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Study on Fuzzy Game Problem in Icosikaitetragonal Fuzzy Number

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ABSTRACT

In this article, we bring in the fuzzy game problem by Icosikaitetragonal fuzzy number. We generate the value of Payoff matrix by Icosikaitetragonal fuzzy number. We modify the fuzzy game problem into crisp valued game problem by using ranking to pay off. The crisp valued game problem can be solved by Oddment method and illustrations are exemplified

Keywords

Icosikaitetragonal fuzzy number, fuzzy game problem, fuzzy ranking, pay off matrix.

1.Introduction

Fuzzy set theory was introduced by Zadeh [1].The concept of fuzzy set theory reveals imprecision and vagueness. We make use of the fuzzy set almost in all business and also in our many day to day life activities. We bring together more information from the environment in each day is fuzzy. We are not able to move our daily life without crossing fuzzy connected circumstance. Fuzzy set is efficient in many ongoing world situations. Jain [2] was the first to propose method of ranking fuzzy numbers for decision making in fuzzy related situation. Raju and Jayagopal [3] was the first to introduce the Icosikaitetragonal fuzzy number. Game theory makes it easier to have interaction of decision makers between the scenario of co operation and spirited approach.

Game theory has extensive range of applications in various fields such as business model development, Economics, diplomacy and military strategy. In a game theory each player is to take good decision by selecting various strategies from the all available strategies. When uncertainties occur in game theory, fuzzy set is the best tool to study this kind of game in which pay off matrices is denoted by fuzzy numbers. In this paper, we have taken two person zero sum game, in which imprecise values are Icosikaitetragonal fuzzy numbers. We have made clear it with converting to crisp valued game problem using ranking technique. We have analyzed fuzzy game problem using Icosikaitetragonal fuzzy number with illustrations and solved the game problem with oddment method

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2. PRELIMINARIES

In this section, we give the preliminaries that are required for this study.

Definition 2.1. *A fuzzy set A is defined by* $A = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$. *Here x is crisp set A and $\mu_A(x)$ is membership function in the interval* $[0,1]$.

Definition 2.2.

The fuzzy number A is a fuzzy set whose membership function must satisfy the following conditions.

(i) A fuzzy set A of the universe of discourse X is convex
(ii) A fuzzy set A of the universe of discourse X is a normal fuzzy set if $x_i \in X$ exists
(iii) $\mu_A(x)$ is piecewise continuous

Definition 2.3.

A fuzzy number $A = (a, b, c)$, where $a \leq b \leq c$, is triangular fuzzy number and its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, for a \leq x \leq b \\ \frac{c-x}{c-b}, for b \leq x \leq c \\ 0, x > c \end{cases}$$

Definition 2.4

A fuzzy number $A = (a, b, c, d)$, where $a \leq b \leq c \leq d$, is trapezoidal fuzzy number and its membership function is given by

$$\mu_A(x) = \begin{cases} 0, for x < a \\ \frac{x-a}{b-a}, for a \leq x \leq b \\ 1, for b \leq x \leq c \\ \frac{d-x}{d-c}, for c \leq x \leq d \\ 0, x > d \end{cases}$$

Definition 2.5

An α-cut of fuzzy set A is classical set defined as ${}^{\alpha}A = \{x \in X | \mu_A(x) \geq \alpha\}$

Definition 2.6

A fuzzy set A is a convex fuzzy set iff each of its α-cut ${}^{\alpha}A$ *is a convex set.*

Definition 2.7

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Game theory provides a mathematical framework for analyzing the decision-making processes and strategies of adversaries (or *players*) in different types of competitive situations. The simplest type of competitive situations is two-person, zero-sum games. These games involve only two players; they are called *zero-sum* games because one player wins whatever the other player loses.

