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Fixed Point Theorem with Fuzzy and Non Archimedean Space

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Abstract:
We started the work with , the reference to George and Veeramani, then we deduce some consequences in complete non Archimedean fuzzy metric space as an common fixed point theorem for non Archimedean fuzzy metric space.

Keywords: Fixed point, fuzzy metric space, non Archimedean fuzzy metric space

1. Introduction

Fixed point theory is an important area in the fast growing fields of non-linear analysis and non-linear operator. It is relatively young but fully developed area of research. Fixed point and fixed point theorem have always been a major theoretical tool in the field widely apart as Differential Equations, Topology, Economics, Game Theory, Dynamics, Optimal Control and Functional Analysis.

In the classical method, a set is characterized by its characteristics function which assigns the real number 1 for membership and 0 for non-membership for each element of any universe of discourse.

The characteristic function χ_A: X→ {0,1} is defined as

$$\chi_A = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases}.$$

here [0,1] is called the valuation set or membership set. In the classical set, the transition from membership to non-membership of every element is abrupt

Zadeh was introduced the concept of Fuzzy set in 1965, which laid the foundation of fuzzy mathematics. Many authors used this concept in topology and analysis and developed the theory of fuzzy sets and its applications. In 1975, Kramosil and Michalekwere introduced the concept of fuzzy metric space by generalizing the concept of probabilistic metric space. George and Veeramani were modified this concept of fuzzy metric space and defined Hausdorff topology on fuzzy metric space. Grabiec was extended the results of Kramosil and Michalek and obtained the fuzzy version of Banach Contraction Principle which is milestone in developing fixed point theory in fuzzy metric spaces.

The foundation of fuzzy mathematics was laid by Lofti A. Zadeh with the introduction of fuzzy sets in 1965, as a way to represent vagueness in everyday life.

2 Definition

2.1.A3-tuple (X, L ,*) is called a fuzzy metric space if X is an arbitrary (non-empty) set, * is continuous t-norm, and L is a Fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions :

(i) L(a,b, u) > 0.

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(ii) L(a, b, u) = 1 if and only if x = y.
(iii) L(a, b, t) = M(b, a, u).
(iv) L(a, b, u)* M(b, c, v) ≤ L(x, c, u + v).
(v) L(a, b, .) : $(0,\infty)\rightarrow[0,1]$ is continuous.
(vi) $\lim_{n\rightarrow\infty}$ M(a, b, u) = 1.

Note that it can be considered as the degree of nearness between x and y with respect to t. we identify a = b with L(a, b, u)= 1,
for all u> 0 and for all ab, c∈ X and v,u> 0.
Let (X, l) be a metric space, and let g*h = min{ g, h}.
Let L(a, b, u) = $u/(u + l(\llbracket a,b \rrbracket))$ for all a,b∈ X and u>0.
Then (X, L ,*) is a fuzzy metric L induced by l is called standard fuzzy metric space .
If we replace (iv) by (vi) L(a, b, t) ∗L(b, c, u) ≤ L(a, c, u), then the triple (X,L, ∗) is called a non-Archimedean fuzzy metric space.
It can be observed that L(a, b,·) is nondecreasing for all a, b∈ X, then (vj) is converted to
L(a, b, u) ∗L(b, c, v) ≤ M(a, c, max{v, u}), that shows (jv). Thus each non-Archimedean fuzzy metric space is a fuzzy metric space, if L(a,b, ·) is nondecreasing for all a, b∈ **X.**

3.Main Result:

Theorem:Let (Y,L, ∗) be a complete non-Archimedean fuzzy metric space and M be the triangular andone self map T : Y→ Y . Than there exist a function £ : $[0, +\infty[\times[0, +\infty[\rightarrow R$ and a lower semi-continuous function ϕ : Y → $[0, +\infty[$ such that

(£1) $£(1/(M(Ta,Tb,t)) - 1 + \phi(Ta) + \phi(Tb), 1/(M(a,b,u)) - 1 + \phi(a) + \phi(b)) \geq 0$
for all a, b∈Y and for all u> 0;
(£2) £ (u, v) < v − u, for all u, v > 0;
(£3) if $\{u_n\}$ and $\{v_n\}$ are sequences in $(0, +\infty)$ such that $\lim_{n\rightarrow+\infty} u_n = \lim_{n\rightarrow+\infty} v_n$ = ` $\in(0, +\infty)$, then lim sup $n\rightarrow+\infty$ $£(u_n, v_n) < 0$. Under these hypotheses, T has a unique fixed point c∈Y with ϕ(c) = 0.

