

A New Approach for Solving the Fixed Charge Transportation Problems with Interval Parameters

G. SUDHA¹, G.Ramesh², K. GANESAN³

^{1, 2,3} Department of Mathematics, SRM Institute of science and Technology,

Kattankulathur, Tamil Nadu, India.

E-Mail: sudhag@srmist.edu.in, rameshapg@gmail.com ganesank@srmist.edu.in

Abstract— The fixed charge shipping difficulty is formulated in the present paper under vagueness, for the most part when parameters are given in interval forms. Both the cost and restriction parameters are considered to be obtained in interval numbers in this situation. The suggested solution's initial feasible simple solution is very close to the optimal solution. To express the efficiency of the planned algorithm, a mathematical illustration is specified.

Keywords— Interval Numbers, Generalized interval arithmetic, Interval arithmetic, Interval initial feasible solution, Fixed charge transportation.

1.INTRODUCTION

The shipping difficulty (TP) is a classic operations research problem in which a homogeneous commodity must be transported by one mode of conveyance from m sources to n destinations. Typically, we must consider the mode of transportation (for example, goods train, freight flights, and trucks), the types of products, and other factors. A TP is applied to a fixed charge transportation issue in such a situation (FCTP). FCTP has developed a variety of extensions and orientations to address the needs of the practise world. When an activity has two types of expenses, a fixed price that must be charged before the commotion can begin and a changeable price that is proportional to the activity's volume, the fixed-charge issue occurs. The fixed-charge shipping difficulty (FCTP) is a variation of the shipping difficulty in which each route that can be opened is correlated with a fixed charge in calculation to the changeable shipping cost relative to the quantity of commodities elated. The aim is to find the most cost-effective delivery strategy. Several algorithms for solving this problem have been suggested in the literature, but the accurate ones are rarely valuable when the difficulty has more than one dimension.

There are many techniques for solving these types of problems in the literature. Hirsch et al [8] proposed The Fixed Charge Problem in Notes on Linear Programming. Balinski [5] has created a transportation dilemma with a fixed cost. Ishibuchi et al. [9] proposed a method for optimising the interval objective function using multi-objective programming. Adlakha et al. [1] have proposed a solution. A straightforward heuristic for resolving small fixed-fee transportation problems. Ganesan et.al [6] developed On Arithmetic Operations of period information. Adlakha et.al [3] studied A Heuristic algorithms for the fixed-charge shipping difficulty. Midya et al. suggested [12]. A rough programming solution to a multi-objective fixed-fee transportation problem. Ganesan [7] have developed On Some Properties of period Matrices. Adlakha et.al [2] discussed A branching method for the fixed charge transportation problem. Ramesh et.al [13] have developed an Interval Linear Programming with generalized interval arithmetic. Safi et.al [15] formulated A Solving fixed charge shipping difficulty with interval parameters. Adlakha et.al [4] have studied On rough calculation of the fixed charge shipping problem. Altassan et.al [3] have proposed A Heuristic Approach for Solving the Fixed Charge shipping troubles. Sudipta Midya et al. [16] used rough programming to formulate a multi-objective fixed-charge

transportation query. In this document, we suggest an better description of Interval Vogel's Approximation Method for solving the problem of fixed charge interval transportation without converting the crisp equivalent query using generalised arithmetic.

The following is the structure of this article: In this section, recall the basic concepts that are related. The basic meanings are covered in Section 2. The fixed charge interval transportation issue and its implications are discussed in Section 3. Under generalised interval calculation, Section 4 presents the Improved Version of Interval Vogel's Approximation process. A numerical illustration is given in Section 5. This paper comes to an end in Section 6.

2. PRELIMINARIES

The purpose of this segment is to provide some observations, ideas and results which are useful in our further consideration .

2.1. Interval numbers

Let $\tilde{a} = [a_1, a_2] = \{x \in \mathbb{R} : a_1 \leq x \leq a_2 \text{ and } a_1, a_2 \in \mathbb{R}\}$ be an interval on the real line $\mathbb{R}$. If $\tilde{a} = a_1 = a_2 = a$, then $\tilde{a} = [a, a] = a$ is a real number (or a degenerate interval). We shall make use of the terms interval and interval number interchangeably. The mid-point and width (or half-width) of an interval number $\tilde{a} = [a_1, a_2]$ are defined as $m(\tilde{a}) = \left(\frac{a_1 + a_2}{2}\right)$ and $w(\tilde{a}) = \left(\frac{a_2 - a_1}{2}\right)$. The interval number $\tilde{a}$ can also be expressed in terms of its midpoint and width as $\tilde{a} = [a_1, a_2] = \langle m(\tilde{a}), w(\tilde{a}) \rangle$. We use $\mathbb{IR}$ to denote the set of all interval numbers defined on the real line $\mathbb{R}$.

