Annals of R.S.C.B., ISSN:1583-6258, Vol. 26, Issue 1, 2022, Pages. 2589 - 2597
Received 08 November 2021; Accepted 15 December 2021.

An Inventory Model for Time Dependent Decaying Items with Exponential Demand under Partial Backlogging

Chandra Prakash Ban
cpgban@gmail.com
Department of Mathematics
Deshbandhu College, University of Delhi

ABSTRACT

In this paper, we considered a deterministic inventory model for production of deteriorating items developed by considering demand dependent production and time dependent demand. It is also assumed that shortages are allowed with partial backlogging. The result is illustrated with numerical example and sensitivity analysis is carried out to discuss the feasibility and applicability of the model.

Keywords: Production Inventory, Time Dependent, Partial backlogging, Decaying Items.

1 Introduction

The recent years there is a state of interest of studying time dependent demand rate. It is observed that the demand rate of newly launched products such as electronics items, mobile phones and fashionable garments increases with time and later it becomes constant. Deterioration of items cannot be avoided in business scenarios. In most of the cases the demand for items increases with time and the items that are stored for future use always loose part of their value with passage of time. In inventory this phenomenon is known as deterioration of items. The rate of deterioration is very small in some items like hardware, glassware, toys and steel. The items such as medicine, vegetables, gasoline alcohol, radioactive chemicals and food grains deteriorate rapidly over time so the effect of deterioration of physical goods cannot be ignored in many inventory systems. The deterioration of goods is a realistic phenomenon in many inventory systems and controlling of deteriorating items becomes a measure problem in any inventory system. Due to deterioration the problem of shortages occurs in any inventory system and shortage is a fraction that is not available to satisfy the demand of the customers in a given period of time.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 26, Issue 1, 2022, Pages. 2589 - 2597
Received 08 November 2021; Accepted 15 December 2021.

2 Literature Review

In the classical inventory model the deterioration is assumed to be a constant. Ghare and Schrader (1963), first derived an EOQ model for exponential decaying inventory with constant deterioration rate. Covert and Philip (1973), extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution. Tadikmalla (1978), developed an economic order quantity model with gamma distributed deterioration rate. Various types of inventory models for items deteriorating at a constant rate were discussed by Roy chowdhury and Chaudhuri (1983), Padmanabhan and Vrat (1995), and Balkhi and Benkherouf (1996), etc. Wee and Wang (1999), studied a production-inventory model for deteriorating items with time varying demand, finite production rate and shortages, over a known planning horizon. In (2001), Wee and Law studied an EOQ model with Weibull deterioration, price dependent demand considering the time value of money. Goyal and Giri (2003), considered the production-inventory problem in which the demand, production and deterioration rates of a product were assumed to vary with time. Shortages of a cycle were allowed to be partially backlogged. The demand was taken to be linear time-varying function. An economic production quantity model for deteriorating items was discussed by Teng and Chang (2005), Sugapriya (2008), studied an EPQ model for non-instantaneous deteriorating item in which holding cost varies with time. It is a production problem of non-instantaneous deteriorating item in which production and demand rate are constant. Manna, Lee and Chiang (2009), developed an EOQ model for non-instantaneous deteriorating items with demand rate as time-dependent. In the model, shortages are allowed and partially backlogged. Singh, Singhal and Gupta (2010), presented a production model for decaying items with multi-variate demand and partial backlogging. Most of the researchers considered the constant production and constant demand rates in their inventory models, which is not realistic. In real business situation, production and demand rate can be variable.

3 Assumptions And Notations

In order to find mathematical form of inventory model under consideration, the following assumptions along with notations are used:

3.1 Assumptions

a. Production rate is taken as demand dependent.

b. Demand rate is taken as exponential.

c. Deterioration rate is time dependent.

d. Shortages are allowed and partially backlogged.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 26, Issue 1, 2022, Pages. 2589 - 2597
Received 08 November 2021; Accepted 15 December 2021.

e. Backlogging rate is waiting time for the next replenishment.

f. Lead time is zero.

