

## G $\eta$ -Homeomorphism in Topological Ordered Spaces

K. Sumathi<sup>1</sup>, T. Arunachalam<sup>2</sup>, D. Subbulakshmi<sup>3</sup>, K. Indirani<sup>4</sup>

<sup>1</sup> Associate Professor, Department of Mathematics, PSGR Krishnammal College for Women, Coimbatore, Tamilnadu, India, [ksumathi@psgrkcw.ac.in](mailto:ksumathi@psgrkcw.ac.in).

<sup>2</sup> Professor, Department of Mathematics, Kumaraguru College of Technology, Coimbatore, Tamilnadu, India, [tarun\\_chalam@yahoo.com](mailto:tarun_chalam@yahoo.com).

<sup>3</sup> Assistant professor, Department of Mathematics, Rathnavel Subramaniam College of Arts and Science, Coimbatore, Tamilnadu, India, [subbulakshmi169@gmail.com](mailto:subbulakshmi169@gmail.com).

<sup>4</sup> Associate Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, Tamilnadu, India, [indirani009@ymail.com](mailto:indirani009@ymail.com).

### Abstract:

The aim of this paper is to introduce a new class of closed map, open map and homeomorphism in topological ordered spaces called  $xg\eta$ -closed map,  $xg\eta$ -open map are obtained. The concept of homeomorphism is called  $xg\eta$ -homeomorphism is defined and obtained some of its properties.

### Keywords

$xg\eta$ -closed map,  $xg\eta$ -open map,  $xg\eta$ -homeomorphism.

### 1. INTRODUCTION

In 1965, Nachbin [13] initiated the study of topological ordered spaces. A new class of  $g\eta$ -closed maps,  $g\eta$ -open maps and  $g\eta$ -homeomorphism has been introduced by Subbulakshmi et al [17]. In 2001, Veera kumar [20] introduced the study of  $i$ -closed,  $d$ -closed and  $b$ -closed sets. In 2017, Amarendra babu [1] introduced  $g^*$ -closed sets in topological ordered spaces. In 2019, Dhanapakyam [7] introduced  $\beta g^*$ -closed sets in topological ordered spaces. In 2002, Veera kumar [20] introduced Homeomorphism in topological ordered spaces. In 2020, Subbulakshmi et al [18] introduced  $g\eta$ -closed, continuity, and contra continuity in topological ordered spaces. In this paper a new class of  $xg\eta$ -homeomorphism in topological ordered spaces are defined and some of their properties are analyzed. Throughout this paper  $[x = i, d, b]$

### 2. PRELIMINARIES

#### Definition : 2.1

A subset  $A$  of a topological space  $(X, \tau)$  is called

- (i)  $\alpha$ -open set [2] if  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ ,  $\alpha$ -closed set if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ .
- (ii) semi-open set [10] if  $A \subseteq \text{cl}(\text{int}(A))$ , semi-closed set if  $\text{int}(\text{cl}(A)) \subseteq A$ .

(iii)  $\eta$ -open set [14] if  $A \subseteq \text{int}(\text{cl}(\text{int}(A))) \cup \text{cl}(\text{int}(A))$ ,  $\eta$ -closed set if  $\text{cl}(\text{int}(\text{cl}(A))) \cap \text{int}(\text{cl}(A)) \subseteq A$ .

**Definition : 2.2** A subset  $A$  of a topological space  $(X, \tau)$  is called

- (i)  $g$ -closed set [11] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
- (ii)  $g^*$ -closed set [19] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ .
- (iii)  $g\eta$ -closed set [15] if  $\eta\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .

**Definition : 2.3** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called

- (i) continuous [3] if  $f^{-1}(V)$  is a closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (ii) semi-continuous [10] if  $f^{-1}(V)$  is a semi-closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (iii)  $\alpha$ -continuous [5] if  $f^{-1}(V)$  is a  $\alpha$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (iii)  $\eta$ -continuous [16] if  $f^{-1}(V)$  is a  $\eta$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (iv)  $g\eta$ -continuous [16] if  $f^{-1}(V)$  is a  $g\eta$ -closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

**Definition: 2.4**

A bijective function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called

- (i) homeomorphism [12] if  $f$  is both continuous map and open map.
- (ii) semi-homeomorphism [4,6] if  $f$  is both semi-continuous map and semi-open map.
- (iii)  $\alpha$ -homeomorphism [5] if  $f$  is both  $\alpha$ -continuous map and  $\alpha$ -open map.
- (iv)  $\eta$ -homeomorphism [17] if  $f$  is both  $\eta$ -continuous map and  $\eta$ -open map.
- (v)  $g\eta$ -homeomorphism [17] if  $f$  is both  $g\eta$ -continuous map and  $g\eta$ -open map.