2.2. Ranking of Interval Numbers

Sengupta and Pal [14] suggested a easy and powerful index to compare any two intervals on $\mathbb{IR}$ through the satisfaction of decision-makers .

Definition 2.2.1. Let $\preceq$ be an extended order relation between the interval numbers $\tilde{a} = [a_1, a_2]$, $\tilde{b} = [b_1, b_2]$ in $\mathbb{IR}$, then for $m(\tilde{a}) < m(\tilde{b})$, we construct a premise $(\tilde{a} \circ \tilde{b})$ which implies that $\tilde{a}$ is inferior to $\tilde{b}$ (or $\tilde{b}$ is superior to $\tilde{a}$).

An *acceptability function* $A_{\preceq} : \mathbb{IR} \times \mathbb{IR} \rightarrow [0, \infty)$ is defined as:

$$A_{\preceq}(\tilde{a}, \tilde{b}) = A(\tilde{a} \preceq \tilde{b}) = \frac{m(\tilde{b}) - m(\tilde{a})}{w(\tilde{b}) + w(\tilde{a})}, \text{ where } w(\tilde{b}) + w(\tilde{a}) \neq 0.$$

$A_{\preceq}$ may be interpreted as the grade of acceptability of the first interval number to be inferior to the second interval number. For any two interval numbers $\tilde{a}$ and $\tilde{b}$ in $\mathbb{IR}$ either $A(\tilde{a} \preceq \tilde{b}) \geq 0$ (or) $A(\tilde{b} \succeq \tilde{a}) \geq 0$ (or) $A(\tilde{a} \preceq \tilde{b}) = 0$ (or) $A(\tilde{b} \succeq \tilde{a}) = 0$ (or) $A(\tilde{a} \preceq \tilde{b}) + A(\tilde{b} \succeq \tilde{a}) = 0$. If $A(\tilde{a} \preceq \tilde{b}) = 0$ and $A(\tilde{b} \succeq \tilde{a}) = 0$, then we say that the interval numbers $\tilde{a}$ and $\tilde{b}$ are equivalent (non-inferior to each other) and we denote it by $\tilde{a} \approx \tilde{b}$. Also if $A(\tilde{a} \preceq \tilde{b}) \geq 0$, then $\tilde{a} \preceq \tilde{b}$ and if $A(\tilde{b} \succeq \tilde{a}) \geq 0$, then $\tilde{b} \succeq \tilde{a}$.

2.3. A New Interval Arithmetic

Ming Ma et al. [11] suggested a new fuzzy arithmetic focused on the index of locations and the index function of fuzziness. For the ordinary arithmetic the position index number is taken, while in the lattice L the fuzziness index functions are assumed to obey the lattice law which is the least upper bound and the greatest lower bound. That is for $a, b \in L$ we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For any two intervals $\tilde{a} = [a_1, a_2]$, $\tilde{b} = [b_1, b_2] \in \mathbb{R}$ and for $*$ $\in \{+, -, \cdot, \div\}$, the arithmetic operations on $\tilde{a}$ and $\tilde{b}$ are defined as:

$$\tilde{a} * \tilde{b} = [a_1, a_2] * [b_1, b_2] = \langle m(\tilde{a}), w(\tilde{a}) \rangle * \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) * m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\} \rangle.$$

In particular here we should use following interval arithmetic operations like addition, subtraction, multiplication and division .

3. MAIN RESULTS

3.1. Mathematical formulation of fixed charge transportation problem

The mathematical model of the FCTP can be represented as follows:

$$\begin{aligned} \text{Minimize } \tilde{Z} &\approx \sum_{i=1}^m \sum_{j=1}^n (\tilde{c}_{ij} \tilde{x}_{ij} + \tilde{f}_{ij} \tilde{y}_{ij}) \\ \text{subject to } \sum_{i=1}^m \tilde{x}_{ij} &\approx \tilde{b}_j, \quad j = 1, 2, 3, \dots, n \\ \sum_{j=1}^n \tilde{x}_{ij} &\approx \tilde{a}_i, \quad i = 1, 2, 3, \dots, m \\ \sum_{i=1}^m \tilde{a}_i &\approx \sum_{j=1}^n \tilde{b}_j, \quad \text{where } i = 1, 2, 3, \dots, m; \quad j = 1, 2, 3, \dots, n \\ \text{and } \tilde{x}_{ij} &\geq \tilde{0} \quad \text{for all } i \text{ and } j, \quad \tilde{y}_{ij} = \begin{cases} 1 & \text{if } \tilde{x}_{ij} > \tilde{0} \\ 0 & \text{if } \tilde{x}_{ij} \leq \tilde{0} \end{cases} \end{aligned} \quad (4.1)$$

Where $\tilde{x}_{ij}$ is the unknown quantity to be transported through the route (i, j) .