3.2 Notations

ae^{bt} : Exponential demand rate

θt : Time dependent deterioration rate

Kae^{bt} : Demand dependent production

$\frac{1}{1+\delta(T-t)}$: Backlogging rate

C_1 : Holding cost per unit per unit time

C_d : Deterioration cost per unit per unit time

C_S : Shortages cost per unit per unit time

C_{LS} : Lost sale cost per unit per unit time

$C^{'}$: Set up cost

4 Development of the Model

The initial inventory of the cycle is zero and production starts at the very beginning of the cycle. As production continues, inventory begins to pile up continuously after meeting demand and deterioration. Production stops at time t_1. The accumulated inventory is just sufficient enough to account for demand and deterioration over the time interval $[t_1, t_2]$. Shortage starts after t_2 with the concept of partial backlogging and reach to maximum shortage level at time t_3. Production restarts after t_3 to fulfill the backlog and demand and the cycle ends with zero inventory. Therefore, the differential equations are given as:

$$\frac{dI(t)}{dt}+\theta tI(t)dt=(K-1)ae^{bt} \quad 0\le t\le t_1 \tag{1}$$

$$\frac{dI(t)}{dt}+\theta tI(t)dt=-ae^{bt} \quad t_1\le t\le t_2 \tag{2}$$

$$\frac{dI(t)}{dt}=-\frac{1}{1+\delta(T-t)}ae^{bt} \quad t_2\le t\le t_3 \tag{3}$$

$$\frac{dI(t)}{dt}=(K-1)ae^{bt} \quad t_3\le t\le T \tag{4}$$

With the boundary conditions: $I(0)=0, I(t_2)=0, I(T)=0$

Solution of equations (1), (2), (3) and (4) are

Annals of R.S.C.B., ISSN:1583-6258, Vol. 26, Issue 1, 2022, Pages. 2589 - 2597
Received 08 November 2021; Accepted 15 December 2021.

$$I(t) = a(K-1)[t + \frac{bt^2}{2} + \frac{\theta t^3}{6}]e^{-\frac{\theta t^2}{2}} \qquad 0 \le t \le t_1 \tag{5}$$

$$I(t) = a[(t_2 - t) + \frac{b(t_2{}^2 - t^2)}{2} + \frac{\theta(t_2{}^3 - t^3)}{6}]e^{-\frac{\theta t^2}{2}} \quad t_1 \le t \le t_2 \tag{6}$$

$$I(t) = a[(t_2 - t) - \delta\{T(t_2 - t) - \frac{(t_2{}^2 - t^2)}{2}\} - b\delta\{\frac{T(t_2{}^2 - t^2)}{2} - \frac{(t_2{}^3 - t^3)}{3}\}] \qquad t_2 \le t \le t_3$$

(7)

$$I(t) = \frac{a(K-1)}{b}[e^{bt} - e^{bT}] \qquad t_3 \le t \le T$$

(8)

From equations (5) and (6), we get:

$$[t_2 + \frac{bt_2{}^2}{2} + \frac{\theta t_2{}^3}{6}] = K[t_1 + \frac{bt_1{}^2}{2} + \frac{\theta t_1{}^3}{6}] \tag{9}$$

Where t_2 is a function of t_1.

$$\frac{dt_2}{dt_1} = \frac{K[1 + bt_1 + \frac{\theta t_1{}^2}{2}]}{[1 + bt_2 + \frac{\theta t_2{}^2}{2}]} \tag{10}$$

From equations (7) and (8), we get:

$$a[(t_2 - t_3) - \delta\{T(t_2 - t_3) - \frac{(t_2{}^2 - t_3{}^2)}{2}\} - b\delta\{\frac{T(t_2{}^2 - t_3{}^2)}{2} - \frac{(t_2{}^3 - t_3{}^3)}{3}\}] = I(t) = a(K-1)[t_3 - T]$$

(11)

Where, t_3 is a function of T.

Holding cost occurs during the interval $(0, t_2)$ is:

$$H.C. = C_1[\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt]$$

$$= C_1[a(K-1)\{\frac{t_1{}^2}{2} + \frac{bt_1{}^3}{6} - \frac{\theta t_1{}^4}{12} - \frac{b\theta t_1{}^5}{20} - \frac{\theta^2 t_1{}^6}{72}\} + a\{\frac{t_2{}^2}{2} - t_2 t_1 + \frac{t_1{}^2}{2} + \frac{2bt_2{}^3}{3} - \frac{bt_2{}^2 t_1}{2}$$

$$- \frac{bt_1{}^3}{6} - \frac{\theta t_2{}^3 t_1}{6} + \frac{\theta t_2{}^4}{12} - \frac{\theta t_1{}^4}{12} + \frac{\theta t_2 t_1{}^3}{6}$$

$$- \frac{b\theta t_2{}^5}{30} + \frac{b\theta t_2{}^2 t_1{}^3}{12} - \frac{b\theta t_1{}^5}{20} - \frac{\theta^2 t_2{}^6}{72} + \frac{\theta^2 t_2{}^3 t_1{}^3}{36} - \frac{\theta^2 t_1{}^6}{72}\}] \tag{12}$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 26, Issue 1, 2022, Pages. 2589 - 2597
Received 08 November 2021; Accepted 15 December 2021.

