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On the Estimation of the Generalized Solution of a Certain Problem of Parabolic type with a Boundary Condition Containing by Time from the Required Function.

[1]**Mamatov AlisherZulunovich**– Doctor of Technical Sciences, Professor of the Department "Mathematics and Computer Science", the Tashkent Institute of Textile and Light Industry, (100100, Tashkent, Shokhjakhon street, 5), E-mail: maz54@mail.ru

[2]**Rakhmanov JamshidTurdaliyevich(Gulistan State University, senior teacher)**

[3]**Turdibaev DilshodKhamidovich(Associate Professor, PhDof Gulistan State University)**

[4]**Bomuratov SherbekNajmiddinovich(Gulistan State University, teacher)**

[5]**Dushaboyev OlimdjanNazarovich(Associate Professor, PhDof Gulistan State University)**

[6]**Dosanov Murtozakul Saidazimovich (Gulistan State University, teacher)**

[7]**GoyibnazarovHabibulloRahmatillayevich(Gulistan State University, senior teacher)**

ABSTRACT. The article considers a parabolic-type boundary value problem with a divergent principal part, when the boundary condition contains the time derivative of the required function.

$$\begin{cases} u_t - \dfrac{d}{dx_i} a_i(x,t,u,\nabla u) + a(x,t,u,\nabla u) = 0 \quad , \\ a_0 u_t + a_i(x,t,u,\nabla u)\cos(\nu, x_i) = g(x,t,u), \quad (x,t) \in S_t \ , \\ u(x,0) = u_0(x) \ , \quad x \in \Omega \end{cases}$$

In such non-classical problems with boundary conditions, in which the electromagnetic wave is located, or if the surface of the body is the same, the temperature is the same at all its points, it is washed off with a good mixed liquid.Such problems have been little studied, therefore, the study of problems of parabolic type, when the boundary condition contains the time derivative of the desired function, is relevant.In this paper, a generalized solution of the problem under consideration is defined in the space$\widetilde{H^{1,1}}(Q_T)$.The aim of the study is to obtain conditions under which the estimate of the error of the approximate solution in the norm $H^1(\Omega)$has order $O(h^{k-1})$. The article first considers an auxiliary problem of elliptic type.Under certain conditions, the conditions are involved in the equation and the boundary condition, estimates are obtained for the solution of the auxiliary elliptic problem under consideration.Further, under the condition that the problem is elliptic, inequalities are obtained for the difference of the generalized solution of the considered parabolic problem with a divergent principal part, when the boundary condition contains

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the time derivative of the desired function and the solution of the auxiliary elliptic problem.These estimates allow obtaining estimates of the error of the approximate solution of the Bubnov-Galerkin method for the solution in the norm$H^1(\Omega)$having order$O\left(h^{k-1}\right)$for the considered nonclassical parabolic problem with divergent principal part, when the boundary condition contains the time derivative from the required function.

Keywords. Mixed problems, quasilinear equation, boundary condition, Galerkin method, generalized solution, parabolic type, approximate solution, error estimate, a priori estimates, monotonicity, inequalities, time derivative, boundary, domain, scalar product, error, dimension of domain, elliptic problem, continuity, the required function.

Introduction. When studying a number of topical technical problems, it becomes necessary to study mixed problems of parabolic type, when the boundary condition contains the time derivative of the required function.Problems of this type arise, for example, when a homogeneous isotropic body is placed in the inductor of an induction furnace and an electromagnetic wave is incident on its surfaceSome nonlinear problems of parabolic type with a boundary condition containing the time derivative of the required function were considered, for example, in [1-3].Many scientists were engaged in the construction of an approximate solution by the Galerkin method and obtaining a priori estimates for an approximate solution for parabolic classical quasilinear problems without a time derivative in the boundary condition: Mikhlin S.G., Douglas J. Jr, Dupont T., Dench JE, Jr, Jutchell L and others [4-9].And quasilinear problems, when the boundary condition contains the time derivative of the required function using the Galerkin method, were studied in [10-13].