The transportation model is a special case of the linear programming problem. It deals with transporting certain product from m sources to n destinations. The sources are production facilities with respective capacities $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \dots, \tilde{a}_m$. and the destinations are warehouses with required levels of demands $\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \dots, \tilde{b}_n$. The penalty for transporting one unit of the given product from the source i to the destination j is $\tilde{c}_{ij}$. In the (FCTP) an additional fixed cost $\tilde{f}_{ij}$ is assumed for opening the route (i, j) and the problem is to determine the amounts to be transported from all sources to all destinations such that the total transportation cost is minimized while satisfying both the supply limits and the demand requirements .

Definition 3.1. An interval feasible solution is defined as a set of non-negative assignments that satisfy (in the context corresponding) the row and the column boundaries .

Definition 3.2. An interval solution to a problem with m sources and n destinations in the transport interval is said to be a basic interval solution if the number of positive allocations is $(m+n-1)$. When the number allocations in a basic interval solution are less than $(m+n-1)$, then it is called the degenerate basic feasible solution .

Definition 3.3. An interval feasible solution is said to be optimal if it minimizes the total cost of transportation .

4. Improved Version of Interval Vogel's Approximation Method

This algorithm provides a more accurate approximation of the IFCTP than the one presented in (Balinski, 1961). The proposed algorithm is an enhanced version of the Vogel Approximation Method (VAM),

which produces a rough solution to the conventional transportation problem. The basic idea behind the algorithm is to modify the cost matrix of the Relaxed Transportation Problem (RTP) after each allocation (iteration) based on supply and demand changes. The proposed algorithm is shown in the steps below:

Step 1: Balance the given interval fixed charge transportation problem if either (total supply > total demand) or (total supply < total demand)

Step 2: Express the all interval parameters supply, demand, variable cost, and fixed charge in terms of midpoint and half width.

That is in the form of $\tilde{a} = [a_1, a_2] = \langle m(\tilde{a}), w(\tilde{a}) \rangle$.

Step 3: Make the Balinski RTP by relaxing the integer form on $\tilde{y}_{ij}$ ($\tilde{y}_{ij} = \tilde{f}_{ij} / \tilde{m}_{ij}$) and the unit shipping price $\tilde{c}_{ij}$ can be represented by $\tilde{c}_{ij} = \tilde{c}_{ij} + \tilde{f}_{ij} / \tilde{m}_{ij}$

$$\text{Where } \tilde{m}_{ij} = \begin{cases} \min(\tilde{a}_i, \tilde{b}_j) & \text{if } \tilde{a}_i, \tilde{b}_j \succ \tilde{0} \\ \tilde{a}_i & \tilde{b}_j \approx \tilde{0} \\ \tilde{b}_j & \tilde{a}_i \approx \tilde{0} \end{cases}$$

Step 4: Determine a penalty for each row (column) by subtracting the smallest unit cost element from the next smallest unit cost element in the same row (column) (column).

Step 5: Determine which row or column has the highest penalty. Breaking ties on the spur of the moment. As a large amount as feasible, go to the unit in the identified row or column by means of the lowest charge. Make the necessary adjustments to deliver and order, and then cross out the happy row or column. Only one of the two is crossed out if a row and a column are both fulfilled at the same time.

Step 6: If only one row (column) with positive supply (demand) remains uncrossed out, distribute this supply (demand) to the remaining uncrossed out cells with their unmet demands (supply) of the uncrossed out column (row) and stop. If not, proceed to phase 7.

Step 7: Recalculate the previous cost matrix for the uncrossed out row (column) found in step 5. Continue to stage 5.

Note: Test out the optimality of the initial basic possible result obtained using the period version of the MODI process.

