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Bipolar Fuzzy AB- Algebra

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Abstract.
In this paper, we have introduced the concept of a bipolar fuzzy AB –ideal of AB algebra and also we proved some relevant characteristics and theories. We also studied the bipolar fuzzy relations on AB algebras , notion of bipolar fuzzy AB-ideals in AB-algebras and some related properties are investigated. Also we introduce the homomorphic image and pre-images of bipolar fuzzy AB-ideals and present some of its properties.

Keywords: Algebra, AB-ideal, fuzzy AB-ideal , bipolar fuzzy set , bipolar fuzzy AB-ideals, Cartesian product of bipolar fuzzy AB ideals, Homomorphism

INTRODUCTION:

The concept of fuzzy set was introduced by Zadeh[1] in 1965. In fuzzy sets the membership degree of elements range over the interval [0,1]. The author Zhang[2] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. In case of Bipolar-valued fuzzy sets membership degree range is enlarged from the interval [0, 1] to [–1, 1]. K.Iseki[3] introduced two classes of abstract algebras:BCK-algebra and BCI-algebras. It is known that the class of BCK-algebras is proper subclass of the class of BCI- algebras. In 2017[4] introduced a notion of AB-ideals in AB–algebras. In this paper we define a new algebraic structure of bipolar fuzzy AB-ideal in AB-algebras and discuss the properties of bipolar fuzzy AB ideals of AB-algebra under homomorphism.

2. Preliminaries

In this section, we site the fundamental definitions that will be used in the sequel.

2.1 Definition

An AB-algebra is a nonempty set X with a constant 0 and a binary operation * satisfying the following axioms

i. $0 * x = 0$

ii. $x * 0 = x$

iii. $(x * y)* (z * y) * (x* z) = 0$ for all $x ,y, z \in X$,

In X, we can define a binary relation $\leq$ by $x \leq y$ if and only if $x * y = 0$. Then $(x, \leq)$ is a partially ordered set.

2.2 Example Let X = { 0,1,2,3 } be a set with the following table

*	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

(X ,* ,0) is an AB-algebra.

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2.3 Definition

In any AB-algebra X , the following axioms holds is a nonempty set X with a constant 0 and a binary operation * satisfying

i. $(x * y) * x = 0$

ii. $x \leq y$ implies $x * z \leq y * z$

iii. $x \leq y$ implies $z * y \leq z * x$, for all $x, y, z \in X$,

Remark: An AB-algebra is satisfies for all $x,y,z \in X$

i. $(x * y) * z = (x * z) * y$

ii. $(x * (x * y)) * y = 0$

2.4 Definition

Let $(X, *, 0)$ be an AB-algebra . A non empty set I of X is called an ideal of X if it satisfies

i. $0 \in I$

ii. $x * (y * z) \in I$ and $y \in I$ imply $x * z \in I$, for all $x, y \in X$.

2.5 Definition

A fuzzy set μ in an AB-algebra X is called a fuzzy AB-subalgebra of X if
$\mu(x*y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$

2.6 Definition

A fuzzy set μ in an AB-algebra X is called a fuzzy AB ideal of X if

i. $\mu(0) \geq \mu(x)$

ii. $\mu(x*z) \geq \min\{\mu(x*(y*z)), \mu(y)\}$ for all $x, y, z \in X$

2.7 Definition

Let X be a non-empty set. A bipolar-valued fuzzy set or bipolar fuzzy set μ in X is an object having the form $\mu = \{\langle x, \mu^+(x), \mu^-(x) \rangle : x \in X\}$,where $\mu^+ : X \to [0,1]$ and $\mu^- : X \to [-1,0]$ are mappings. For the sake of simplicity, we shall use the symbol $\mu = (\mu^+, \mu^-)$ for the bipolar-valued fuzzy set $\mu = \{\langle x, \mu^+(x), \mu^-(x)\rangle : x \in X\}$.

2.8 Definition

A bipolar fuzzy set μ of an AB-algebra X is a bipolar fuzzy subalgebra of X if for all $x, y \in X$,

i. $\mu^+(x*y) \geq \min\{\mu^+(x), \mu^+(y)\}$

iii. $\mu^-(x*y) \leq \max\{\mu^-(x), \mu^-(y)\}$.