Proof: We construct the so-called Picard sequence at initial point x_0, where x_0 is an arbitrary point in Y and $a_n = T_{an-1}$ for all n ∈ N. its obvious, that whenever there exists an index m such that $a_m = a_{m+1}$, then the equalities $a_m = a_{m+1} = T a_m$ lead to the avail that a_m is a fixed point of T.
In order to continue the proof, we consider that $a_{n-1} = a_n$ for all n ∈ N and prove that
$\lim_{n\rightarrow+\infty} L(a_n, a_{n+1}, u) = 1$ for all u> 0.
letting, that there exista pointu_0 such that $\lim_{n\rightarrow+\infty} L(a_n, a_{n+1}, u_0) < 1$.
.using relation (ii) we conclude that L(an, a_{n+1}, u_0) < 1 for all n ∈ N. This shows that

$$S(an-1, an, u0;\ \phi) := \frac{1}{M(an-1, an, u0)} - 1 + \phi(an-1) + \phi(an) > 0, for\ all\ n \in N,$$

Then by using (£1) and (£2), with a= an−1 and b = an, we have
$0 \leq £ (S(a_n, a_{n+1}, u0; \phi), S(a_{n-1}, a_n, u0; \phi))$
$< S(a_{n-1}, a_n, u0; \phi) - S(a_n, a_{n+1}, u0; \phi)$,
for all n ∈ N.
we can rewrite as $S(a_n, a_{n+1}, u_0; \phi) < S(a_{n-1}, a_n, u_0; \phi)$, for all n ∈ N,
shows that $\{S(x_{n-1}, x_n, t_0; \phi)\}$ is a decreasing sequence of positive real numbers.

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Then we could say that there exists a limit point $l > 0$ such that
$\lim_{n\to+\infty} S(a_{n-1}, a_n, u_0; \phi) = l$, (3.1)
and claiming for contradiction, show that $l = 1$.
hence we let $l > 0$ and use the condition (£3),
$\limsup_{n\to+\infty}$ £ $(S(a_n, a_{n+1}, u_0; \phi), S(a_{n-1}, a_n, u_0; \phi)) < 0$
$t_n = S(a_n, a_{n+1}, u_0; \phi)$, and $s_n = S(a_{n-1}, a_n, u_0; \phi)$, to conclude that
But its contradiction to $l = 0$. Now, since the function ϕ has only non-negative values, from (3.1) we get
$\lim_{n\to+\infty} L(a_{n-1}, a_n, u_0) = 1$ and $\lim_{n\to+\infty} \phi(a_n) = 0$. (3.2)
The main point of the proof is in establishing that the sequence $\{a_n\}$ is Cauchy in Y. Again, we obtain the claim by contradiction. Therefore, we assume that the sequence is not Cauchy, that is, $\liminf_{m,n\to+\infty} L(a_m, a_n, u_0) < 1$ for some $u_0 > 0$. We give a standard reasoning, in fact, we suppose there exist $0 < \varepsilon < 1$ and two subsequences $\{a_{mk}\}$ and $\{a_{nk}\}$ of $\{a_n\}$ such that a_{nk} is the smallest index for which
$nk > mk > k$ and $L(a_{mk}, a_{nk}, t_0) \leq 1 - \varepsilon$, (3.3) and
$L(a_{mk}, a_{nk}-1, t0) > 1 - \varepsilon$. (3.4)
With respect to the inequalities (3.3) and (3.4), by using the triangular inequality (vi),
we have
$1 - \varepsilon > L(a_{mk}, a_{nk}, u_0) > L(a_{mk}, a_{nk-1}, u_0) * L(a_{nk-1}, a_{nk}, u_0) > (1 - \varepsilon) * L(a_{nk-1}, a_{nk}, u_0)$
We have just to recall the first limit in (3.2) and by letting k to infinity, we deduce
$\lim_{k\to+\infty} L(a_{nk}, ank, u_0) = 1 - \varepsilon$. (3.5)
By the same reason as above, we obtain
$1 - \varepsilon > L(a_{mk}, a_{nk}, u_0)$
$> L(a_{mk}, a_{mk-1}, u_0) * L(a_{mk-1}, a_{nk-1}, u_0) * L(a_{nk-1}, a_{nk}, u_0)$,
$L(a_{mk-1}, a_{nk-1}, u_0) > L(a_{mk-1}, a_{mk}, u_0) * L(a_{mk}, a_{nk}, u_0) * L(a_{nk}, a_{nk-1}, u_0)$
From the last inequalities, by letting k to infinity,
we get $\lim_{k\to+\infty} L(a_{mk-1}, a_{nk-1}, u_0) = 1 - \varepsilon$. (3.6)
Moreover, by letting k to infinity and using (3.2), (3.5) and (3.6), we obtain
$\lim_{k\to+\infty} S(a_{mk}, a_{nk}, u_0; \phi) = \varepsilon^*(1 - \varepsilon)$, $\lim_{k\to+\infty} S(a_{mk-1}, a_{nk-1}, u_0; \phi) = \varepsilon^*(1 - \varepsilon .)$
Finally, we work with the condition (ζ3), with $u_k = S(a_{mk}, a_{nk}, u_0; \phi)$, and
$ak = S(a_{mk-1}, a_{nk-1}, t_0; \phi)$, so that
we deduce 0 $\limsup_{k\to+\infty}$ £ $S(a_{mk}, a_{nk}, u_0; \phi), S(a_{mk-1}, a_{nk-1}, u_0; \phi)) < 0$
the above inequality can not be true and so $\{a_n\}$ is a Cauchy sequence in X. Now, by using the axiom of completeness of Y, to we wiil prove a point c∈Ysuch that $\lim_{n\to+\infty} L(a_n, c, u) = 1$ for all $u > 0$. Then by using second limit in (3.2) andwith help of lower semi-continuity of the function ϕ to have $0 \leq \phi(c) \leq \liminf_{n\to+\infty} \phi(a_n) = 0$,
now it left to show that c is a fixed ppint in Y.In that case we must show there is a subsequence
$\{a_{nk}\}$ of $\{a_n\}$ such that $T a_{nk} = Tc$, for all $k \in N$, then it shows completeness
. On other side, if this condition does not met, , then we can assume the contra positive of that.an $\neq$c and T an $\neq$ Tc, for all $n \in N \cup \{0\}$.,this implies that $L(an, c, u) < 1$ and $L(an, Tc, u) < 1$, for all $n \in N$. In this context, by using (£1) and (£2) with $a = an$, $b = c$ and $u > 0$, we conclude that