Deterioration cost occurs during the interval (0, t_2) is:

$$D.C. = C_d[\int_0^{t_1} \theta t I(t)dt + \int_{t_1}^{t_2} \theta t I(t)dt]$$

$$= C_d\theta[a(K-1)\{\frac{t_1^3}{3} + \frac{bt_1^4}{8} - \frac{\theta t_1^5}{15} - \frac{b\theta t_1^6}{24} - \frac{\theta^2 t_1^7}{84}\}$$

$$+a\{\frac{t_2^3}{6} - \frac{t_2 t_1^2}{2} + \frac{t_1^3}{3} + \frac{3bt_2^4}{8} - \frac{bt_2^2 t_1^2}{4}$$

$$-\frac{bt_1^4}{8} - \frac{\theta t_2^5}{24} - \frac{\theta t_2^3 t_1^2}{12} + \frac{\theta t_2^5}{15} - \frac{\theta t_1^5}{15} + \frac{\theta t_2 t_1^4}{8}$$

$$-\frac{b\theta t_2^6}{48} + \frac{b\theta t_2^2 t_1^4}{16} - \frac{b\theta t_1^6}{24} - \frac{\theta^2 t_2^7}{112} + \frac{\theta^2 t_2^3 t_1^4}{48} - \frac{\theta^2 t_1^7}{84}\}] \quad (13)$$

Shortage cost occurs during the interval (t_2, T) is:

$$S.C. = C_S[\int_{t_2}^{t_3} (-I(t))dt + \int_{t_3}^{T} (-I(t))dt]$$

$$= C_S[a\{\frac{t_3^2}{2} - \frac{t_2^2}{2} - t_2 t_3 - t_2^2 + \delta(Tt_2 t_3 - Tt_2^2 - \frac{Tt_3^2}{2} + \frac{Tt_2^2}{2} - \frac{t_2^2 t_3}{2}$$

$$+\frac{t_2^3}{3} - \frac{t_3^3}{6}) + b\delta(\frac{Tt_2^2 t_3}{2} - \frac{Tt_2^3}{3} - \frac{Tt_3^3}{6} - \frac{t_2^3 t_3}{3} + \frac{t_2^4}{4} + \frac{t_3^4}{12})\}$$

$$+\frac{a(K-1)}{b}\{\frac{bT^2}{2} - bTt_3 + \frac{b^2 T^3}{3} - \frac{b^2 T^2 t_3}{2} + \frac{bt_3^2}{2} + \frac{b^2 t_3^3}{6}\}]$$

(14)

Lost sale cost occurs during the interval (t_2, t_3) is:

$$L.S.C. = C_{LS}[\int_{t_2}^{t_3} (1 - \frac{1}{1+\delta(T-t)})ae^{bt}dt]$$

$$= aC_{LS}[\delta(Tt_3 - Tt_2 - \frac{t_3^2}{2} + \frac{t_2^2}{2}) + b\delta(\frac{Tt_3^2}{2} - \frac{Tt_2^2}{2} - \frac{t_3^3}{3} + \frac{t_2^3}{3})]$$

(15)

Total average cost of the system during (0,T) is:

$$T.C. = \frac{1}{T}[H.C. + D.C. + S.C. + L.S.C. + St.C.]$$

$$= \frac{1}{T}[C_1[a(K-1)\{\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\theta t_1^4}{12} - \frac{b\theta t_1^5}{20} - \frac{\theta^2 t_1^6}{72}\}$$

$$+a\{\frac{t_2^2}{2} - t_2 t_1 + \frac{t_1^2}{2} + \frac{2bt_2^3}{3} - \frac{bt_2^2 t_1}{2} - \frac{bt_1^3}{6} - \frac{\theta t_2^3 t_1}{6} + \frac{\theta t_2^4}{12} - \frac{\theta t_1^4}{12} + \frac{\theta t_2 t_1^3}{6}$$

Annals of R.S.C.B., ISSN:1583-6258, Vol. 26, Issue 1, 2022, Pages. 2589 - 2597
Received 08 November 2021; Accepted 15 December 2021.

