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Determining Hydraulic Resistance of Stationary Flow of Blood in Vessels with Permeable Walls

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Abstract: The paper considers stationary flow of blood in vessels with permeable walls. Since, the outflow of blood in the branches departing from the main arterial trunk can be approximately mathematically modeled by introducing an equivalent wall permeability coefficient. To determine the hydraulic resistance in an arterial vessel, the blood is considered to be a Newtonian viscous fluid, and the flows are stationary.It should be noted that in addition to wave flows in the arterial vessel, it is advisable to study the steady flow of blood in a vessel with permeable walls. However, this case does not follow from the solution for wave flows by the limiting transition at. Therefore, in this paper we consider stationary problems in a separate formulation. In solving problems, formulas are obtained for determining the corresponding hydrodynamic parameters, such as velocity, fluid flow, and pressure gradient.Impedance method determined the hydraulic resistance. With a steady flow of hydraulic resistance in a permeable vessel, it depends significantly on the permeability coefficient: with increasing this coefficient, it decreases.

Keywords: Newton fluids, continuity, oscillation parameter, pulsating motions.

Introduction

In work [1-3], the formulation of the problem of pulsating blood flow in the arterial bed is considered, where the flow of blood is mathematically modeled as permeable Walls of blood vessels. It should be noted that in addition to wave flows, it is advisable to study the steady flow of blood in a vessel with permeable walls. However, this case does not follow from the solution for wave flows by a limiting transition when [4-8], therefore the stationary problem should be considered in a separate formulation. For a steadyflow of a fluid,

$$\frac{\partial v_x}{\partial t} = 0, \frac{\partial v_r}{\partial t} = 0, \frac{\partial u_x}{\partial t} = \frac{\partial u_r}{\partial t} = 0$$

Taking them into account, we obtain from (1), (2), (3) a system of equations

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$$\begin{cases} \frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\left(\frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r}\frac{\partial v_x}{\partial r} + \frac{\partial^2 v_x}{\partial x^2}\right) \\ \frac{1}{\rho}\frac{\partial p}{\partial r} = \nu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{1}{r^2}v_r + \frac{\partial^2 v_r}{\partial x^2}\right) \\ \frac{\partial v_r}{\partial r} + \frac{1}{r}v_r + \frac{\partial v_x}{\partial x} = 0 \end{cases} \quad (1)$$

And boundary conditions

$$\begin{cases} v_r = \frac{R\gamma_*}{\mu}\left(p - p_c\right), v_x = 0, r = R \\ \frac{\partial v_x}{\partial r} = 0,\ v_r = 0,\ r = 0 \\ p = p_{0,},\ Q = Q_0, x = 0 \end{cases} \quad (2)$$

From the system (1), after making some calculations, we find the equation for the pressure

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial x^2} = 0 \qquad (3)$$

According to this, in a stationary flow the pressure distribution obeys the Laplace equation. We seek the solution of (3) in the form

$$p = p_1\left(r\right)\exp\left(i\gamma_{nx}\right) + p_2\left(r\right)\exp\left(-i\gamma_{nx}\right) \quad (4)$$

This method follows from the general solution of [3] under the condition that. In this case, the complex argument is transformed to the real one, i.e.

$$p = p_1\left(r\right)\exp\left(\frac{\lambda}{R}x\right) + p_2\left(r\right)\exp\left(-\frac{\lambda}{R}x\right) \quad (5)$$

Here all functions are real. Therefore, in the stationary problem, there is no need for a complex solution.

Substitution of (5) into (3) leads to the Bessley equation

$$\frac{d^2 p_{1,2}}{dr^2} + \frac{1}{r}\frac{dp_{1,2}}{dr} + \frac{\lambda^2}{R^2}p_{1,2} = 0$$

Its fundamental solutions will be functions:

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$$J_0\left(\frac{\lambda}{R}r\right) \;\; u \;\; Y_0\left(\frac{\lambda}{R}r\right)$$

The general solution has the form

$$p_{1,2} = c_{1,2}\; J_0\left(\frac{\lambda}{R}r\right) + d_{1,2}\; Y_0(\frac{\lambda}{R}r) \quad (6)$$