Definition 2.8 [3] A fuzzy number A = $\left(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, ... a_{24}\right)$ is Icosikaitetragonal fuzzy number and its membership function is given by

$$\mu_A(x) = \begin{cases} 0,\ for\ x < a_1 \\ k_1\left(\dfrac{x-a_1}{a_2-a_1}\right),\ for\ a_1 \le x \le a_2 \\ k_1,\ for\ a_2 \le x \le a_3 \\ k_1 + (k_2 - k_1)\left(\dfrac{x-a_3}{a_4-a_3}\right),\ for\ a_3 \le x \le a_4 \\ k_2,\ for\ a_4 \le x \le a_5 \\ k_2 + (k_3 - k_2)\left(\dfrac{x-a_5}{a_6-a_5}\right),\ for\ a_5 \le x \le a_6 \\ k_3,\ a_6 \le x \le a_7 \\ k_3 + (k_4 - k_3)\left(\dfrac{x-a_7}{a_8-a_7}\right),\ for\ a_7 \le x \le a_{8/} \\ k_4,\ for\ a_8 \le x \le a_9 \\ k_4 + (k_5 - k_4)\left(\dfrac{x-a_9}{a_{10}-a_9}\right),\ for\ a_9 \le x \le a_{10} \\ k_5,\ for\ a_{10} \le x \le a_{11} \\ k_5 + (1 - k_5)\left(\dfrac{x-a_{11}}{a_{12}-a_{11}}\right),\ for\ a_{11} \le x \le a_{12} \\ 1,\ for\ a_{12} \le x \le a_{13} \\ k_5 + (1 - k_5)\left(\dfrac{a_{14}-x}{a_{14}-a_{13}}\right),\ for\ a_{13} \le x \le a_{14} \\ k_5,\ for\ a_{14} \le x \le a_{15} \\ k_4 + (k_5 - k_4)\left(\dfrac{a_{16}-x}{a_{16}-a_{15}}\right),\ for\ a_{15} \le x \le a_{16} \\ k_4,\ for\ a_{16} \le x \le a_{17} \\ k_3 + (k_4 - k_3)\left(\dfrac{a_{18}-x}{a_{18}-a_{17}}\right),\ for\ a_{17} \le x \le a_{18} \\ k_3,\ for\ a_{18} \le x \le a_{19} \\ k_2 + (k_3 - k_2)\left(\dfrac{a_{20}-x}{a_{20}-a_{19}}\right),\ for\ a_{19} \le x \le a_{20} \\ k_2,\ for\ \quad a_{20} \le x \le a_{21} \\ k_1 + (k_2 - k_1)\left(\dfrac{a_{22}-x}{a_{22}-a_{21}}\right),\ for\ a_{21} \le x \le a_{22} \\ k_1,\ for\ a_{22} \le x \le a_{23} \\ k_1\left(\dfrac{a_{24}-x}{a_{24}-a_{23}}\right),\ for\ a_{23} \le x \le a_{24} \\ 0,\ for\ x > a_{24} \end{cases}$$

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3. Mathematical formulation of Fuzzy Game problem:

Consider a two person zero sum game in which all the entries of the payoff matrix are Icosikaitetragonal fuzzy numbers. Let us take the player P has m strategies and player Q has n strategies. We will assume that each player has to choose from the pure strategies. Player P is always considered to be gainer and the player Q is always loser. Then the payoff matrix m x n is

$$G= \begin{pmatrix} p_{11} & p_{12} & . & p_{1n} \\ p_{21} & p_{22} & . & p_{2n} \\ . & . & . & . \\ p_{m1} & p_{m2} & . & p_{mn} \end{pmatrix}$$

3.1 Numerical Examples:

Consider the fuzzy game problem with payoff matrix as Icosikaitetragonal fuzzy numbers. This problem is worked out by taking the values $k_1 = \frac{1}{6}, k_2 = \frac{2}{6}, k_3 = \frac{3}{6}, k_4 = \frac{4}{6}, k_5 = \frac{5}{6}$