$$-\frac{b\theta t_2^{\,5}}{30}+\frac{b\theta t_2^{\,2}t_1^{\,3}}{12}-\frac{b\theta t_1^{\,5}}{20}-\frac{\theta^2 t_2^{\,6}}{72}+\frac{\theta^2 t_2^{\,3}t_1^{\,3}}{36}-\frac{\theta^2 t_1^{\,6}}{72}\}]$$

$$+C_d\theta[a(K-1)\{\frac{t_1^{\,3}}{3}+\frac{bt_1^{\,4}}{8}-\frac{\theta t_1^{\,5}}{15}-\frac{b\theta t_1^{\,6}}{24}-\frac{\theta^2 t_1^{\,7}}{84}\}+a\{\frac{t_2^{\,3}}{6}-\frac{t_2 t_1^{\,2}}{2}+\frac{t_1^{\,3}}{3}$$

$$+\frac{3bt_2^{\,4}}{8}-\frac{bt_2^{\,2}t_1^{\,2}}{4}-\frac{bt_1^{\,4}}{8}-\frac{\theta t_2^{\,5}}{24}-\frac{\theta t_2^{\,3}t_1^{\,2}}{12}+\frac{\theta t_2^{\,5}}{15}-\frac{\theta t_1^{\,5}}{15}+\frac{\theta t_2 t_1^{\,4}}{8}$$

$$-\frac{b\theta t_2^{\,6}}{48}+\frac{b\theta t_2^{\,2}t_1^{\,4}}{16}-\frac{b\theta t_1^{\,6}}{24}-\frac{\theta^2 t_2^{\,7}}{112}+\frac{\theta^2 t_2^{\,3}t_1^{\,4}}{48}-\frac{\theta^2 t_1^{\,7}}{84}\}]$$

$$+C_S[a\{\frac{t_3^{\,2}}{2}-\frac{t_2^{\,2}}{2}-t_2t_3-t_2^{\,2}+\delta(Tt_2t_3-Tt_2^{\,2}-\frac{Tt_3^{\,2}}{2}+\frac{Tt_2^{\,2}}{2}-\frac{t_2^{\,2}t_3}{2}$$

$$+\frac{t_2^{\,3}}{3}-\frac{t_3^{\,3}}{6})+b\delta(\frac{Tt_2^{\,2}t_3}{2}-\frac{Tt_2^{\,3}}{3}-\frac{Tt_3^{\,3}}{6}-\frac{t_2^{\,3}t_3}{3}+\frac{t_2^{\,4}}{4}+\frac{t_3^{\,4}}{12})\}$$

$$+\frac{a(K-1)}{b}\{\frac{bT^2}{2}-bTt_3+\frac{b^2T^3}{3}-\frac{b^2T^2t_3}{2}+\frac{bt_3^{\,2}}{2}+\frac{b^2t_3^{\,3}}{6}\}]$$

$$+aC_{LS}[\delta(Tt_3-Tt_2-\frac{t_3^{\,2}}{2}+\frac{t_2^{\,2}}{2})+b\delta(\frac{Tt_3^{\,2}}{2}-\frac{Tt_2^{\,2}}{2}-\frac{t_3^{\,3}}{3}+\frac{t_2^{\,3}}{3})]+C^{'}]$$

(16)

5 Numerical example

For the computation point of view, following values of parameters are used:

K = 3, C' = Rs. 240 per order,

C_1 = Rs. 1.7 per unit per month, C_S = Rs. 5.0 per unit per month,

C_S = Rs. 5.0 per unit per month, C_{LS} = Rs. 2.8 per unit, a = 100, b = 0.6 unit,

t_2 = 3 months, θ = 0.04 unit, δ = 0.06 unit

The optimum solutions are: t_1=2.271, t_3=4.682, T=6.639, TC=145.26

6 Sensitivity Analysis

In order to study how various costs affect the optimal solution of underlying inventory model, Cost sensitivity analysis is performed the values of change in various costs areas:

Δ_1 = 3, Δ_2 = 3.5, Δ_3 = 4.5,

Δ_4 = 4.9, Δ_5 = 5, Δ_6 = 5.5,

Δ_7 = 3, Δ_8 = 2.5

The results of the sensitivity analysis with these parameters are described in following table 1:

Annals of R.S.C.B., ISSN:1583-6258, Vol. 26, Issue 1, 2022, Pages. 2589 - 2597
Received 08 November 2021; Accepted 15 December 2021.