At, the pressure turns to infinity, so the coefficients in front of the function $Y_0\left(\frac{\lambda}{R}r\right)$ Must equal zero and in this case the solution (6) takes the form

$$p_{1,2} = c_{1,2} J_0\left(\frac{\lambda}{R}r\right) \quad (7)$$

Or finally, we get a solution

$$p - p_c = J_0\left(\frac{\lambda}{R}r\right)\left[c_1 \exp\left(\frac{\lambda}{R}x\right) + c_2 \exp\left(-\frac{\lambda}{R}x\right)\right] \quad (8)$$

Where the coefficients c_1 u c_2 Are determined from the boundary conditions (2). The account (8) for the first equation of system (1) gives the equation

$$\frac{\partial^2 v_x}{\partial r^2} + \frac{1}{r}\frac{\partial v_x}{\partial r} + \frac{\partial^2 v_x}{\partial x^2} = \frac{\lambda}{\mu R} J_0\left(\frac{\lambda}{R}r\right)\left[c_1 \exp\left(\frac{\lambda}{R}x\right) - c_2 \exp\left(-\frac{\lambda}{R}x\right)\right] \quad (9)$$

Whose solution is sought in the form

$$v_x = v_{1x}(r)\left[c_1 \exp\left(\frac{\lambda}{R}x\right) - c_2 \exp\left(-\frac{\lambda}{R}x\right)\right] \quad (10)$$

Then

$$\frac{d^2 v_{1x}(r)}{dr^2} + \frac{1}{r}\frac{dv_{1x}(r)}{dr} + \frac{\lambda^2}{R^2} v_{1x} = \frac{\lambda}{\mu R} J_0\left(\frac{\lambda}{R}r\right) \quad (11)$$

Equation (11) is solved under the condition that the liquid adheres to the wall of the tube, i.e. $v_x = 0$ at $r = R$ And limitations $v_x < \infty$ on the axis of the pipe.

The fundamental solutions (16) of the Bessel function of the zero order are expressed in this way

$$J_0\left(\frac{\lambda}{R}r\right) \;\; and \;\; Y_0\left(\frac{\lambda}{R}r\right) \quad (12)$$

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Equation (11) is inhomogeneous, so we find its particular solution from the formula [1]

$$v^*_{1x}=\frac{\pi}{2}Y_0(x)\int xJ_0(x)\,f(x)dx-\frac{\pi}{2}J_0(x)\int xY_0(x)f(x)dx \qquad (13)$$

Where

$$f(x)=\frac{\lambda}{\mu R}J_0(x),\quad x=\frac{\lambda}{R}r$$

Sometimes, taking into account the relation

$$J_0(x)Y_0'(x)-J_0'(x)Y_0(x)=\frac{2}{\pi x} \quad (14)$$

We get

$$v*_{1x}=\frac{1}{2\mu}rJ_1\left(\frac{\lambda}{R}r\right) \quad (15)$$

The use of the fundamental solution (12) and the limited velocity on the axis of the tube gives

$$v_{1x}=c_1J_0\left(\frac{\lambda}{R}r\right)+\frac{1}{2\mu}rJ_1\left(\frac{\lambda}{R}r\right) \quad (16)$$

From condition $v_x=0$ at $r=R$ Define

$$\overline{c_1}=-\frac{1}{2\mu}\frac{RJ_1(\lambda)}{J_0(\lambda)} \quad (17)$$

Substituting (17) into (16), we find

$$v_x=\frac{R}{2\mu}\left\{\frac{r}{R}J_1\left(\frac{\lambda}{R}r\right)-\frac{J_1(\lambda)}{J_o(\lambda)}J_0\left(\frac{\lambda}{R}r\right)\times\left\{\ c_1\exp\left(\frac{\lambda}{R}x\right)-c_2\exp\left(-\frac{\lambda}{R}x\right)\right\}\right. \quad (18)$$

In the same way, using the solution (8) and the second equation of the system (1), we obtain a formula for the distribution of the transverse velocity:

$$v_r=\frac{R}{2\mu\lambda}\left\{\frac{\lambda r}{R}J_0\left(\frac{\lambda}{R}r\right)+\left[\frac{\lambda J_1(\lambda)}{J_0(\lambda)}-2\right]J_1\left(\frac{\lambda}{R}r\right)\right\}\times\left(\ \mathrm{c}_1\exp\left(\frac{\lambda}{\mathrm{R}}x\right)\ +\mathrm{c}_2\exp\left(-\frac{\lambda}{R}x\right)\right) \quad (19)$$