**Definition 2.5: [20]** A topological ordered space is a triple  $(X, \tau, \leq)$ , where  $\tau$  is a topology on  $X$  and  $\leq$  is a partial order on  $X$ .

Let  $A$  be a subset of topological ordered space  $(X, \tau, \leq)$ .

For any  $x \in X$ ,

- (i)  $[x, \rightarrow] = \{y \in X/x \leq y\}$  and
- (ii)  $[\leftarrow, x] = \{y \in X/y \leq x\}$ .

The subset  $A$  is said to be

- (i) increasing if  $A = i(A)$ , where  $i(A) = \bigcup_{a \in A} [a, \rightarrow]$  and
- (ii) decreasing if  $A = d(A)$ , where  $d(A) = \bigcup_{a \in A} [\leftarrow, a]$
- (iii) balanced if it is both increasing and decreasing.

The complement of an increasing set is a decreasing set and the complement of a decreasing set is an increasing set.

**Definition: 2.6 [20]** A subset  $A$  of a topological ordered space  $(X, \tau, \leq)$  is called

(i)  $x$ -closed set [18] if it is both increasing (resp. decreasing, increasing and decreasing) set and closed set.

(ii)  $x\alpha$ -closed set [18] if it is both increasing (resp. decreasing, increasing and decreasing) set and  $\alpha$ -closed set.

(iii)  $x$ semi-closed set [18] if it is both increasing (resp. decreasing, increasing and decreasing) set and semi-closed set.

**Definition: 2.7** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be

(i)  $x$ closed map [20] if the image of every closed set in  $(X, \tau, \leq)$  is an  $x$ -closed set in  $(Y, \sigma, \leq)$ .

(ii)  $x\alpha$ -closed map [20] if the image of every closed set in  $(X, \tau, \leq)$  is an  $x\alpha$ -closed set in  $(Y, \sigma, \leq)$ .

(iii)  $x$ semi-closed map [20] if the image of every closed set in  $(X, \tau, \leq)$  is an  $x$ semi-closed set in  $(Y, \sigma, \leq)$ .

**Definition: 2.8** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be

(i)  $x$ open map [8] if the image of every open set in  $(X, \tau, \leq)$  is an  $x$ -open set in  $(Y, \sigma, \leq)$ .

(ii)  $x\alpha$ -open map [8] if the image of every open set in  $(X, \tau, \leq)$  is an  $x\alpha$ -open set in  $(Y, \sigma, \leq)$ .

(iii)  $x$ semi-open map [8] if the image of every closed set in  $(X, \tau, \leq)$  is an  $x$ semi-open set in  $(Y, \sigma, \leq)$ .

**Definition: 2.9** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be

(i)  $x$ -homeomorphism [9] if  $f$  is both  $x$ -continuous function and  $x$ -open map.

(ii)  $x\alpha$ -homeomorphism [9] if  $f$  is both  $x\alpha$ -continuous function and  $x$ -open map.

(iii)  $x$ semi-homeomorphism [9] if  $f$  is both  $x$ semi-continuous function and  $x$ -open map.

### 3. $ig\eta$ -closed map

**Definition : 3.1** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be an  $ig\eta$ -closed map if the image of every closed set in  $(X, \tau, \leq)$  is an  $ig\eta$ -closed set in  $(Y, \sigma, \leq)$ .

**Definition : 3.2** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be an  $ig\eta$ -closed map if the image of every closed set in  $(X, \tau, \leq)$  is an  $ig\eta$ -closed set in  $(Y, \sigma, \leq)$ .

**Theorem 3.3:** Every  $i$ -closed,  $i$ semi-closed,  $i\alpha$ -closed,  $ig\eta$ -closed maps are  $ig\eta$ -closed map, but not conversely.

**Proof:** The proof follows from the fact that every closed, semi-closed,  $\alpha$ -closed,  $\eta$ -closed maps are  $g\eta$ -closed maps. [17]. Then every  $i$ -closed,  $i$ semi-closed,  $i\alpha$ -closed,  $i\eta$ -closed maps are  $ig\eta$ -closed map.