Table 1

Sensitivity Analysis of Optimal Solution w.r.t. System Parameters

Parameter	% Change	-30%	-20%	-10%	10%	20%	30%
Δ_1	**TC**	2.772	1.894	1.392	-1.392	-1.894	-2.772
Δ_2	**TC**	-4.541	-3.682	-2.309	+2.309	+3.682	+4.541
Δ_3	**TC**	0.576	0.352	0.231	-0.231	-0.352	-0.576
Δ_4	**TC**	-0.456	-0.128	-0.112	+0.112	+0.128	+0.456
Δ_5	**TC**	0.051	0.039	0.028	-0.028	-0.039	-0.051
Δ_6	**TC**	-0.068	-0.042	-0.016	+0.016	+0.042	+0.068
Δ_7	**TC**	+0.179	+0.154	+0.122	-0.122	-0.154	-0.179
Δ_8	**TC**	-0.245	-0.216	-0.189	+0.189	+0.216	+0.245

7 Observations

From the table 1 it has been observed that

1. As holding cost interval increases in left i.e., Δ_1 increase. Holding cost decreases, TC decreases
2. As Δ_2 increases, total cost is increases and as Δ_2 decreases, total cost is decreases.
3. On increasing the Δ_3, total cost is decreases and on decreasing the Δ_3, total cost is decreases.
4. If Δ_4 is increases then total cost is increases and if Δ_4 is decreases then total cost is decreases.
5. As Δ_5 increases, total cost is decreases and as Δ_5 decreases, total cost is increases.
6. On increasing the Δ_6, total cost is increases and on decreasing the Δ_3, total cost is decreases.
7. Δ_7 as increases, total cost is decreases and Δ_7 as decreases, total cost is increases.
8. As Δ_8 increases, total cost is increases and as Δ_8 decreases, total cost is decreases.

8 Conclusion

The Inventory model for time dependent decaying item with exponential demand has been considered. A numerical assessment of the theoretical model has been carried out to illustrate the theory and solution has also been for sensitivity analysis with the result of the

Annals of R.S.C.B., ISSN:1583-6258, Vol. 26, Issue 1, 2022, Pages. 2589 - 2597
Received 08 November 2021; Accepted 15 December 2021.

model are found quite suitable and stable. The variations in the system statistics with a variation in system parameters has also been illustrated graphically. All these facts together make this study very unique and matter-of-fact.

References

[1] Tadikamalla, P.R. (1978), "An EOQ inventory model for items with Gamma distribution", AIIE Transaction 5, 100–103.

[2] J.T. Teng, L.Y. Ouyang and C.T. Chang (2005), Deterministic economic production quantity models with time-varying demand and cost, Applied Mathematical Modeling, 29, 10, 987-1003.

[3] Wee, H. M., Wang W.T. (1999), "A variable production scheduling policy for deteriorating items with time varying demand"' Computers and Operations Research 26, 237-254.

[4] Wee, H.M. and Law, S-T. (2001), **"**Replenishment and pricing policy for deteriorating items taking into account the time value of money:" International Journal of Production Economics, 71, 213-220.

[5] S.K. Manna, C.C. Lee, C. Chiang (2009), "EOQ model for non-instantaneous deteriorating items with time-varying demand and partial backlogging"**,** International Journal of Industrial and Systems Engineering, 4, 3, 241 – 254.

[6] Ghare, P. M., Schrader, G. F., (1963), "A model for exponential decaying inventory. Journal of Industrial Engineering" 14, 238–243.

[7] Giri, B.C., Jalan, A.K., Chaudhuri, K.S. (2003), **"**Economic Order Quantity model with Weibull deterioration distribution, shortage and ramp-type demand. International Journal of Systems Science", 34 (4), 237-243.

[8] Covert, R. P., Philip, G.C., (1973), "An EOQ model for items with Weibull distribution deterioration",AIIE Transaction 5, 323–326.

[9] C. Sugapriya and K. Jeyaraman (2008), "An EPQ model for non-instantaneous deteriorating item in which holding cost varies with time", Electronic Journal of Applied Statistical Analysis, 1, 19 – 27.

[10] Balkhi, Z.T. and Benkherouf, L. (1996), "A production lot size inventory model for deteriorating items and arbitrary production and demand rate". E.J.O.R., 92, 302-309.

[11] Padmanabhan, G. and Vrat, P. (1995), "EOQ models for perishable items under stock dependent selling rate". E.J.O.R., 86, 281-292.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 26, Issue 1, 2022, Pages. 2589 - 2597
Received 08 November 2021; Accepted 15 December 2021.

[12] Roychowdhury, M. and Chaudhuri, K.S. (1983), "An order level inventory model for deteriorating items with finite rate of replenishment". Opsearch, 20, 99-106.

[13] Singh, S.R., Singhal, S. and Gupta, P.K. (2010), "A volume flexible inventory model for Defective Items with Multi-Variate Demand and Partial Backlogging", International Journal of Operations Research and Optimization, 1, 4, 54-68.