Statement of the problem. In this paper, we consider a quasilinear problem of parabolic type, when the boundary condition contains the time derivative of the required function:

$$\begin{cases} u_t - \frac{d}{dx_i} a_i(x,t,u,\nabla u) + a(x,t,u,\nabla u) = 0 \quad , \\ a_0 u_t + a_i(x,t,u,\nabla u)\cos(\nu, x_i) = g(x,t,u), \quad (x,t) \in S_t \ , \\ u(x,0) = u_0(x) \ , \ x \in \Omega \end{cases} \tag{1}$$

$where\ \Omega - bounded\ domain\ in, m =$ dim $-$ dimension of domain $\Omega,$ $\quad Q_T = \left\{\Omega \times [0,T]\right\},$

$$S_T = \left\{\partial\Omega \times [0,T]\right\}, \qquad a_0 = const > 0$$

Definition.A generalized solution from the space$\widetilde{H^{1,1}}(Q_T) = \{u \in H^{1,1}(Q_T) : a_0 u_t \in$ $L2(ST)$to problem (1) is a function from$H1,1QT.$satisfying the identity

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$$\int_{Q_T} \left(u_t\eta + a_i(x,t,u,\nabla u)\eta_{x_i} + a(x,t,u,\nabla u)\eta\right) dxdt +$$

$$+ \int_{S_T} (a_0 u_t + g(x,t,u)))\eta)\, dxdt = 0 \qquad (2)$$

$$\forall\, \eta \in H^{1,1}(Q_T)$$

Here$\hat{L}_2(\Omega)$ – is the space of a function with scalar product

$$(u,v)_{\hat{L}_2} = (u,v)_\Omega + a_0(u,v)_s$$

The purpose of this article is to find out the conditions under which the estimate of the error of the approximate solution in the norm$H^1(\Omega)$is of order$O\left(h^{k-1}\right), 1 \le k \le r$we assume that the set$M = M_h$is chosen from the familyS_h^r.

Main results:

We first investigate the following elliptic problem:Find a function W (x, t) ∈M satisfying the integral identity [14-15].

$$\left(a_i(x,t,u,\nabla u) - a_i(x,t,u,\nabla W), V_{x_i}\right)_\Omega + \lambda(u - W, v)_\Omega =$$

$$(g(x,t,u) - -g(x,t,W), v)_s, \ \ \forall v\epsilon M \quad (3)$$

where$u$-the solution of the problem (2).

Suppose λ is such a sufficiently large positive number that problem (3) has a unique solution.

In what follows, assume that the following norms are bounded$\|u\|_{\left(L_\infty\left(o,T;L_\infty(\overline{\Omega})\right)\right)}, \|\nabla u\|_{(L_\infty(o,T;L_\infty(\Omega))}, \|W\|_{\left(L_\infty\left(o,T;L_\infty(\overline{\Omega})\right)\right)}, \le \|\nabla W\|_{\left(L_\infty\left(o,T;L_\infty(\overline{\Omega})\right)\right)} \le$ $K = const,$ (4)

Note that the boundedness of the norms in (4) for W (x, t) was proved in Wheeler [16], under the following assumptions:

1) Domain$\Omega = X_{i-1}^m(a_i, d_i), \quad a_i < d_i$-parallelepiped in$E_m$

2) The spaceM_h-is the tensor product of Hermitian polynomials of degree $2k-1$,which are defined on an almost uniform partition $\Delta = X_{i=1}^m \Delta_i$of the domain$\overline{\Omega}$.Here$\Delta_i$- partition $[a_i, d_i]$. ПочтиравномернымWheeler calls an almost uniform partition for which there exists a constant C such that

$$\overline{\rho_i} \ / \ \underline{\rho_i} \le C, \qquad 1 \le i, \qquad j \le m,$$

where$\overline{\rho_i}$and $\underline{\rho_i}$ -largest and smallest length of intervals in$\Delta_i$respectively

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3) $u \epsilon L_\infty(O,T;K^{r,\infty}(\overline{\Omega}))$, $r \geq \frac{m+2}{2}$.

where $K^{r,\infty}$-space of functions and with the following properties: $u \in c^{r-1}(\overline{\Omega})$, $D^{r-1}u$ - absolute continuous and $D^r u \epsilon L_\infty(\Omega)$.