**Example 3.4:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{Y, \varphi, \{a\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, b), (c, b)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = a, f(b) = c, f(c) = b$ . This map is  $ig\eta$ -closed map, but not  $i$ -closed,  $i$ semi-closed,  $i\alpha$ -closed,  $ig\alpha$ -closed,  $ig^*$ -closed,  $isg$ -closed,  $i\eta$ -closed map. Since for the closed set  $V = \{a, c\}$  in  $(X, \tau, \leq)$ . Then  $f(V) = \{a, b\}$  is  $ig\eta$ -closed but not  $i$ -closed,  $i$ semi-closed,  $i\alpha$ -closed,  $ig\alpha$ -closed,  $ig^*$ -closed,  $isg$ -closed,  $i\eta$ -closed in  $(Y, \sigma, \leq)$ .

#### 4. $dg\eta$ -closed map

**Definition : 4.1** A function  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be a  $d\eta$ -closed map if the image of every closed set in  $(X, \tau, \leq)$  is a  $d\eta$ -closed set in  $(Y, \sigma, \leq)$ .

**Definition : 4.2** A function  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be a  $dg\eta$ -closed map if the image of every closed set in  $(X, \tau, \leq)$  is a  $dg\eta$ -closed set in  $(Y, \sigma, \leq)$ .

**Theorem 4.3:** Every  $d$ -closed,  $d$ semi-closed,  $d\alpha$ -closed,  $d\eta$ -closed maps are  $dg\eta$ -closed map, but not conversely.

**Proof:** The proof follows from the fact that every closed, semi-closed,  $\alpha$ -closed,  $\eta$ -closed maps are  $g\eta$ -closed map [17]. Then every  $d$ -closed,  $d$ semi-closed,  $d\alpha$ -closed,  $d\eta$ -closed maps are  $dg\eta$ -closed map.

**Example 4.4 :** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \varphi, \{a\}\}$  and  $\sigma = \{Y, \varphi, \{a\}, \{b, c\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = c, f(b) = b, f(c) = a$ . This map is  $dg\eta$ -closed map but not  $d$ -closed,  $d$ semi-closed,  $d\alpha$ -closed,  $d\eta$ -closed map. Since for the closed set  $V = \{b, c\}$  in  $(X, \tau, \leq)$ . Then  $f(V) = \{a, b\}$  is  $dg\eta$ -closed but not  $d$ -closed,  $d$ semi-closed,  $d\alpha$ -closed,  $d\eta$ -closed in  $(Y, \sigma, \leq)$ .

#### 5. $bg\eta$ -closed map

**Definition : 5.1** A function  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be a  $b\eta$ -closed map if the image of every closed set in  $(X, \tau, \leq)$  is a  $b\eta$ -closed set in  $(Y, \sigma, \leq)$ .

**Definition : 5.2** A function  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be a  $bg\eta$ -closed map if the image of every closed set in  $(X, \tau, \leq)$  is a  $bg\eta$ -closed set in  $(Y, \sigma, \leq)$ .

**Theorem 5.3:** Every  $b$ -closed,  $b\alpha$ -closed maps are  $bg\eta$ -closed map, but not conversely.

**Proof:** The proof follows from the fact that every closed,  $\alpha$ -closed maps are  $g\eta$ -closed map [17]. Then every  $b$ -closed,  $b\alpha$ -closed maps are  $bg\eta$ -closed map.

**Example 5.4:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{b, c\}\}$  and  $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\} . \leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b, f(b) = a, f(c) = c$ . This map is  $b g\eta$ -closed map but not  $b$ -closed,  $b\alpha$ -closed map. Since for the closed set  $V = \{a\}$  in  $(X, \tau, \leq)$ . Then  $f(V) = \{b\}$  is  $bg\eta$ -closed but not  $b$ -closed,  $b\alpha$ -closed in  $(Y, \sigma, \leq)$ .

**Theorem 5.5:** Every  $b$ semi-closed,  $b\eta$ -closed maps are  $bg\eta$ -closed map, but not conversely.

**Proof:** The proof follows from the fact that every semi-closed,  $\eta$ -closed maps are  $bg\eta$ -closed map [17]. Then every  $b$ semi-closed,  $b\eta$ -closed maps are  $bg\eta$ -closed map.