We get the values of $\mu_{Icsktetra}\ (a_{ij})$

a_{11}	-13,-12,-11,-10,-9,-8,-7,-6,-4,-3,-2, 0,1,2,4,6,8,10,12,14,16,18,20,22	$\mu_{Icsktetra}\ (a_{11}) = 2$
a_{12}	1,2,3,4,5,6,7,8,9,10,11,12,13,14, 15,16,17,18,19,20,21,22,23,24	$\mu_{Icsktetra}\ (a_{12}) = 12.5$
a_{21}	-12,-11,,-10,-9,-8,-7,-6,-5,-4,-3, -2,-1,0,3,5,6,7,8,10,12,14,15,16,18	$\mu_{Icsktetra}\ (a_{21}) = 1.67$
a_{22}	2,4,6,8,10,12,14,16,18,20,22,24,26, 28,30,32,34,36,38,40,42,44,46,48	$\mu_{Icsktetra}\ (a_{22}) = 25$
a_{31}	0,1,2,3,4,5,6,7,8,9,10,11,13,15, 17,19,21,23,25,27,29,30,31,33	$\mu_{Icsktetra}\ (a_{31}) = 14.54$
a_{32}	1,3,5,7,9,11,13,15,17,19,21,23,25, 27,29,31,33,35,37,39,41,43,45,47	$\mu_{Icsktetra}\ (a_{32}) = 24$

Fuzzy game problem is reduced to the following payoff matrix $A=\begin{pmatrix} 2 & 12.5 \\ 1.67 & 25 \\ 14.54 & 24 \end{pmatrix}$

Minimum of first row is 2
Minimum of second row is 1.67
Minimum of third row is 14.54
Maximum of first column is 14.54
Maximum of second column is 25

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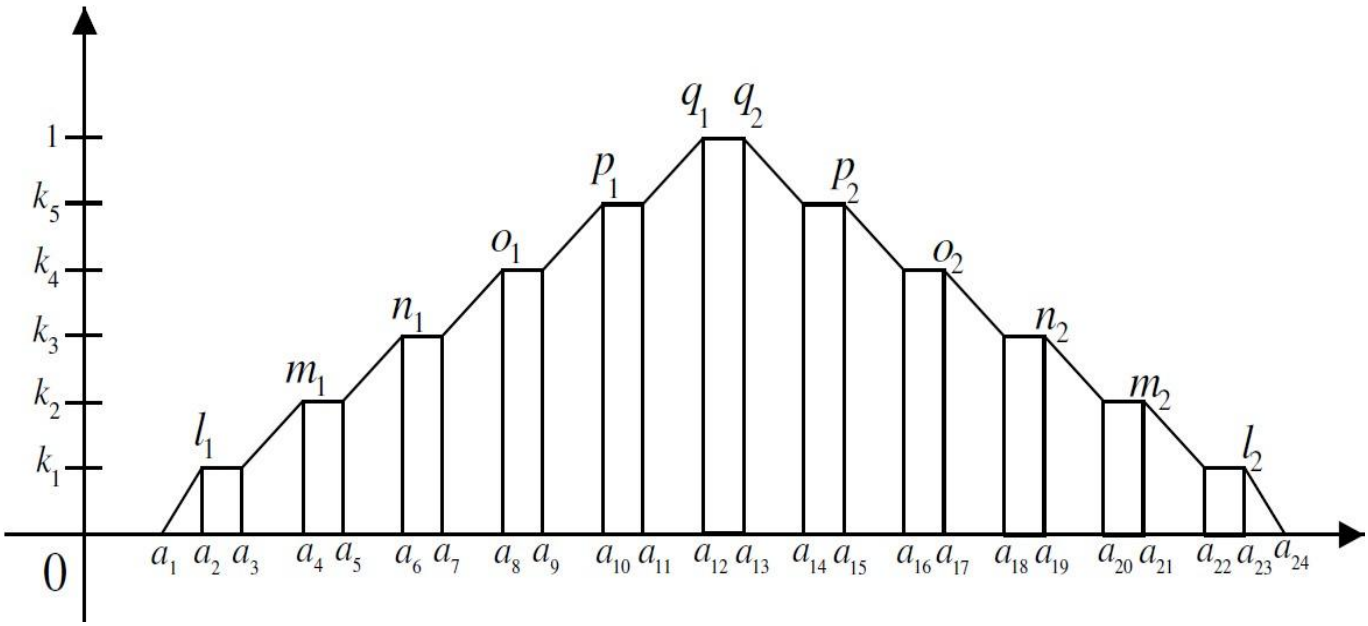
Max(min) = 14.54
Min(max) = 14.54
It has a saddle point
Value of the game is 14.54