Theorem I. Let $u \in H^k(\Omega), 1 \leq k \leq r$, and $W(x,t)$- the solution of the problem (2) и (3) respectivly. Set $M = M_h$ selected from the family S_h^r . Moreover, let the condition is performed:

$(p_i - q_i)(a_i(x,t,u,\mathrm{p}) - a_i(x,t,u,\mathrm{q})) \geq c(p-q)^2$, c=const> 0(5)

and functions $a_1(x,t,u,\nabla u)$ satisfy the Lipschitz condition with respect to ∇u, and the functions g (x, t, u) with respect to u.Then the inequality is true

$$\|\eta\|_{H'(\Omega)} \leq C\|u\|_{H^k(\Omega)} h^{k-1}, \qquad 1 \leq k \leq r \qquad (6)$$

Proof. In equality (3), we take as a test function $v = \tilde{u} - W = \eta - -\delta u,\ \delta u = u - \tilde{u},\ \forall \tilde{u} \in M$. Due to the ellipticity condition (5), from equality (3) we have

$$\begin{aligned} v_1\|\nabla\eta\|^2_{L_2(\Omega)} + \lambda\|\eta\|^2_{L_2(\Omega)} & \\ &\leq \left(a_i(x,t,u,\nabla u) - a_i(x,t,u,\nabla W), (\delta u)_{x_i}\right)_\Omega + \lambda(\eta, \delta u) \\ &+ (g(x,t,u) - g(x,t,W), \eta - \delta u)_S \end{aligned}$$

Estimating the last terms from above using the Cauchy inequality with ε and $\|u\|_{L_2(S)} \leq \varepsilon\|\nabla u\|^2_{L_2(\Omega)} + C_\varepsilon\|u\|^2_{L_2(\Omega)}$, (7)

we get[17-21].

$$\left(\frac{v_1}{2} - \frac{3}{2} g_0 \varepsilon\right)\|\nabla\eta\|^2_{L_2(\Omega)} + \left(\frac{\lambda}{2} - \frac{3}{2} C_\varepsilon y_i\right)\|\eta\|^2_{L_2(\Omega)} \leq C\|\delta u\|^2_{H^1(\Omega)},$$

where C_ε is determined from the Cauchy inequality. We take here $\varepsilon = \frac{v_1}{6g_0}$. Therefore, if $\lambda > 3C_\varepsilon g_0$, then $\|\eta\|^2_{H^1(\Omega)} \leq C\|\delta u\|^2_{H^1(\Omega)}$

Hence, due to the arbitrariness of the function $\tilde{u} \in M_h$ and the definition of the family S_h^r we obtain the estimate (6).

Theorem 2. Let the solution of problem (2) satisfy

$$u(x,t) \in L_{\bar{q}}\left(O,T,H^k(\Omega)\right),$$

$$u_{\ t} \in L_2\left(O,T,H^k(\Omega)\right) \cap L_{\bar{q}}\left(O,T,L_{\bar{p}}(\Omega)\right) \cap L_{\bar{q}}\left(O,T,W_q^1(\Omega)\right), \qquad 1 \leq k \leq r \quad (8)$$

where

$$\bar{p} = \frac{2\bar{q}}{\bar{q} - 2\gamma}, \qquad \bar{q} \geq 2; \qquad q = 2 + \frac{2\gamma}{1-\gamma};$$