**Example 5.6:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{b, c\}\}$  and  $\sigma = \{Y, \phi, \{a\}\} . \leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b, f(b) = a, f(c) = c$ . This map is  $bg\eta$ -closed map but not  $b$ semi-closed,  $b\eta$ -closed map. Since for the closed set  $V = \{b, c\}$  in  $(X, \tau, \leq)$ . Then  $f(V) = \{a, c\}$  is  $bg\eta$ -closed but not  $b$ semi-closed,  $b\eta$ -closed in  $(Y, \sigma, \leq)$ .

## 6. $ig\eta$ open map

**Definition :6.1** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be an  $ig\eta$ -open map if the image of every open set in  $(X, \tau, \leq)$  is an  $ig\eta$ -open set in  $(Y, \sigma, \leq)$ .

**Definition :6.2** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be an  $ig\eta$ -open map if the image of every open set in  $(X, \tau, \leq)$  is an  $ig\eta$ -open set in  $(Y, \sigma, \leq)$ .

**Theorem 6.3:** Every  $i$ -open,  $i$ semi-open,  $i\alpha$ -open,  $i\eta$ -open maps are  $ig\eta$ -open map, but not conversely.

**Proof:** The proof follows from the fact that every open, semi-open,  $\alpha$ -open,  $\eta$ -open maps are  $ig\eta$ -open map [17]. Then every  $i$ -open,  $i$ semi-open,  $i\alpha$ -open,  $i\eta$ -open maps are  $ig\eta$ -open map.

**Example 6.4:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{b, c\}\}$  and  $\sigma = \{Y, \phi, \{a\}, \{b, c\}\} . \leq = \{(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = c, f(b) = b, f(c) = a$ . This map is  $ig\eta$ -open map, but not  $i$ -open,  $i$ semi-open,  $i\alpha$ -open,  $i\eta$ -open map. Since for the open set  $V = \{b, c\}$  in  $(X, \tau, \leq)$ . Then  $f(V) = \{a, b\}$  is  $ig\eta$ -open but not  $i$ -open,  $i$ semi-open,  $i\alpha$ -open,  $i\eta$ -open in  $(Y, \sigma, \leq)$ .

## 7. $dg\eta$ open map

**Definition :7.1** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be a  $dg\eta$ -open map if the image of every open set in  $(X, \tau, \leq)$  is a  $dg\eta$ -open set in  $(Y, \sigma, \leq)$ .

**Definition :7.2** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be a  $d\eta$ -open map if the image of every open set in  $(X, \tau, \leq)$  is a  $d\eta$ -open set in  $(Y, \sigma, \leq)$ .

**Theorem 7.3:** Every  $d$ -open,  $d\alpha$ -open maps are  $d\eta$ -open map, but not conversely.

**Proof:** The proof follows from the fact that every open,  $\alpha$ -open maps are  $\eta$ -open map [17]. Then every  $d$ -open,  $d\alpha$ -open maps are  $d\eta$ -open map.

**Example 7.4:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a, b\}\}$  and  $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, b), (c, b)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = c, f(b) = b, f(c) = a$ . This map is  $d\eta$ -open map, but not  $d$ -open,  $d\alpha$ -open, map. Since for the open set  $V = \{a, b\}$  in  $(X, \tau, \leq)$ . Then  $f(V) = \{b, c\}$  is  $d\eta$ -open but not  $d$ -open,  $d\alpha$ -open in  $(Y, \sigma, \leq)$ .

**Theorem 7.5:** Every  $d$ semi-open,  $d\eta$ -open maps are  $d\eta$ -open map, but not conversely.

**Proof:** The proof follows from the fact that every semi-open,  $\eta$ -open maps are  $d\eta$ -open map [17]. Then every  $d$ semi-open,  $d\eta$ -open maps are  $d\eta$ -open map.

**Example 7.6:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{c\}\}$  and  $\sigma = \{Y, \phi, \{a\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, b), (c, b)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = c, f(b) = a, f(c) = b$ . This map is  $d\eta$ -open map, but not  $d$ semi-open,  $d\eta$ -open map. Since for the open set  $V = \{c\}$  in  $(X, \tau, \leq)$ . Then  $f(V) = \{b\}$  is  $d\eta$ -open but not  $d$ semi-open,  $d\eta$ -open in  $(Y, \sigma, \leq)$ .

## 8. $b\eta$ open map

**Definition :8.1** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be a  $b\eta$ -open map if the image of every open set in  $(X, \tau, \leq)$  is a  $b\eta$ -open set in  $(Y, \sigma, \leq)$ .