Using the wall permeability condition (2) leads to an equation for determining the eigenvalue λ

$$\gamma*=\frac{1}{2\lambda}\left\{\lambda+\frac{J_1(\lambda)}{J_0(\lambda)}\left[\lambda\frac{J_1(\lambda)}{J_0(\lambda)}-2\right]\right\} \quad (20)$$

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If $j^* << I$ Then, expanding the functions $J_0(\lambda)$, $J_1(\lambda)$ In convergent series

$$\begin{cases} J_0(z) = 1 - \dfrac{\left(\dfrac{z}{2}\right)^2}{1} + \dfrac{\left(\dfrac{z}{2}\right)^4}{1^2 2^2} - \dfrac{\left(\dfrac{z}{2}\right)^6}{1^2 2^2 3^2} + \dots, \\ J_1(z) = \dfrac{z}{2} - \dfrac{\left(\dfrac{z}{2}\right)^3}{1^2 2} + \dfrac{\left(\dfrac{z}{2}\right)^5}{1^2 2^2 3^2} + \dots, \end{cases} \tag{21}$$

From (20), after some calculations, we find

$$\lambda = 4\sqrt{\gamma *} \tag{22}$$

Equation (20) is transcendental and has an infinite set of roots λ_n Summation over eigenvalues λ_n Gives formulas for pressure, radial and axial velocities

$$p - p_c = \sum_{n=1}^{\infty} J_0\left(\frac{\lambda_n}{R} r\right)\left[A_n \exp\left(\frac{\lambda_n}{R} x\right) + B_n \exp\left(-\frac{\lambda_n}{R} x\right)\right]$$

$$\begin{cases} v_x = \sum\limits_{n=1}^{\infty} \dfrac{R}{2\mu}\left\{\dfrac{r}{R} J_1\left(\dfrac{\lambda_n}{R} r\right) - \dfrac{J_1(\lambda_n)}{J_0(\lambda_n)} J_0\left(\dfrac{\lambda_n}{R} r\right)\right\} \times \left\{A_n \exp\left(\dfrac{\lambda_n}{R} x\right) - B_n \exp\left(-\dfrac{\lambda_n}{R} x\right)\right\} \\ v_r = \sum\limits_{n=1}^{\infty} \dfrac{R}{2\mu\lambda_n}\left\{\dfrac{\lambda_n r}{R} J_0\left(\dfrac{\lambda_n}{R} r\right) + \left[\dfrac{\lambda_n J_1(\lambda_n)}{J_0(\lambda_n)} - 2\right] J_0\left(\dfrac{\lambda_n}{R} r\right)\right\} \times \\ \times \left\{A_n \exp\left(\dfrac{\lambda_n}{R} x\right) + B_n \exp\left(-\dfrac{\lambda_n}{R} x\right)\right\} \end{cases} \tag{23}$$

Where

$$\begin{cases} A_n = \dfrac{\lambda_n(\bar{p} - p_c)}{4J_1(\lambda)} - \dfrac{\dfrac{\mu\lambda_n^{\ 2} Q_0}{\pi R^3}}{2\left(\left(\lambda_n J_0(\lambda_n) + J_1(\lambda_n)\left[\dfrac{\lambda_n J_1(\lambda_n)}{J_0(\lambda_n)} - 2\right]\right)\right)} \\ B_n = \dfrac{\lambda_n(\bar{p} - p_c)}{4J_1(\lambda_n)} + \dfrac{\dfrac{\mu\lambda_n^{\ 2} Q_0}{\pi R^3}}{2\left(\left(\lambda_n J_0(\lambda_n) + J_1(\lambda_n)\left[\dfrac{\lambda_n J_1(\lambda_n)}{J_0(\lambda_n)} - 2\right]\right)\right)}. \end{cases} \tag{24}$$