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$$0 < \gamma \le \alpha \le 1, \qquad \tilde{p} = \frac{2\tilde{q}}{\tilde{q} - 2\gamma},$$

$$\tilde{q} \in \begin{cases} \left[2, \dfrac{2(m-1)}{m-2}\right], m \ge 3 \\ [2, \infty) m = 2 \end{cases} \tag{9}$$

Let conditions (4) and the ellipticity conditionis performed

$$v \sum_{i=1}^{m} p_i^2 \le \frac{\partial a_i}{\partial p_j} p_i p_j \le \mu \sum_{i=1}^{n} p_i^2, \quad \forall p \in E_m \tag{10}$$

In addition, we assume that the functions $\frac{\partial a_1}{\partial t}, \frac{\partial a_2}{\partial u}, \frac{\partial a_3}{\partial p}$satisfy the Hölder condition with respect to ∇u with exponent α, and the functions $\frac{\partial g}{\partial t}, \frac{\partial g}{\partial u}$- by$u$.

If the set $M = M_h$belongs to the familyS_h^r, then occurs the estimate

$$\left\| \frac{\partial \eta}{\partial t} \right\|_{L_2\left(O,T,H^1(\Omega)\right)} \le C h^{\gamma(k-1)} \tag{11}$$

where

$$C = C(R) \left(1 + \left\| \frac{\partial u}{\partial t} \right\|_{L_{\tilde{p}}\left(O,T,L_{\tilde{p}}(\mathrm{S})\right)} + \left\| \frac{\partial u}{\partial t} \right\|_{L_{\tilde{p}}\left(O,T,W_{\tilde{q}}^1(\Omega)\right)} \right)$$
$$\times \left(\|u\|_{L\tilde{q}\left(O,T,H^k(\Omega)\right)} + \left\| \frac{\partial u}{\partial t} \right\|_{L_2\left(O,T,H^k(\Omega)\right)} \right) \tag{12}$$

Proof. Let us differentiate equation (3) with respect to t.Taking into account that$v_t$belongs to the set M andintegrating the resulting identity from zero to t, we get

$$\int_0^t \left\{ \left(\frac{\partial a_i}{\partial t}(x,t,u,\nabla u) - \frac{\partial a_i}{\partial t}(x,t,u,\nabla W), v_{x_i} \right)_\Omega \right.$$
$$+ \left(\left(\frac{\partial a_i}{\partial u}(x,t,u,\nabla u) - \frac{\partial a_i}{\partial u}(x,t,u,\nabla W) \right) u_t, v_{x_i} \right)_\Omega$$
$$\left. + \left(\frac{\partial a_i}{\partial p_j}(x,t,u,\nabla u) u_{x_j t} - \frac{\partial a_i}{\partial p_j}(x,t,u,\nabla W) W_{x_j t}, v_{x_i} \right)_\Omega + \lambda \left(\frac{\partial \eta}{\partial t}, v \right)_\Omega \right\} dt =$$
$$= \int_0^t \left\{ \left(\frac{\partial g}{\partial u}(x,t,u) u_t - \frac{\partial g}{\partial u}(x,t,W) W_t, v \right)_s \right.$$
$$\left. + \left(\frac{\partial g}{\partial t}(x,t,u) - \frac{\partial g}{\partial t}(x,t,W), v \right)_s \right\} dt \tag{13}$$

Asv we take

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$$v=\frac{\partial\eta}{\partial t}-\frac{\partial(\delta u)}{\partial t},\qquad \frac{\partial(\delta u)}{\partial t}=\frac{\partial u}{\partial t}-\tilde{u},\qquad \forall\,\tilde{u}\in M$$