**Definition :8.2** A function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is said to be a  $bg\eta$ -open map if the image of every open set in  $(X, \tau, \leq)$  is a  $bg\eta$ -open set in  $(Y, \sigma, \leq)$ .

**Theorem 8.3:** Every  $b$ -open,  $b\alpha$ -open maps are  $bg\eta$ -open map, but not conversely.

**Proof:** The proof follows from the fact that every open,  $\alpha$ -open maps are  $bg\eta$ -open map [17]. Then every  $b$ -open,  $b\alpha$ -open maps are  $bg\eta$ -open map.

**Example 8.4:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{b, c\}\}$  and  $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b, f(b) = a, f(c) = c$ . This map is  $bg\eta$ -open map, but not  $b$ -open,  $b\alpha$ -open map. Since for the open set  $V = \{b, c\}$  in  $(X, \tau, \leq)$ . Then  $f(V) = \{a, c\}$  is  $bg\eta$ -open but not  $b$ -open,  $b\alpha$ -open in  $(Y, \sigma, \leq)$ .

**Theorem 8.5:** Every  $b$ semi-open,  $b\eta$ -open maps are  $bg\eta$ -open map, but not conversely.

**Proof:** The proof follows from the fact that every semi-open,  $\eta$ -open maps are  $g\eta$ -open map [17]. Then every bsemi-open,  $b\eta$ -open maps are  $bg\eta$ -open map.

**Example 8.6:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \varphi, \{a\}, \{b, c\}\}$  and  $\sigma = \{Y, \varphi, \{a\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b, f(b) = a, f(c) = c$ . This map is  $bg\eta$ -open map, but not bsemi-open,  $b\eta$ -open map. Since for the open set  $V = \{a\}$  in  $(X, \tau, \leq)$ . Then  $f(V) = \{b\}$  is  $bg\eta$ -open but not bsemi-open,  $b\eta$ -open in  $(Y, \sigma, \leq)$ .

### 9. $i\eta$ -Homeomorphism:

**Definition: 9.1** A bijection function  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is called a  $i\eta$ -homeomorphism if  $f$  is both  $i\eta$ -continuous function and  $i\eta$ -open map.

**Definition: 9.2** A bijection function  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is called a  $ig\eta$ -homeomorphism if  $f$  is both  $ig\eta$ -continuous function and  $ig\eta$ -open map.

**Theorem 9.3:** Every  $i$ -homeomorphism,  $i\alpha$ -homeomorphism are  $ig\eta$ -homeomorphism but not conversely.

**Proof:** The proof follows from the fact that every  $i$ -continuous,  $i\alpha$ -continuous functions are  $ig\eta$ -continuous [18]. Also every  $i$ -open,  $i\alpha$ -open maps are  $ig\eta$ -open map. By theorem [6.3].

**Example 9.4:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{Y, \varphi, \{a\}, \{b, c\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b, f(b) = a, f(c) = c$ . This map is  $ig\eta$ -homeomorphism, but not  $i$ -homeomorphism,  $i\alpha$ -homeomorphism. Since for the closed set  $V = \{a\}$  in  $(Y, \sigma, \leq)$ . Then  $f^{-1}(V) = \{b\}$  is  $ig\eta$ -closed but not  $i$ -closed,  $i\alpha$ -closed in  $(X, \tau, \leq)$ .

**Theorem 9.5:** Every  $i$ semi-homeomorphism,  $i\eta$ -homeomorphism are  $ig\eta$ -homeomorphism but not conversely.

**Proof:** The proof follows from the fact that every  $i$ semi-continuous and  $i\eta$ -continuous functions are  $ig\eta$ -continuous [18]. Also every  $i$ semi-open,  $i\eta$ -open maps are  $ig\eta$ -open map. By theorem [6.3].

**Example 9.6:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \varphi, \{a\}\}$  and  $\sigma = \{Y, \varphi, \{a\}, \{b, c\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b, f(b) = a, f(c) = c$ . This map is  $ig\eta$ -homeomorphism, but not  $i$ semi-homeomorphism,  $i\eta$ -homeomorphism. Since for the closed set  $V = \{b, c\}$  in  $(Y, \sigma, \leq)$ . Then  $f^{-1}(V) = \{a, c\}$  is  $ig\eta$ -closed but not  $i$ semi-closed,  $i\eta$ -closed in  $(X, \tau, \leq)$ .