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Formulas (23) allow us to investigate the characteristics of the internal structure of the flow of viscous blood in a pipe with permeable walls: hydrodynamic resistances on the permeable wall, fluid flow along the length of the tube, and a number of other hydrodynamic parameters. Using (23), we find the distribution of pressure drop, blood flow and shear stress on the wall:

$$-\frac{\partial p}{\partial x} = -\sum_{n=1}^{\infty} J_0\left(\frac{\lambda_n}{R} r\right)\left[A_n \frac{\lambda_n}{R}\exp\left(\frac{\lambda_n}{R}x\right) - \frac{\lambda_n}{R}B_n \exp\left(-\frac{\lambda_n}{R}x\right)\right] \qquad (25)$$

$$Q = 2\pi\int_0^R r v_x(r,x)dr =$$

$$= \frac{\pi R^3}{\mu}\sum_{n=1}^{\infty}\left\{\frac{1}{\lambda_n^{\ 2}} J_0(\lambda_n)\times\left\{\lambda + \frac{J_1(\lambda_n)}{J_0(\lambda_n)}\left[\frac{J_1(\lambda_n)\lambda_n}{J_0(\lambda_n)} - 2\right]\right\}\left(B_n \exp\left(-\frac{\lambda_n}{R}x\right) - A_n \exp\left(\frac{\lambda_n}{R}x\right)\right)\right\} (26)$$

$$\tau_{average} = \frac{1}{L}\int_0^L \mu\left(\frac{\partial v_x}{\partial r} + \frac{\partial v_r}{\partial x}\right)\Big|_{r=R} dx =$$

$$= \sum_{n=1}^{\infty}\left\{\frac{R}{L\lambda_n} J_0(\lambda_n)\left(2\lambda_n\gamma * + \frac{J_1(\lambda_n)}{J_0(\lambda_n)}\right)\left[\left(A_n \exp\left(\frac{\lambda_n}{R}L\right) + B_n \exp\left(-\frac{\lambda_n}{R}L\right) - (A_n + B_n)\right)\right]\right\} (27)$$

Using formulas (25) and (26), we determine the ratio of the pressure drop per unit length to the blood flow:

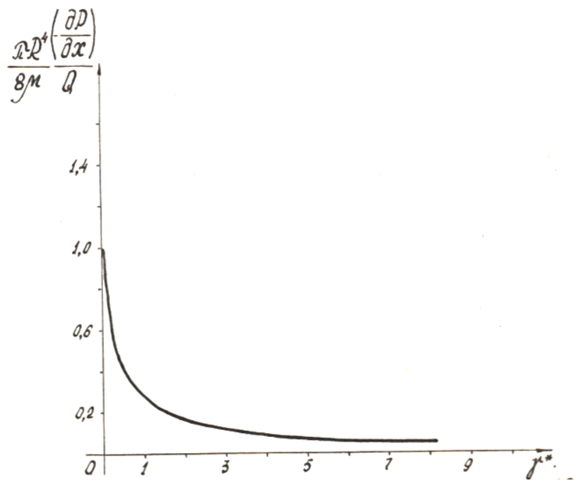

Figure1. The ratio of the pressure drop per unit length to the blood flow

$$\frac{\left(-\frac{\partial \overline{p}}{\partial x}\right)}{Q} = \frac{8\mu}{\pi R^4}\sum_{n=1}^{\infty}\frac{\lambda_n}{8\gamma *}\frac{J_1(\lambda_n)}{J_0(\lambda_n)} (28)$$

Or, in a dimensionless form,

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$$\frac{\pi R^4}{8\mu}\frac{\left(-\frac{\partial\overline{p}}{\partial x}\right)}{Q}=\sum_{n=1}^{\infty}\frac{\lambda_n}{8\gamma *}\frac{J_1(\lambda_n)}{J_0(\lambda_n)} \quad (29)$$

Conclusion

This relation in the literature is called the effective impedance or impedance. According to (29), the effective impedance at a stationary flow in a permeable pipe essentially depends on the permeability coefficient: with an increase in this coefficient, it decreases (Fig. With pulsating blood flow in the arterial channel, where the flow of blood is mathematically modeled as a permeable wall of blood vessels. Here for a normal person, the permeability coefficient is approximately equal to 0.1 unit. For this case, it can be seen from Fig. 1 that in the arterial vessel the hydraulic resistance decreases due to the permeability of the wall somewhere 35-45 percent.

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