2.9 Definition

A bipolar fuzzy set μ of an AB-algebra X is a bipolar fuzzy AB ideal of X if for all $x, y \in X$,

i. $\mu^+(0) \geq \mu^+(x)$, $\mu^-(0) \leq \mu^-(x)$,

ii. $\mu^+(x * z) \geq \min\{\mu^+(x * (y * z)), \mu^+(y)\}$,

iii. $\mu^-(x * z) \leq \max\{\mu^-(x * (y * z)), \mu^-(y)\}$.

2.10 Definition

Let $\mu = (\mu^+, \mu^-)$, $\varphi = (\varphi^+, \varphi^-)$ are bipolar fuzzy subsets of an AB -algebra X Then a product $\mu \times \varphi = ((\mu \times \varphi)^+, (\mu \times \varphi)^-)$ where $(\mu \times \varphi)^+ : X \times X \to [0,1]$ and $(\mu \times \varphi)^- : X \times X \to [-1,0]$ are mappings defined by

i. $(\mu \times \varphi)^+(x,y) = \min\{(\mu^+(x), \varphi^+(y)) /$ for all $x, y \in X\}$.

ii. $(\mu \times \varphi)^-(x,y) = \max\{(\mu^-(x), \varphi^-(y)) /$ for all $x, y \in X\}$.

2.11 Theorem

If $\mu = (\mu^+, \mu^-)$ and $\varphi = (\varphi^+, \varphi^-)$ are two bipolar fuzzy AB-ideals of an AB-algebra X, then $(\mu \cap \varphi)$ is a bipolar fuzzy AB-ideal of an AB-algebra X.

Proof: Let $\mu = (\mu^+, \mu^-)$ and $\varphi = (\varphi^+, \varphi^-)$ are two bipolar fuzzy AB-ideals of an AB-algebra X. For any $x, y \in X$

i. $(\mu \cap \varphi)^+(0) = (\mu \cap \varphi)^+(0 * x)$
$= \min\{\mu^+(0 * x), \varphi^+(0 * x)\}$

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$\geq \min\{\min\{\mu^{+}(0), \mu^{+}(x)\}, \min\{\mu^{+}(0), \varphi^{+}(x)\}\}$
$= \min\{\mu^{+}(x), \varphi^{+}(x)\}$
$= (\mu \cap \varphi)^{+}(x)$

$(\mu \cap \varphi)^{+}(0) \geq (\mu \cap \varphi)^{+}(x)$ and

$(\mu \cap \varphi)^{-}(0,0) = (\mu \cap \varphi)^{-}(0 * x)$
$= \max\{\mu^{-}(0 * x), \varphi^{-}(0 * x)\}$
$\leq \max\{\max\{\mu^{-}(0), \mu^{-}(x)\}, \max\{\varphi^{-}(0), \varphi^{-}(x)\}\}$
$= \max\{\mu^{-}(x), \varphi^{-}(x)\}$
$= (\mu \cap \varphi)^{-}(x, x)$

$(\mu \cap \varphi)^{-}(0) \leq (\mu \cap \varphi)^{-}(x)$

ii. $(\mu \cap \varphi)^{+}(x * z) = \min\{\mu^{+}(x * z), \varphi^{+}(x * z)\}$
$\geq \min\{\min\{\mu^{+}(x * (y * z)), \mu^{+}(y)\}, \min\{\varphi^{+}(x * (y * z)), \varphi^{+}(y)\}\}$
$= \min\{\min\{\mu^{+}(x * (y * z)), \varphi^{+}(x * (y * z))\}, \min\{\mu^{+}(y), \varphi^{+}(y)\}\}$
$= \min\{(\mu \cap \varphi)^{+}(x * (y * z)), (\mu \cap \varphi)^{+}(y)\}$

$(\mu \cap \varphi)^{+}(x * z) \geq \min\{(\mu \cap \varphi)^{+}(x * (y * z)), (\mu \cap \varphi)^{+}(y)\}$

iii. $(\mu \cap \varphi)^{-}(x * z) = \max\{\mu^{-}(x * z), \varphi^{-}(x * z)\}$
$\leq \max\{\max\{\mu^{-}(x * (y * z)), \mu^{-}(y)\}, \max\{\varphi^{-}(x * (y * z)), \varphi^{-}(y)\}\}$
$= \max\{\max\{\mu^{-}(x * (y * z)), \varphi^{-}(x * (y * z))\}, \max\{\mu^{-}(y), \varphi^{-}(y)\}\}$
$= \max\{(\mu \cap \varphi)^{-}(x * (y * z)), (\mu \cap \varphi)^{-}(y)\}$