We estimate each term separately, using the Cauchy inequality with ε, we obtain

$$\left|\int_0^t\int_\Omega\left(\frac{\partial a_i}{\partial t}(x,t,u,\nabla u)-\frac{\partial a_i}{\partial t}(x,t,u,\nabla W)\right)v_{x_i}dxdt\right|\le$$

$$\le C(R)\int_0^t\int_\Omega|\nabla\eta|^\gamma|\nabla v|dxdt\le C_1\int_0^t\int_\Omega|\nabla\eta|^{2\gamma}dxdt+\frac{\varepsilon}{2}\|\nabla v\|^2_{L_2(0,t,L_2(\Omega))}$$

$$\le C_2\int_0^t\int_\Omega\|\nabla v\|^{2\gamma}_{L_2(\Omega)}dt+\varepsilon\|\nabla\eta_t\|^2_{L_2(0,t,L_2(\Omega))}+\varepsilon\left\|\nabla\frac{\partial(\delta u)}{\partial t}\right\|^2_{L_2(0,t,L_2(\Omega))}$$

where$C_2=C_2(R,diam\Omega)$

Further,

$$\left|\int_0^t\int_\Omega\left(\frac{\partial a_i}{\partial u}(x,t,u,\nabla u)-\frac{\partial a_i}{\partial u}(x,t,u,\nabla W)\right)u_tv_{x_i}dxdt\right|\le \tag{14}$$

$$\le C\int_0^t\int_\Omega|\nabla\eta|^\gamma|u_t||\nabla v|dxdt\le C_1\int_0^t\int_\Omega|\nabla\eta|^{2\gamma}U_t^2dxdt+\frac{\varepsilon}{2}\|\nabla v\|^2_{L_2(0,t,L_2(\Omega))}$$

To estimate the first term on the right, we use Hölder's inequality with exponents$\frac{q}{2}$in$x$and$\frac{\bar{q}}{2\gamma}$in t

$$\int_0^t\int_\Omega|\nabla\eta|^{2\gamma}U_t^2dxdt+\int_0^t\|\nabla v\|^{2\gamma}_{L_2(\Omega)}\|u_t\|^2_{L_q(\Omega)}dt\le\|\nabla\eta\|^{2\gamma}_{L_{\bar{q}}(0,t,L_2(\Omega))}\|u_t\|^2_{L_{\bar{p}}(0,t,L_q(\Omega))}$$

Here$q,\bar{q},\bar{p}$satisfy relation (9).

Второе слагаемое в правой части (14) не превосходит

$$\varepsilon\left\|\nabla\frac{\partial\eta}{\partial t}\right\|^2_{L_2(0,t,L_2(\Omega))}+\varepsilon\left\|\nabla\frac{\partial(\delta u)}{\partial t}\right\|^2_{L_2(0,t,L_2(\Omega))}$$

Hence,

$$\left|\int_0^t\int_\Omega\left(\frac{\partial a_i}{\partial u}(x,t,u,\nabla u)-\frac{\partial a_i}{\partial u}(x,t,u,\nabla W)\right)u_tv_{x_i}dxdt\right|\le$$

$$\le C\|\nabla\eta\|^{2\gamma}_{L_{\bar{q}}(0,t,L_2(\Omega))}\|u_t\|^2_{L_{\bar{p}}(0,t,L_q(\Omega))}+\varepsilon\left\|\nabla\frac{\partial\eta}{\partial t}\right\|^2_{L_2(0,t,L_2(\Omega))}+\varepsilon\left\|\nabla\frac{\partial(\delta u)}{\partial t}\right\|^2_{L_2(0,t,L_2(\Omega))}$$

Similarly, using assumptions (10), the Cauchy inequality, referring small ε to factors containing$(\nabla\eta)_t$, we obtain

$$\int_0^t\int_\Omega\left\{\left[\frac{\partial a_i}{\partial p_j}(x,t,u,\nabla u)-\frac{\partial a_i}{\partial p_j}(x,t,u,\nabla W)\right]u_{x_jt}+\frac{\partial a_i}{\partial p_j}(x,t,u,\nabla W)\left[u_{x_jt}-W_{x_jt}\right]\right\}v_{x_it}dxdt\ge$$