3.2 Diagram of Icosikaitetragonal fuzzy number:



3.3 Ranking of Icosikaitetragonal fuzzy number:

Let I be a normal Icosikaitetragonal fuzzy number. The value $M\ (I)$, called as measure of I is calculated as

$$M(I)=\frac{1}{2}\int_{1}^{k_1}(\ell_1+\ell_2)d\ell+\frac{1}{2}\int_{k_1}^{k_2}(m_1+m_2)dm+\int_{k_2}^{k_3}(n_1+n_2)dn+\int_{k3}^{k_4}(o_1+o_2)do+\int_{k4}^{k_5}(p_1+p_2)dp+\int_{k5}^{1}(q_1+q_2)dq$$

$where\ \ 0\le k_1\le k_2\le k_3\le k_4\le k_5\le 1$

$$M(L)=\frac{1}{4}\left[\begin{array}{l}(a_1+a_2+a_{23}+a_{24})k_1+(a_3+a_4+a_{21}+a_{22})(k_2-k_1)+(a_5+a_6+a_{19}+a_{20})(k_3-k_2)+\\(a_7+a_8+a_{17}+a_{18})(k_4-k_3)+(a_9+a_{10}+a_{15}+a_{16})(k_5-k_4)+(a_{11}+a_{12}+a_{13}+a_{14})(1-k_5)\end{array}\right]$$

$where\ \ 0\le k_1\le k_2\le k_3\le k_4\le k_5\le 1$

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$$we\ take\ the\ values\ for\quad k_1=\frac{1}{6},k_2=\frac{2}{6},k_3=\frac{3}{6},k_4=\frac{4}{6},k_5=\frac{5}{6}$$

3.4 Numerical Examples:

Let us consider the matrix

$$\begin{pmatrix} \begin{pmatrix}-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,\\6,7,8,9,10,11,12,13,14,15,16,17\end{pmatrix} & \begin{pmatrix}-8,-7,-6,-5,-4,-3,-2,-1,0,1,\\2,3,4,5,6,7,8,9,10,11,12,13,14,15\end{pmatrix} & \begin{pmatrix}1,2,3,4,5,6,7,8,9,10,11,12,13,14,\\15,16,17,18,19,20,21,22,23,24\end{pmatrix} \\ \begin{pmatrix}-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,\\6,7,8,9,10,11,12,13,14,15,16,18\end{pmatrix} & \begin{pmatrix}-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,\\10,11,12,13,14,15,16,17,18,19\end{pmatrix} & \begin{pmatrix}0,1,2,4,5,6,7,8,9,10,11,12,13,14,\\16,18,20,22,24,26,27,28,29,30\end{pmatrix} \\ \begin{pmatrix}0,1,2,3,4,5,6,7,9,10,11,13,14,15,\\17,19,21,22,24,25,26,27,28,29\end{pmatrix} & \begin{pmatrix}1,2,3,6,8,9,10,12,13,15,16,17,19,20,\\22,23,25,28,30,32,34,35,37,39\end{pmatrix} & \begin{pmatrix}-5,-4,-3,-2,-1,0,1,2,3,4,5,67,\\8,9,10,11,12,13,14,15,16,17,18\end{pmatrix} \end{pmatrix}$$

Step 1:

We obtain the values of $\mu_{Icskoct}(\ a_{ij})$ of the given fuzzy game problem and convert the fuzzy game problem into crisp valued problem which is shown in the given table.