## 10. $d\eta$ -Homeomorphism:

**Definition 10.1** A bijection function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is called a  $d\eta$ -homeomorphism if  $f$  is both  $d\eta$ -continuous function and  $d\eta$ -open map.

**Definition 10.2** A bijection function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is called a  $d\eta$ -homeomorphism if  $f$  is both  $d\eta$ -continuous function and  $d\eta$ -open map.

**Theorem 10.3:** Every  $d$ -homeomorphism,  $d\alpha$ -homeomorphism are  $d\eta$ -homeomorphism but not conversely.

**Proof:** The proof follows from the fact that every  $d$ -continuous,  $d\alpha$ -continuous functions are  $d\eta$ -continuous [18]. Also every  $d$ -open,  $d\alpha$ -open maps are  $d\eta$ -open map. By theorem [7.3].

**Example 10.4:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{Y, \varphi, \{a\}, \{b, c\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b$ ,  $f(b) = a$ ,  $f(c) = c$ . This map is  $d\eta$ -homeomorphism, but not  $d$ -homeomorphism,  $d\alpha$ -homeomorphism. Since for the closed set  $V = \{a\}$  in  $(Y, \sigma, \leq)$ . Then  $f^{-1}(V) = \{b\}$  is  $d\eta$ -closed but not  $d$ -closed,  $d\alpha$ -closed in  $(X, \tau, \leq)$ .

**Theorem 10.5:** Every  $d$ semi-homeomorphism,  $d\eta$ -homeomorphism are  $d\eta$ -homeomorphism but not conversely.

**Proof:** The proof follows from the fact that every  $d$ semi-continuous,  $d\eta$ -continuous functions are  $d\eta$ -continuous [18]. Also every  $d$ semi-open,  $d\eta$ -open maps are  $d\eta$ -open map. By theorem [7.5].

**Example 10.6:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \varphi, \{a\}\}$  and  $\sigma = \{Y, \varphi, \{a\}, \{b, c\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b$ ,  $f(b) = a$ ,  $f(c) = c$ . This map is  $d\eta$ -homeomorphism, but not  $d$ semi-homeomorphism,  $d\eta$ -homeomorphism. Since for the closed set  $V = \{b, c\}$  in  $(Y, \sigma, \leq)$ . Then  $f^{-1}(V) = \{a, c\}$  is  $d\eta$ -closed but not  $d$ semi-closed,  $d\eta$ -closed in  $(X, \tau, \leq)$ .

## 11. $b\eta$ -Homeomorphism:

**Definition 11.1** A bijection function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is called a  $b\eta$ -homeomorphism if  $f$  is both  $b\eta$ -continuous function and  $b\eta$ -open map.

**Definition 11.2** A bijection function  $f : (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  is called a  $b\eta$ -homeomorphism if  $f$  is both  $b\eta$ -continuous function and  $b\eta$ -open map.

**Theorem 11.3:** Every bsemi-homeomorphism,  $b\alpha$ -homeomorphism,  $b\eta$ -homeomorphism, are  $b\eta$ -homeomorphism but not conversely.

**Proof:** The proof follows from the fact that every bsemi-continuous,  $b\alpha$ -continuous,  $b\eta$ -continuous functions are  $b\eta$ -continuous [18]. Also every bsemi-open,  $b\alpha$ -open,  $b\eta$ -open, maps are  $b\eta$ -open map. By theorem [8.3 and 8.5].

**Example 11.4:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \varphi, \{a\}\}$  and  $\sigma = \{Y, \varphi, \{a\}, \{b, c\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b, f(b) = a, f(c) = c$ . This map is  $b\eta$ -homeomorphism, but not bsemi-homeomorphism,  $b\alpha$ -homeomorphism,  $b\eta$ -homeomorphism. Since for the closed set  $V = \{b, c\}$  in  $(Y, \sigma, \leq)$ . Then  $f^{-1}(V) = \{a, c\}$  is  $b\eta$ -closed but not bsemi-closed,  $b\alpha$ -closed,  $b\eta$ -closed in  $(X, \tau, \leq)$ .

**Theorem 11.5 :** Every b-homeomorphism is  $b\eta$ -homeomorphism but not conversely.

**Proof:** The proof follows from the fact that every b-continuous functions is  $b\eta$ -continuous [18]. Also every b-open map is  $b\eta$ -open map. By theorem [8.3].