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$$\geq -\int_0^t \int_\Omega \left|\frac{\partial a_i}{\partial p_j}(x,t,u,\nabla u) - \frac{\partial a_i}{\partial p_j}(x,t,u,\nabla W)\right| \left|u_{x_j t}\right| \left|\eta_{x_i t} + (\partial u)_{x_i t}\right| dxdt +$$

$$+\int_0^t \frac{\partial a_i}{\partial p_j}(x,t,u,\nabla W)\, \eta_{x_i t}\eta_{x_j t} dxdt - \int_0^t \int_\Omega \left|\frac{\partial a_i}{\partial p_j}(x,t,u,\nabla W)\eta_{x_i t}(\delta u)_{x_i t}\right| dxdt$$

$$\geq C\|\nabla u_t\|^2_{L_{\bar{p}}\left(0,T,L_q(\Omega)\right)} \|\nabla\eta\|^{2\gamma}_{L_{\bar{q}}(0,T,L_2(\Omega))} - \varepsilon\left\|\nabla\frac{\partial\eta}{\partial t}\right\|^2_{L_2\left(0,t,L_2(\Omega)\right)}$$

$$+ C_1\left\|\nabla\frac{\partial(\delta u)}{\partial t}\right\|^2_{L_2\left(0,t,L_2(\Omega)\right)} + \gamma\left\|\nabla\frac{\partial\eta}{\partial t}\right\|^2_{L_2\left(0,t,L_2(\Omega)\right)}$$

Now let us estimate from above the terms on the right-hand side of (13). By the assumptions of the theorem on the function g (x, t, u), we have

$$\left|\int_0^t \int_s \left(\frac{\partial g}{\partial u}(x,t,u)u_t - \frac{\partial g}{\partial u}(x,t,W)\right) u_t v dxdt + \int_0^t \int_s \frac{\partial g}{\partial u}(x,t,W)(u_t - W_t)\, v dxdt\right| \leq$$

$$\leq C\int_0^t \int_s \left(\left|\frac{\partial\eta}{\partial t}\right| + |\eta|^\gamma |u_t|\right)\left(\left|\frac{\partial\eta}{\partial t}\right| + \left|\frac{\partial(\delta u)}{\partial t}\right|\right) dxdt \leq$$

$$\leq 2C\int_0^t \int_s \left(\frac{\partial\eta}{\partial t}\right)^2 dxdt + C\int_0^t \int_s |\eta|^{2\gamma}\, u_t^2 dxdt + C\int_0^t \int_s \left(\frac{\partial(\delta u)}{\partial t}\right)^2 dxdt \qquad (15)$$

The first term, by inequality (7), does not exceed

$$\varepsilon\left\|\nabla\frac{\partial\eta}{\partial t}\right\|^2_{L_2(\Omega)} + C_\varepsilon\left\|\frac{\partial\eta}{\partial t}\right\|^2_{L_2(\Omega)} \qquad (16)$$

Let us estimate the second term on the right-hand side of (15).We use Hölder's inequality with the exponents$\frac{\widetilde{q}}{2\gamma}$, $\frac{\bar{q}}{2\gamma}$. As a result, we get

$$\int_0^t \int_s |\eta|^{2\gamma}\, u_t^2 dxdt \leq \|u_t\|^2_{L_{\bar{p}}\left(0,t,L_{\widetilde{p}}(s)\right)} * \|\eta\|^{2\gamma}_{L_{\bar{q}}\left(0,t,L_{\widetilde{q}}(s)\right)},$$

the exponents$\bar{p}, \bar{q}, \widetilde{p}и\widetilde{q}$and satisfy relation (9).