a_{11}	$-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,$ $6,7,8,9,10,11,12,13,14,15,16,17$	$\mu_R(\ a_{11}) = 5.5$
a_{12}	$-8,-7,-6,-5,-4,-3,-2,-1,0,1,$ $2,3,4,5,6,7,8,9,10,11,12,13,14,15$	$\mu_R(\ a_{12}) = 3.5$
a_{13}	$1,2,3,4,5,6,7,8,9,10,11,12,13,14,$ $15,16,17,18,19,20,21,22,23,24$	$\mu_R(\ a_{13}) = 12.5$
a_{21}	$-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,$ $6,7,8,9,10,11,12,13,14,15,16,18$	$\mu_R(\ a_{21}) = 5.54$
a_{22}	$-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,$ $10,11,12,13,14,15,16,17,18,19$	$\mu_R(\ a_{22}) = 7.5$
a_{23}	$0,1,2,4,5,6,7,8,9,10,11,12,13,14,$ $16,18,20,22,24,26,27,28,29,30$	$\mu_R(\ a_{23}) = 14.3$
a_{31}	$0,1,2,3,4,5,6,7,9,10,11,13,14,15,$ $17,19,21,22,24,25,26,27,28,29$	$\mu_R(\ a_{31}) =14.25$
a_{32}	$1,2,3,6,8,9,10,12,13,15,16,17,19,$ $20,22,23,25,28,30,32,34,35,37,39$	$\mu_R(\ a_{32}) = 19$
a_{33}	$-5,-4,-3,-2,-1,0,1,2,3,4,5,6$ $7,8,9,10,11,12,13,14,15,16,17,18$	$\mu_R(\ a_{33}) = 6.5$

Step 2 : The given fuzzy game problem is reduced to the following payoff matrix

B

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$$A \begin{pmatrix} 5.5 & 3.5 & 12.5 \\ 5.54 & 7.5 & 14.3 \\ 14.25 & 19 & 6.5 \end{pmatrix}$$

Minimum of first row = 3.5
Minimum of second row = 5.54
Minimum of third row = 6.5
Maximum of first column = 14.25
Maximum of second column = 19
Maximum of third column = 14.3
Max (min) = 6.5
Min (max) = 14.25
Max (min) ≠ Min (max)
6.5 ≠14.5
There is no saddle point

Step 3: To solve the value of the problem, we apply dominance method. Clearly first row is dominated by second row as all the elements of first row are less than second row. Hence eliminate first row, we get

$$\begin{matrix} & B \\ A & \begin{pmatrix} 5.54 & 7.5 & 14.3 \\ 14.25 & 19 & 6.5 \end{pmatrix} \end{matrix}$$

Again first column is dominated by second column. So eliminate first column.

$$\begin{matrix} & B \\ A & \begin{pmatrix} 7.5 & 14.3 \\ 19 & 6.5 \end{pmatrix} \end{matrix}$$

Now we obtain 2x2 payoff matrix. To solve the reduced matrix, which won't have any saddle point. So we apply oddment method. Hence the augmented payoff matrix is

$$\begin{matrix} & B \\ A & \begin{pmatrix} 7.5 & 14.3 \\ 19 & 6.5 \end{pmatrix} \end{matrix}$$

$(7.5P_1) + 14.3(1- P_1) = 19 P_1 + 6.5(1- P_1)$
$P_1= 0.4041$
$1- P_1 = 0.5959$

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$7.5\ Q_1 + 19\ (1- Q_1) = 14.3\ Q_1 + 6.5(1- Q_1)$
$Q_1 = 0.6477$
$1- Q_1 = 0.3523$

Strategy for B = (0.4041 ,0, 0.5959)
Strategy for A = (0.6477, 0, 0.3523)

Value of the game = $(7.5\ P_1) + 14.3(1- P_1)$
$= (7.5 \times 0.4041) + (14.3 \times 0.5959) = 11.55212$

Conclusion: In this article, we have examined and solved 3×3 fuzzy pay off matrix whose elements are Icosikaitetragonal fuzzy number. We have illustrated the optimal solution of the fuzzy valued game problem converting to crisp valued game problem using ranking techniques. The Crisp valued game problem is solved by oddment method

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