5. NUMERICAL EXAMPLE

Consider a corporation that has three factories in cities S1, S2, and S3 all manufacturing the same type of product. D1, D2, D3, and D4 are the four other cities that receive this commodity as customers. The product's unique manufacturing conditions resulted in confusion and ambiguity in the supply and demand values in factories and destinations, correspondingly. Previous experiences have shown that these ideals can be communicated in intervals. Another company is in charge of getting the goods from the manufacturers to the customers. Special care is required when shipping through each rout, which necessitates the use of a technician and results in a fixed charge cost for opening each rout. Obviously, the amount of goods transported via a route has no bearing on this. In interval types, variable shipping expenses are also obtainable. The generating corporation finds the most

cost-effective shipping strategy that meets all restrictions while lowering the over all

	D1	D2	D3	D4	Supply
S1	[4,8],[10,30]	[8,12],[19,25]	[9,11],[19,25]	[8,10],[20,30]	[30,33]
S2	[10,18],[16,20]	[10,12],[15,25]	[11,15],[25,55]	[5,7],[38,40]	[27,28]
S3	[7,19],[10,20]	[8,12],[22,30]	[8,14],[30,50]	[13,17],[20,22]	[22,25]
Demand	[20,21]	[19,24]	[23,24]	[20,22]	

transportation cost

TABLE 1. INTERVAL TRANSPORTATION COST AND FIXED - CHARGE

	D1	D2	D3	D4	Supply
S1	<6,2>,<20,10>	<10,2>,<22,3>	<10,1>,<22,3>	<9,1>,<25,5>	<31.5,1.5>
S2	<14,4>,<18,2>	<11,1>,<20,5>	<13,2>,<40,15>	<6,1>,<39,1>	<27.5,0.5>
S3	<13,6>,<15,5>	<10,2>,<26,4>	<11,3>,<40,10>	<15,2>,<21,1>	<23.5,1.5>
S4	<0,0>,<0,0>	<0,0>,<0,0>	<0,0>,<0,0>	<0,0>,<0,0>	<4,1.5>
Demand	<20.5,0.5>	<21.5,2.5>	<23.5,0.5>	<21,1>	

Since $\sum_{i=1}^m \tilde{a}_i \neq \sum_{j=1}^n \tilde{b}_j$, the given interval shipping difficulty is unbalanced.

The provided interval shipping difficulty is balanced by adding a dummy Supply S4 with zero cost and fixed charge.

TABLE 2. INTERVAL TRANSPORTATION COST AND FIXED - CHARGE IN TERMS OF MID POINT AND WIDTH FORM

TABLE 3. BALINSKI'S RTP MATRIX

By applying the planned algorithm, the original essential possible explanation is obtained as

TABLE 4. INITIAL BASIC FEASIBLE SOLUTION

	D1	D2	D3	D4	Supply
S1	<6.9756,10>	<11.0233,3>	<10.9362,3>	<10.1905,5>	<31.5,1.5>
S2	<14.8780,4>	<11.9302,5>	<14.7021,15>	<7.8571,1>	<27.5,0.5>
S3	<13.7317,6>	<11.2093,4>	<12.7021,10>	<16,2>	<23.5,1.5>
S4	<0,0>	<0,0>	<0,0>	<0,0>	<4,1.5>
Demand	<20.5,0.5>	<21.5,2.5>	<23.5,0.5>	<21,1>	

	D1	D2	D3	D4	Supply
S1	<6,2>,<20,10> <20.5,0.5>	<10,2>,<22,3>	<10,1>,<22,3> <11,1.5>	<9,1>,<25,5>	<31.5,1.5>
S2	<14,4>,<18,2>	<11,1>,<20,5> <6.5,1>	<13,2>,<40,15>	<6,1>,<39,1> <21,1>	<27.5,0.5>
S3	<13,6>,<15,5>	<10,2>,<26,4> <11,2.5>	<11,3>,<40,10> <12.5,1.5>	<15,2>,<21,1>	<23.5,1.5>
S4	<0,0>,<0,0>	<0,0>,<0,0> <4,1.5>	<0,0>,<0,0>	<0,0>,<0,0>	<4,1.5>
Demand	<20.5,0.5>	<21.5,2.5>	<23.5,0.5>	<21,1>	

The

interval variant of the MODI process is used to test out the optimality of the current solution, which is found to be optimal since the quantity of optimistic allocations is $(m+n-1)=7$.