**Example 11.6:** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \varphi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{Y, \varphi, \{a\}, \{b, c\}\}$ .  $\leq = \{(a, a), (b, b), (c, c), (a, c)\}$ . Define a map  $f: (X, \tau, \leq) \rightarrow (Y, \sigma, \leq)$  by  $f(a) = b, f(b) = a, f(c) = c$ . This map is  $b\eta$ -homeomorphism, but not b-homeomorphism. Since for the closed set  $V = \{a\}$  in  $(Y, \sigma, \leq)$ . Then  $f^{-1}(V) = \{b\}$  is  $b\eta$ -closed but not b-closed in  $(X, \tau, \leq)$ .

## 12. REFERENCES:

- [1] Amarendra babu. V., Aswini. J.,  $g^*$ -closed sets in topological ordered spaces, International journal of Advanced in Management, Technology and Engineering Sciences, 7 (12) (2017), 113-125.
- [2] Andrijevic D. "Some properties of the topology of  $\alpha$ -sets", Mat. Vesnik 36(1984).
- [3] Balachandran. K, Sundaram. P & Maki. H, On generalised continuous maps in topological spaces, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math, 12(1991), 5-13.
- [4] Biswas. N, "On some mappings in topological spaces" Bull. Calcutta Math. Soc.1 (1969), 127-135.
- [5] Devi. R & Balachandran. K (2001), Some generalizations of  $\alpha$ -homeomorphisms in topological spaces , Indian J.Pure Appl.Math, 32(4) : 551-563
- [6] Devi. R, Balachandran. K, and Maki. H, semi generalized homeomorphisms and generalized semi homeomorphisms in topological spaces, Indian J. Pure Appl. Math., 26(3) :271:284,

1995.

- [7] Dhanapakyam C., Indirani K., On  $\beta g^*$  Closed Sets in Topological Ordered Spaces, International Journal of Mathematics Trends and Technology, 65(1), (2019). 7-8.
- [8] Krishna rao K., Some Concepts In Topological Ordered Spaces using semi-open sets, pre-open sets,  $\alpha$ -open sets and  $\beta$ -open sets, Acharya Nagarjuna University Thesis, 2014.
- [9] Krishna rao K., Chudamani R.,  $\alpha$ -homeomorphism in topological ordered spaces, International Journal of Mathematical Sciences, Technology and Humanities, 52 (2012) 541-560.
- [10] Levine N., Semi open sets and semi continuity in Topological spaces, Amer. Math. Monthly, 70(1963), 36-41.
- [11] Levine. N, Generalized closed sets in topology, Rend. Circ. Mat. Ser. III, 10, (1975), 347 – 350.
- [12] Maki. H, Sundaram. P and Balachandran. K, On Generalized Homeomorphisms in Topological Spaces, Bull. Fukuoka Univ. Ed, part-III, 40(1991), 13-21.
- [13] Nachbin. L, Topology and order, Van Nostrand. D, Inc., Princeton, New Jersey [1965].
- [14] Subbulakshmi. D, Sumathi. K, Indirani. K.,  $\eta$ -open sets in topological spaces, International Journal of Innovative Technology and Exploring Engineering, 8(10s) (2019), 276-282.
- [15] Subbulakshmi. D, Sumathi. K, Indirani. K.,  $g\eta$ -closed sets in topological spaces, International Journal of Recent Technology and Engineering, 8(3) (2019), 8863-8866.
- [16] Subbulakshmi. D, Sumathi. K, Indirani. K.,  $g\eta$ -continuous in topological spaces, Advances in Mathematics: Scientific Journal 8 (2019), no.3, 677-682.
- [17] Subbulakshmi. D, Sumathi. K, Indirani. K.,  $g\eta$ -homeomorphism in topological spaces, Advances in Mathematics: Scientific Journal 8 (2019), no.3, 705-713.
- [18] Subbulakshmi. D, Sumathi. K, Indirani. K.,  $g\eta$ -closed, continuity, and contra continuity in topological ordered spaces, communicated.....
- [19] Veerakumar. M. K. R. S, Between  $g^*$  closed sets and  $g$  closed sets Antartica J. Math, Reprint.
- [20] Veera Kumar. M.K.R.S., Homeomorphisms in topological ordered spaces, Acta Ciencia Indica, XXVIII(M), No.1.(2002), 67-76.p