + \varphi + \mu_1)I = 0$$

$$\alpha SI - (\mu_1 + \varphi + \mu)I = 0 \text{with} E = 0$$

$$R_0 \left[\frac{(\mu_1 + \beta)(\mu_1 + w)\ (\mu_2 + \varphi + \mu_1)}{\beta\mu}\right] IS - (\mu_2 + \varphi + \mu_1)I = 0$$

$$(\mu_2 + \varphi + \mu_1)\left[\frac{R_0(\mu_1 + \beta)(\mu_1 + w)}{\beta\mu} S - 1\right] = 0$$

$$(\mu_2 + \varphi + \mu_1)\left[\frac{R_0(\mu_1 + \beta)(\mu_1 + w)}{\beta w} - 1\right] = 0$$

It denotes $(\mu_2 + \varphi + \mu_1)[R_0 - 1] \approx 0$

It can then be shown that, if $R_0 < 1$, then $\frac{dL_0}{dt} < 0$

This ensures the system's asymptotic global stability.

On the other hand, if $\mathrm{R}_0 < 1$ , then $\frac{\mathrm{dL}_0}{\mathrm{dt}} > 0$, the vector fields' continuity means that the system in D is in a neighborhood.

According to the Lyapunov stability principle, the system in Eqs. (5)–(8) is unstable.The persistent principle [22], which is close to the proof of Theorem 2.5 in [23], can be used to prove the last part of the theorem.

**Conclusion**

On the basis of the investigation's results, the SEIR model may be used as a guideline for COVID-19 second wave distribution in India. The model's study offers a global overview of

COVID-19 spread as well as details on whether India is endemic for the virus. The recreation results then provide an early indication of the number of COVID-19 cases in India, as well as demonstrating that immunizations can hasten COVID-19 recovery and that the detachment time frame can monitor COVID-19 spread in India. Suppose the second wave is reached high, the third wave becomes very small and very soon this disease will end otherwise third wave becomes very high.

References

[1] SuwardiAnnas, Muh. Isbarpratama, Muh. Rifandi. Stability analysis and numerical simulation of SEIR model for pandemic COVID-19 spread in Indonesia. Chaos, Solitons and Fractals 139 (2020) 110072

[2] Anonim,2020. Ungkap. carapenyebaran virus Cororna di Dunia. https://www.cnbcindonesia.com/tech/20200323104158-37-146860/simak-nihwho-ungkap-cara-penyebaran-virus-corona-di-dunia., [Accessed 24th March, 2020].

[3] Briantika A. Positif COVID-19 di Indonesia Jadi 514 Kasus. Meninggal 2020;48. https://tirto.id/positif-covid-19-di-indonesia-jadi-514-kasus-48- meninggal-eGUa [Accessed, 23rd March, 2020].

[4] M.F. Vina2020. 331.273 Orang Recovered, 97.847 Orang Sembuh", https: //www.kompas.com/tren/read/2020/03/23/072649465/update-virus-coronadi-dunia-331273-orang-Recovered-97847-orang-sembuh?page=2. [Accessed, 23rd, March 2020].

[5] Abdallah S Waziri, Estomih S Massawe, Oluwole Daniel Makinde. Mathematical modelling of HIV/AIDS dynamics with treatment and vertical transmission. Appl Math 2012;2(3):77–89.

[6] T. Ashley, S. Jacqueline S. John. 2010. Modeling the spread of tuberculosis in a closed population. http://educ.jmu.edu/strawbem/math_201/final_reports/ Scotti_Takahashi_Spreadbury_Final.pdf [Diakses, 20 April 2018].

[7] Tang B, Wang X, Li Q, Bragazzi NL, Tang S, Xiao Y, et al. Estimation of the transmission risk of 2019-nCoV and its implication for public health interventions. J Clin Med 2020;9:462.

[8] Syafruddin S, Mulbar U, Sidjara S, Sanusi W. A SEIR Model for transmission of tuberculosis. AIP conference proceedings, 1830; 2017.

[9] Diekmann O, Heesterbeek JAP, Roberts MG. The construction of next-generation matrices for compartmental epidemic models. J R Soc Interface Vol 2010;7:873–85.

[10] Dontwi IK, Obeng WD, Andam EA, Obiri LA. A mathematical model to predict the prevalence and transmission dynamics of tuberculosis in amansie west district, Ghana. Br J Math Comp Sci 2014;4(3):402–4025.

[11] Spencer JA, Shutt DP, Moser SK, Clegg H, Wearing HJ, Mukundan H. et al., Epidemiological parameter review and comparative dynamics of influenza, respiratory syncytial virus,rhinovirus, human coronvirus, and adenovirus. medRxiv 2020.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 6, 2021, Pages. 1944 - 1953
Received 25 April 2021; Accepted 08 May 2021.

[12] Rusliza A, Budin H. Stability analysis of mutualism population model with time delay. Int J Math ComputPhysElectrComputEng 2012;6(2):151–5.

[13] Ofosuhene O. Modelling the spread of HIV and AIDS epidemic trends in male and female populations. World J Model Simul 2017;13(3):183–92 (2017).

[14] Elif D, Arzu U, Nuri O. A fractional order SEIR model with density dependent death rate. Hacet J Math Stat 2011;40(2):287–95.

[15] Rangkuti YM, Side S, Noorani MSM. Numerical analytic solution of SIR model of dengue fever disease in south Sulawesi using homotopy perturbation method and variational iteration method. ITBC J Sci (J Math FundamSci) 2014;46A(1):91–105.

[16] Syafruddin Side, Irwan T, Usman Mulbar, WahidahSanusi. SEIR model simulation for Hepatitis B, 1885. AIP Conference Proceedings; 2017. p. 20185.

[17] Syafruddin S. A susceptible-infected-recovered model and simulation for transmission of tuberculosis. AdvSciLettVol 2015;21(2):137–9.

[18] Syafruddin S, Noorani MSM. Lyapunov function of SIR and SEIR model for transmission of dengue fever disease. Int J Simul Process Model (IJSPM) 2013;8:177–84 (2/3).

[19] Syafruddin S, Noorani MSM. A SIR model for spread of dengue fever disease (simulation for south sulawesi Indonesia and selangor Malaysia). World Journal Modeling Simulation 2013;9(2):96–105.

[20] Yang C, Wang J. A mathematical model for the novel coronavirus epidemic in Wuhan, China. Math BiosciEng 2020;17(3):2708–24.

[21] Anonim, 2020. SituasiKasus Indonesia. https://covid19.kemkes.go.id/. [diakses 5 April 2020]

[22] Thieme HR. Persistence under relaxed point-dissipativity (with application to an endemic model). SIAM J Math Anal 1993;24:407–35.

[23] Gao D, Ruan S. An SIS patch model with variable transmission coefficients. Math Biosci 2011;232:110–15.

[24] Anonim, 2020. ProyeksiJumlahPenduduk Indonesia 2020. https: //databoks.katadata.co.id/datapublish/2020/01/02/inilah-proyeksi-jumlahpenduduk-indonesia-2020. [diakses 29 maret 2020]

[25] Anonim, 2020. Daftar Negara MenurutPertambahanJumlahPenduduk. https:// id.wikipedia.org/wiki/Daftar_negara_menurut_pertambahan_jumlah_penduduk [diakses 29 maret 2020]