Hence the solution of the given interval transportation fixed charge problem is

$$x_{11} = \langle 20.5, 0.5 \rangle, x_{13} = \langle 11, 1.5 \rangle, x_{22} = \langle 6.5, 1 \rangle, x_{24} = \langle 21, 1 \rangle \\ x_{32} = \langle 11, 2.5 \rangle, x_{33} = \langle 12.5, 1.5 \rangle, x_{42} = \langle 4, 1.5 \rangle$$

The total interval variable cost $\sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \tilde{x}_{ij} = \langle 678, 3 \rangle$ and

The total interval fixed cost $\sum_{i=1}^m \sum_{j=1}^n \tilde{f}_{ij} \tilde{y}_{ij} = \langle 167, 10 \rangle$

The interval transportation minimum total cost is $= \sum_{i=1}^m \sum_{j=1}^n (\tilde{c}_{ij} \tilde{x}_{ij} + \tilde{f}_{ij} \tilde{y}_{ij})$
 $= \langle 678, 3 \rangle + \langle 167, 10 \rangle$
 $= \langle 845, 10 \rangle$
 $= [835, 855]$

TABLE 5. COMPARISON OF SOLUTIONS

S.No	Solution by the proposed method	Solution by M. R. Safi and A. Razmjoo[14]
1	[835,855]	[640,1020]

6. CONCLUSIONS

Where the definitions are unclear and ambiguous in nature, the current occupation has been established on Interval fixed charge shipping difficulty (IFCTP). FCTP is widely used in industries, fitness care, farming, and other fields. A comparison of the results obtained from our proposed model has been made. Finally, we announced that the best result was obtained in the final event. Finally, we came to the conclusion that the proposed representation in this paper is highly applicable in the real world (decision making) FCTP where the data is unclear. The filling of this document may be applied to multi-objective fixed-charge shipping troubles in the upcoming. Furthermore, the paper's definition generates a broad spectrum of unknown FCTP.

References

- [1]. Adlakha, V., & Kowalski, K. A simple heuristic for solving small fixed-charge transportation problems. *Omega*, 31(3),(2003), 205-211.
- [2]. Adlakha, V., Kowalski, K., & Lev, B. A branching method for the fixed charge transportation problem. *Omega*, 38(5),(2010), 393-397.
- [3]. Adlakha, V., Kowalski, K., & Vemuganti, R.. Heuristic algorithms for the fixed-charge transportation problem. *Opsearch*, 43(2), (2006), 132-151.
- [4]. Adlakha, V., Kowalski, K., Wang, S., Lev, B., & Shen, W. On approximation of the fixed charge transportation problem. *Omega*, 43, (2014), 64-70.
- [5]. Balinski, M. Fixed cost transportation problems. *Naval Research Logistics Quarterly*, 8, (1961), 41–54.
- [6]. K. Ganesan and P. Veeramani, On Arithmetic Operations of Interval Numbers, International Journal of Uncertainty, Fuzziness and Knowledge - Based Systems, 13 (6) (2005), 619 - 631.
- [7]. K. Ganesan, On Some Properties of Interval Matrices, International Journal of Computational and Mathematical Sciences, 1 (2) (2007), 92 - 99.
- [8]. Hirsch, W. M., & Dantzig, G. B. *Notes on Linear Programming, Part XIX: The Fixed Charge Problem*: Rand Corporation. (1954).
- [9]. Ishibuchi, H., Tanaka, H., Multi objective programming in optimization of the interval objective function. *European Journal of Operational Research*, 48 (1990), 219–225.
- [10]. Khalid M. Altassan, Mahmoud M. El-sherbiny, A Heuristic Approach for Solving the Fixed Charge Transportation Problems, International Review of Management and Business Research, 7(2), (2018).
- [11]. Ming Ma, Menahem Friedman and Abraham Kandel. 1999. A new fuzzy arithmetic. *Fuzzy sets and systems*. 108: 83-90.

- [12]. S. Midya and S. K. Roy, Multi-objective fixed-charge transportation problem using rough programming, *Int. J. Oper. Res.* 9(3) (2017).
- [13]. G. Ramesh and K. Ganesan, Interval Linear Programming with generalized interval arithmetic, *International Journal of Scientific & Engineering Research*, 2 (11), November-2011.
- [14]. A. Sengupta and T.K. Pal, Interval-valued transportation problem with multiple penalty factors, *VU Journal of Physical Sciences*, 9, (2003), 71 – 81.
- [15]. M. R. Safi and A. Razmjoo, Solving fixed charge transportation problem with interval parameters, *Appl. Math. Model.* 37 (2013) 8341–8347.
- [16]. Sudipta Midya, Sankar Kumar Roy, Multi-objective fixed-charge transportation problem using rough programming, *International Journal of Operational Research*, 37(3) (2020).