$(\mu \cap \varphi)^{-}(x * z) \leq \max\{(\mu \cap \varphi)^{-}(x * (y * z)), (\mu \cap \varphi)^{-}(y)\}$

2.12 Theorem

If $\mu = (\mu^{+}, \mu^{-})$ and $\varphi = (\varphi^{+}, \varphi^{-})$ are two bipolar fuzzy AB-ideals of an AB-algebra X, then $(\mu \times \varphi)$ is a bipolar fuzzy AB-ideal of an AB-algebra X×X

Proof: Let $\mu = (\mu^{+}, \mu^{-})$ and $\varphi = (\varphi^{+}, \varphi^{-})$ are two bipolar fuzzy AB-ideals of an AB-algebra X×X. For any $(x,y) \in X \times X$

i. $(\mu \times \varphi)^{+}(0,0) = (\mu \times \varphi)^{+}((0,0) * (x_1,x_2))$
$= (\mu \times \varphi)^{+}((0 * x_1), (0 * x_2))$
$= \min\{\mu^{+}(0 * x_1), \varphi^{+}(0 * x_2)\}$
$\geq \min\{\min\{\mu^{+}(0), \mu^{+}(x_1)\}, \min\{\mu^{+}(0), \varphi^{+}(x_2)\}\}$
$= \min\{\mu^{+}(x_1), \varphi^{+}(x_2)\}$
$= (\mu \times \varphi)^{+}(x_1, x_2)$

$(\mu \times \varphi)^{+}(0,0) \geq (\mu \times \varphi)^{+}(x_1, x_2)$ and

$(\mu \times \varphi)^{-}(0,0) = (\mu \times \varphi)^{-}((0,0) * (x_1,x_2))$
$= (\mu \times \varphi)^{-}((0 * x_1), (0 * x_2))$
$= \max\{\mu^{-}(0 * x_1), \varphi^{-}(0 * x_2)\}$
$\leq \max\{\max\{\mu^{-}(0), \mu^{-}(x_1)\}, \max\{\varphi^{-}(0), \varphi^{-}(x_2)\}\}$
$= \max\{\mu^{-}(x_1), \varphi^{-}(x_2)\}$
$= (\mu \times \varphi)^{-}(x_1, x_2)$

$(\mu \times \varphi)^{-}(0,0) \leq (\mu \times \varphi)^{-}(x_1, x_2)$

ii. $\min\{(\mu \times \varphi)^{+}((x_1,x_2) * ((y_1,y_2) * (z_1,z_2))), (\mu \times \varphi)^{+}(y_1,y_2)\}$
$= \min\{(\mu \times \varphi)^{+}((x_1,x_2) * (y_1 * z_1, y_2 * z_2)), (\mu \times \varphi)^{+}(y_1,y_2)\}$
$= \min\{(\mu \times \varphi)^{+}(x_1 * (y_1 * z_1), x_2 * (y_2 * z_2)), (\mu \times \varphi)^{+}(y_1,y_2)\}$
$= \min\{\min\{\mu^{+}(x_1 * (y_1 * z_1)), \varphi^{+}(x_2 * (y_2 * z_2))\}, \min\{\mu^{+}(y_1), \varphi^{+}(y_2)\}\}$
$= \min\{\min\{\mu^{+}(x_1 * (y_1 * z_1)), \mu^{+}(y_1)\}, \min\{\varphi^{+}(x_2 * (y_2 * z_2)), \varphi^{+}(y_2)\}\}$
$\leq \min\{\mu^{+}(x_1 * z_1), \varphi^{+}(x_2 * z_2)\}$
$= (\mu \times \varphi)^{+}(x_1 * z_1, x_2 * z_2)$
$= (\mu \times \varphi)^{+}((x_1, x_2) * (z_1, z_2))$

$(\mu \times \varphi)^{+}((x_1, x_2) * (z_1,z_2)) \geq \min\{(\mu \times \varphi)^{+}((x_1,x_2) * ((y_1,y_2) * (z_1,z_2))), (\mu \times \varphi)^{+}(y_1,y_2)\}$