$0 \le £ ((T\, an, Tc, u; \phi), S(an, c, u; \phi)) < S(an, c, u; \phi) - S(T\, an, Tc, u; \phi)$.
Starting from $S(T\, a_n, Tc, u; \phi) < S(a_n, c, u; \phi)$, for all $n \in N$
and expliciting the notation, one can be written $1 \le M(c, Tc, u) - 1 \le L(c, a_{n+1}, u) - L(T\, an, Tc, u) - 1$

$$\begin{aligned} 1/L(c, Tc, u) - 1 &\le 1/L(c, an + 1, u) = 1 + 1/L(T\, an, Tc, u) - 1 \\ &\le 1/L(c, an + 1, u) = 1 + S(T\, an, Tc, u; \phi) \\ &< 1/(L(c, an + 1, u)) - 1 + S(an, c, u; \phi), \end{aligned}$$

for all $n \in N$.
now taking the limits upto infinite
$M(c, Tc, u) = 1$, that is, $c = Tc$. Then, the existence part is established, but we have to prove the uniqueness part. The proof of this claim is obtained by contradiction:
if the fixed point c is not unique, then there exists $w \in X$ such that $w = Tw$ and $c \neq w$. It follows by (jj) of Definition 2.3 that $L(c, w, u) < 1$. Trivially, by using (£1) and (£2) with $a = w$, $b= c$ and $u> 0$, we get that
$0 \le £ (S(Tw, Tc, u_0; \phi), S(w, c, u; \phi)) < S(w, c, u; \phi) - S(w, c, u; \phi) = 0$,
which is a contradiction and hence $w = c$
Conclusions :
The fuzzy metric and modular metric spaces represent two interesting way of enlarging the mathematical research in (classical) metric spaces, by focusing on vagueness and function spaces, respectively. Here, we work with methods of fixed point theory to establishing an existence and uniqueness theorem for a self-mapping in a complete non-Archimedean fuzzy metric space. Then, we extend our approach to a modular metric space. This procedure may be useful to generalize and relate to each other various results in the existing literature..
The fuzzy term explains about the vagueness, which means “ not clear”. In the same context the fuzzy metric space and modular metric space also explains the same concept, by focussing on function space. we have proved a theorem by using the notion of modular metric space and non archimedean metric space, with fixed point to generate new result with self mapping and uniqueness of fixed point.
This method can be referred for better generalization of fixed point theorems.

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