Substituting the estimates obtained in (15) and using the embeddings$H^1(\Omega) \subset L_{\widetilde{q}}(S),\ L_2(S)$we conclude that the first term on the right-hand side of (13) is bounded from above byquantity

$$\varepsilon\left\|\nabla\frac{\partial\eta}{\partial t}\right\|^2_{L_2\left(0,t,L_2(\Omega)\right)} + C_\varepsilon\left\|\frac{\partial\eta}{\partial t}\right\|^2_{L_2\left(0,t,L_2(\Omega)\right)} + C\|u_t\|^2_{L_{\bar{p}}\left(0,t,L_{\widetilde{p}}(s)\right)}\|\eta\|^{2\gamma}_{L_{\bar{q}}\left(0,t,H^1(\Omega)\right)} +$$

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$$+C_2\left\|\nabla\frac{\partial(\delta u)}{\partial t}\right\|^2_{L_2\left(0,t,H^1(\Omega)\right)}$$

And finally

$$\left|\int_0^t\int_s\left(\frac{\partial g}{\partial t}(x,t,u)-\frac{\partial g}{\partial t}(x,t,W)\right)vdxdt\right|\le$$

$$\le C\int_0^t\int_s|\eta|^\gamma|v|dxdt\le C\left(|\eta|^{2\gamma}dxdt+\int_0^t\int_s\left(\frac{\partial\eta}{\partial t}\right)^2dxdt+\int_0^t\int_s\left(\frac{\partial(\delta u)}{\partial t}\right)^2dxdt\right)$$

The second term, as we have seen, does not exceed the value (16).We estimate the remaining terms from above using the embeddings$H^1(\Omega)\subset L_2(S),\ L_{2\gamma}(S)$. This will lead us to inequality

$$\left|\int_0^t\int_s\left(\frac{\partial g}{\partial t}(x,t,u)-\frac{\partial g}{\partial t}(x,t,W)\right)vdxdt\right|\le$$

$$\le\varepsilon\left\|\nabla\frac{\partial\eta}{\partial t}\right\|^2_{L_2(0,t,L_2(\Omega))}+C\left(\|\eta\|^{2\gamma}_{L_2\left(0,t,H^1(\Omega)\right)}+\left\|\frac{\partial(\delta u)}{\partial t}\right\|^2_{L_2\left(0,t,H^1(\Omega)\right)}\right)+$$

$$+C_\varepsilon\left\|\frac{\partial\eta}{\partial t}\right\|^2_{L_2(0,t,L_2(\Omega))}$$

Substituting the obtained inequalities into (13) and reducing similar terms, we arrive at the inequality

$$(\gamma-5\varepsilon)\left\|\nabla\frac{\partial\eta}{\partial t}\right\|^2_{L_2(0,t,L_2(\Omega))}+\left(\frac{\lambda}{2}-2C_\varepsilon\right)\left\|\frac{\partial\eta}{\partial t}\right\|^2_{L_2(0,t,L_2(\Omega))}\le$$

$$\le C\left(1+\|u_t\|^2_{L_{\overline{p}}\left(0,t,L_{\widetilde{p}}(s)\right)}+\|u_t\|^2_{L_{\overline{p}}\left(0,t,W_q^1(\Omega)\right)}\right)*$$

$$*\|\eta\|^{2\gamma}_{L_{\overline{q}}\left(0,t,H^1(\Omega)\right)}+C_1\left\|\frac{\partial(\delta u)}{\partial t}\right\|^2_{L_2\left(0,t,H^1(\Omega)\right)}$$

Setting$\varepsilon=\frac{\gamma}{10},\lambda>4C_\varepsilon$, taking into account estimate (6), the arbitrariness of the element

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$\tilde{u}$and$M$and the definition of the familyS_h^rwe obtain the desired estimate (11). Thetheoremisproved.

Conclusion. Under certain conditions for the function involved in the equation and the boundary condition, estimates are obtained for solving the auxiliary elliptic problem under considerationFurther, under the condition that the problem is elliptic, inequalities are obtained for the difference of the generalized solution of the considered parabolic problem with a divergent principal part and for the solution of the auxiliary elliptic problem.

These estimates allow obtaining estimates of the error of the approximate solution of the Bubnov-Galerkin method for the solution in the norm $H^1(\Omega)$ having order $O\left(h^{k-1}\right)$ for the considered nonclassical parabolic problem with divergent principal part, when the boundary condition contains the time derivative from the required function.

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