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iii. $\max\{(\mu\times\varphi)^-((x_1,x_2)*((y_1,y_2)*(z_1,z_2))),(\mu\times\varphi)^-(y_1,y_2)\}$

$= \max\{(\mu\times\varphi)^-((x_1,x_2)*(y_1*z_1, y_2*z_2)),(\mu\times\varphi)^-(y_1,y_2)\}$

$= \max\{(\mu\times\varphi)^-(x_1*(y_1*z_1), x_2*(y_2*z_2)),(\mu\times\varphi)^-(y_1,y_2)\}$

$= \max\{\max\{\mu^-(x_1*(y_1*z_1)),\varphi^-(x_2*(y_2*z_2))\}, \max\{\mu^-(y_1),\varphi^-(y_2)\}\}$

$= \max\{\max\{\mu^-(x_1*(y_1*z_1)),\mu^-(y_1)\}, \max\{\varphi^-(x_2*(y_2*z_2)),\varphi^-(y_2)\}\}$

$\geq \max\{\mu^-(x_1*z_1), \varphi^-(x_2*z_2)\}$

$= (\mu\times\varphi)^-(x_1*z_1, x_2*z_2)$

$= (\mu\times\varphi)^-((x_1,x_2)*(z_1,z_2))$

$(\mu\times\varphi)^-((x_1,x_2)*(z_1,z_2)) \geq \min\{(\mu\times\varphi)^-(((x_1,x_2)*(y_1,y_2))*(z_1,z_2)),(\mu\times\varphi)^-(y_1,y_2)\}$

2.13 Theorem

Let $\mu=(\mu^+,\mu^-)$ and $\varphi=(\varphi^+,\varphi^-)$ be two bipolar fuzzy sets in AB-algebra X such that $\mu\times\varphi$ is a bipolar fuzzy AB-ideal of AB-algebra X×X then

(i) Either $\mu^+(0)\geq\mu^+(x)$, $\mu^-(0)\leq\mu^-(x)$ or
$\varphi^+(0)\geq\varphi^+(x)$, $\varphi^-(0)\leq\varphi^-(x)$ for all $x\in X$

(ii) If $\mu^+(0)\geq\mu^+(x)$, $\mu^-(0)\leq\mu^-(x)$ for all $x\in X$, then
either $\varphi^+(0)\geq\mu^+(x)$ and $\varphi^-(0)\leq\mu^-(x)$ or $\varphi^+(0)\geq\varphi^+(x)$, $\varphi^-(0)\leq\varphi^-(x)$

(iii) If $\varphi^+(0)\geq\varphi^+(x)$, $\varphi^-(0)\leq\varphi^-(x)$ for all $x\in X$,
then either $\mu^+(0)\geq\mu^+(x)$, $\mu^-(0)\leq\mu^-(x)$ or $\mu^+(0)\geq\varphi^+(x)$, $\mu^-(0)\leq\varphi^-(x)$

Proof: Let $\mu=(\mu^+,\mu^-)$ and $\varphi=(\varphi^+,\varphi^-)$ be two bipolar fuzzy sets of an AB-algebra X×X.

i. Assume that $\mu(x)>\mu(0) \Rightarrow \mu^+(x)>\mu^+(0)$, $\mu^-(x)<\mu^-(0)$ or
$\varphi(y)>\varphi(0)\Rightarrow\varphi^+(0)>\varphi^+(x)$, $\varphi^-(0)<\varphi^-(x)$, for some $x,y\in X$, Then

$(\mu\times\varphi)^+(x,y) = \min\{\mu^+(x),\varphi^+(y)\}$
$> \min\{\mu^+(0),\varphi^+(0)\}$
$= (\mu\times\varphi)^+(0,0)$ and

$(\mu\times\varphi)^-(x,y) = \max\{\mu^-(x),\varphi^-(y)\}$
$< \max\{\mu^-(0),\varphi^-(0)\}$
$= (\mu\times\varphi)^-(0,0)$

i.e. $(\mu\times\varphi)^+(x,y)>(\mu\times\varphi)^+(0,0)$ and $(\mu\times\varphi)^-(x,y)<(\mu\times\varphi)^-(0,0)$
for all $x,y\in X$,
which is a contradiction. Hence (i) proved.

ii. Assume that $\mu(x)>\mu(0) \Rightarrow \mu^+(x)>\mu^+(0)$, $\mu^-(x)<\mu^-(0)$ or
$\varphi(y)>\varphi(0)\Rightarrow\varphi^+(0)>\varphi^+(x)$, $\varphi^-(0)<\varphi^-(x)$, for some $x,y\in X$, Then

$(\mu\times\varphi)^+(x,y) = \min\{\mu^+(x),\varphi^+(y)\}$
$> \min\{\mu^+(0),\varphi^+(0)\}$
$= (\mu\times\varphi)^+(0,0)$ and

$(\mu\times\varphi)^-(x,y) = \max\{\mu^-(x),\varphi^-(y)\}$
$< \max\{\mu^-(0),\varphi^-(0)\}$
$= (\mu\times\varphi)^-(0,0)$

i.e. $(\mu\times\varphi)^+(x,y)>(\mu\times\varphi)^+(0,0)$ and $(\mu\times\varphi)^-(x,y)<(\mu\times\varphi)^-(0,0)$
for all $x,y\in X$,
which is a contradiction. Hence (ii) proved.

(iii) The proof is similar to proof (ii) .

Remark: Converse of the above theorem is not true in general.

2.14 Definition

A fuzzy subset μ of X is said to have sup property if, for any subset A of X, there exist $a_0\in A$ such that $\mu(a_0)=\max\{\mu(a); a\in A\}$.

2.15 Definition

Let X and Y be any two AB-algebras. Let $\mu=(\mu^+,\mu^-)$ and $\varphi=(\varphi^+,\varphi^-)$ are bipolar fuzzy subsets in X and Y

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respectively. Let $f : X \to Y$ be a mapping , then $f(\mu)$,the image of μ is the bipolar fuzzy subset $f(\mu) = ((f(\mu))^+ , (f(\mu))^-)$ of Y defined by for all $f(x) = y\in Y$,where $x\in X$

$$(f(\mu))^+ (f(x)) = \begin{cases} \max \{\mu^+ (x) : x \in f^{-1}(y) \} , & \text{if } f^{-1}(y) \neq \phi \\ 0 & , \text{ otherwise} \end{cases}$$

and

$$(f(\mu))^- (f(x)) = \begin{cases} \max \{\mu^- (x) : x \in f^{-1}(y) \} , & \text{if } f^{-1}(y) \neq \phi \\ 0 & , \text{ otherwise} \end{cases}$$

also the pre-image $f^{-1}(\sigma_\varphi)$ of σ_φ under f is a bipolar fuzzy subset of X defined by $(f^{-1}(\varphi))^+ (x) = \varphi^+ (f(x))$, $(f^{-1}(\varphi))^- (x) = \varphi^- (f(x))$.

2.16 Theorem : Let $f : X \to Y$ be a homomorphism of an AB-algebra .If μ is a bipolar fuzzy AB-ideal of X with sup property. Then the image $f(\mu)$ is a bipolar fuzzy AB-ideal of Y.

Proof: Let $f : X \to Y$ be a homomorphism .

Let μ be a bipolar fuzzy AB-ideal For any $x,y\in X$

i. $(f(\mu))^+ f(0) = \mu^+ (0) \geq \mu^+ (x) = (f(\mu))^+ f(x)$ and
$(f(\mu))^- f(0) = \mu^- (0) \leq \mu^- (x) = (f(\mu))^- f(x)$

ii.
$$\begin{aligned}(f(\mu))^+ (f(x)*f(z)) &= (f(\mu))^+ f(x*z)\\ &= \mu^+ (x*z)\\ &\geq \min \{ \mu^+ (x*(y*z)) , \mu^+ (y)\}\\ &= \min \{ (f(\mu))^+ f(x*(y*z)) , (f(\mu))^+ f(y)\}\\ &= \min \{ (f(\mu))^+ (f(x) * f(y *z)) , (f(\mu))^+ f(y)\}\\ &\geq \min \{ (f(\mu))^+ (f(x)* (f(y) * f(z)) , (f(\mu))^+ f(y)\}\end{aligned}$$
$$(f(\mu))^+ (f(x)*f(z)) \geq \min \{ (f(\mu))^+ (f(x)* (f(y) * f(z)) , (f(\mu))^+ f(y)\}$$

and
$$\begin{aligned}(f(\mu))^- (f(x)*f(z)) &= (f(\mu))^- f(x*z)\\ &= \mu^- (x*z)\\ &\geq \max \{ \mu^- (x*(y*z)) , \mu^- (y)\}\\ &= \max \{ (f(\mu))^- f(x*(y*z)) , (f(\mu))^- f(y)\}\\ &= \max \{ (f(\mu))^- (f(x) * f(y *z)) , (f(\mu))^- f(y)\}\\ &= \max \{ (f(\mu))^- (f(x)* (f(y) * f(z)) , (f(\mu))^- f(y)\}\end{aligned}$$
$$(f(\mu))^- (f(x)*f(z)) \geq \max \{ (f(\mu))^- (f(x)* (f(y) * f(z)) , (f(\mu))^- f(y)\}$$

Hence, $f(\mu)$ is a bipolar fuzzy AB-ideal in Y.

2.17 Theorem : Let $f : X \to Y$ be a homomorphism of an AB-algebra .If φ is a bipolar fuzzy AB-ideal of Y, then $f^{-1}(\varphi)$ is a bipolar fuzzy AB-ideal of X.

Proof: For any $x\in X$

(i). $(f^{-1}(\varphi))^+ (x) = \varphi^+ (f(x)) \leq \varphi^+ (f(0)) = (f^{-1}(\varphi))^+ (0)$
$(f^{-1}(\varphi))^- (x) = \varphi^- (f(x)) \geq \varphi^- (f(0)) = (f^{-1}(\varphi))^{+-} (0)$
$(f^{-1}(\varphi))^+ (x) = \varphi^+ (f(x))$, $(f^{-1}(\varphi))^- (x) = \varphi^- (f(x))$.

(ii)
$$\begin{aligned}(f^{-1}(\varphi))^+ (x*z) &= \varphi^+ (f(x*z))\\ &= \varphi^+ (f(x) * f(z))\\ &\geq \min \{ \varphi^+ (f(x) * (f(y)*f(z)), \varphi^+ (f(y))\}\\ &= \min \{ \varphi^+ (f(x)* f(y * z)) , \varphi^+ (f(y))\}\\ &= \min \{ \varphi^+ (f(x * (y *z)) , \varphi^+ (f(y))\}\\ &= \min \{ (f^{-1}(\varphi))^+ (x * (y * z)) , (f^{-1}(\varphi))^+ (y)\}\end{aligned}$$
$(f^{-1}(\varphi))^+ (x*z) \geq \min \{ (f^{-1}(\varphi))^+ (x * (y * z)) , (f^{-1}(\varphi))^+ (y)\}$ and
$$\begin{aligned}(f^{-1}(\varphi))^- (x*z) &= \varphi^- (f(x*z))\\ &= \varphi^- (f(x) * f(z))\\ &\leq \max \{ \varphi^- (f(x) * (f(y)*f(z)), \varphi^- (f(y))\}\\ &= \max \{ \varphi^- (f(x)* f(y * z)) , \varphi^- (f(y))\}\\ &= \max \{ \varphi^- (f(x * (y *z)) , \varphi^- (f(y))\}\\ &= \max \{ (f^{-1}(\varphi))^- (x * (y * z)) , (f^{-1}(\varphi))^- (y)\}\end{aligned}$$

$(f^{-1}(\varphi))^{-}(x*z) \leq \max\{(f^{-1}(\varphi))^{-}(x*(y*z)), (f^{-1}(\varphi))^{-}(y)\}$

$f^{-1}(B)$ is a bipolar fuzzy AB- ideal in X.

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The author have declared no conflicts of interest

REFERENCES

[1] L.A. Zadeh, Fuzzy Sets, Information and Control, 8 (1965), pp 338-365.
[2] W.R. Zhang , Bipolar fuzzy sets, Proc. of FUZZ-IEEE, pp 835-840, 1998.
[3] K.Iseki,S.Tanaka, A.Rosenfeld, An introduction to theory of BCK- algebras , Math.Japonica, 23 (1978) , pp.1-26.
[4] Areej Tawfeeq Hameed Baneen Najah Abbas, AB –Ideals of AB –Algebras, Applied Mathematical Sciences, Volume 11, 2017, no.35, pp 1715-1723.