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Mathematical Model of Unsteady Gas-Liquid Flows Flows in the Formation-Well System

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Abstract

This paper proposes a mathematical model of gas-liquid flow in the formation-well system, based on the laws of conservation of mass and momentum and representations of pipe and underground hydromechanics. Unsteady flows of liquid and gas arising during the gas-entrainment and start-up of the gas-lift well are investigated.

Keywords: gas-liquid flow, mathematical model, unsteady filtration, multiphase flows, oil and gas industry, bubbling, cork, emulsion, dispersed-ring and dispersed flow modes, well, oil reservoir, gas elevator.

This study contains a physically meaningful mathematical model of gas-liquid flow in the well, coupled with unsteady filtration model in the formation. The model of gas-liquid flow in the well is based on the laws of conservation of mass and momentum of phases. These models were used from thermal and nuclear power engineering, where the problems associated with the movement of gas-liquid mixtures in pipes are investigated in more detail and versatile in theoretical terms [1-9]. A numerical calculation scheme based on the finite difference method is constructed.

Gas-liquid flows are associated with many problems in power engineering, mechanical engineering, chemical and oil and gas industries. The multiphase flows significantly complicate research, so the hydrodynamics of gas-liquid flows is still under development, in contrast, to single-phase flows' hydrodynamics.

In the oil and gas industry, gas-liquid flows can occur at all stages from well drilling to hydrocarbons' transportation to consumers. A qualitative description of gas-liquid flows in pipes and quantification of their main parameters (pressure drop, the volume concentration of gas and liquid, phase velocities, flow friction forces with the pipeline wall) are necessary for designing production well structures.

Once a well has been drilled and developed, the oil must be produced from it. Oil is underground under such pressure that it rushes to the surface when the well is routed to it. As a rule, wells only gush at the beginning, i.e. immediately after drilling. After some time, the pressure in the reservoir decreases and the fountain runs out.

When flowing stops due to lack of formation energy, a mechanized method of profitable operation is used, in which additional energy is injected from the outside (from the surface). One of such methods, in which energy is injected in the form of compressed gas, is a gaslift. Gaslift - a system consisting of production tubing and tubing string (tubing string) lowered into it, where the fluid is lifted with compressed gas (air).

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High-pressure gas is injected into the annulus, causing the fluid to drop and the tubing to rise. When the liquid level drops to the tubing's bottom end, the compressed gas will flow into the tubing and mix with the liquid. As a result, this gas-liquid mixture's density becomes lower than the density of liquid coming from the formation and the level in the tubing will increase. The more gas is injected, the lower the mixture's density will be and the higher it will rise [9].

Five main flow modes are distinguished in the vertical upward flow: bubble, cork, emulsion, dispersed-ring and dispersed flow, which replace each other with an increase in the volume concentration of gas.

In bubble flow, gas moves in the form of individual gas bubbles in the bound liquid phase. The bubbles' size is small compared to the diameter of the tube, and their distribution in the cross section is close to uniform. Typically, bubble flow exists at the volume concentration of gas Cork mode is a flow of large gas bubbles separated by liquid plugs. The bubbles have axial symmetry and are projectile shaped. There is a liquid film between the bubble and the pipe wall. The emulsion flow resembles foam flow with large irregularly shaped gas inclusions. The size and shape of the gas inclusions frequently change; the flow has a chaotic character.

Dispersed-ring flow behavior is determined by the distribution of mass flow between its three components: gas and droplets in the flow core and the wall film. This mode usually occurs at high gas contents $\alpha > 0.6 - 0.8.$ If there are no droplets in the gas core, the flow is called annular. At high gas flow rates, the fraction of liquid carried away in the core increases and the film on the pipe surface becomes thinner. If all liquid moves in the form of droplets in the gas and there are no liquid film, such flow is called dispersed flow.

In this work, the gas-liquid flow is considered pseudo-uniform, its parameters are averaged over the pipe cross-section. All the effects of non-uniform parameter distribution over the pipeline cross-section are taken into account in the dependences for the averaged parameters, temperature effects are not taken into account. It is supposed that viscosity of phases is shown only in processes of interphase interaction and is not shown in macroscopic momentum transfer, pressure in phases is the same, flows occur in pipes with the constant area.

Bubble, cork and emulsion structures are combined into a single flow structure and generally accepted single-velocity models were used to calculate them [1]. Usually, these modes exist at gas content $0.2 - 0.3 < \alpha < 0.6 - 0.8.$ When modeling disperse-ring flow, it was assumed that gas and droplet velocities are equal to each other and different from the film velocity. Parameters of this mixture flow regime were calculated using a system of differential equations [1], consisting of three continuity equations (for gas, droplets and liquid) and two momentum equations (for film and gas-droplet flow core), which account for the intensity of droplet deposition on film, the intensity of droplet dynamic carry-over from the film surface.

$$\frac{\partial}{\partial t}(\rho_1^o \alpha_1) + \frac{\partial}{\partial z}(\rho_1^o \alpha_1 v_1) = J_{21} + J_{31},$$

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$$\frac{\partial}{\partial t}(\rho_2^o\alpha_2)+\frac{\partial}{\partial z}(\rho_2^o\alpha_2 v_1)=-J_{21}-J_{23}+J_{32},$$
$$\frac{\partial}{\partial t}(\rho_2^o\alpha_3)+\frac{\partial}{\partial z}(\rho_2^o\alpha_3 v_3)=-J_{31}+J_{23}-J_{32},$$
$$\frac{\partial}{\partial z}(\rho_2^o\alpha_3 v_3)+\frac{\partial}{\partial z}(\rho_2^o\alpha_3 v_3^2)=-\alpha_3\frac{dP}{dz}-F_w+F_{13}-\rho_2^o\alpha_3 g-(J_{32}+J_{31})v_3+J_{23}v_1 \qquad (1)$$
$$\frac{\partial}{\partial t}\left[(\rho_1^o\alpha_1+\rho_2^o\alpha_2)v_1\right]+\frac{\partial}{\partial z}\left[(\rho_1^o\alpha_1+\rho_2^o\alpha_2)v_1^2\right]=$$
$$-(\alpha_1+\alpha_2)\frac{\partial P}{\partial z}-F_{13}-(\rho_1^o\alpha_1+\rho_2^o\alpha_2)g-J_{23}v_1+(J_{31}+J_{32})v_3.$$

Here parameters related to the gas phase, droplets and liquid film contain lower indices 1, 2, 3 respectively, t - time, z - coordinate, P - pressure, α_i - volume concentration of i -th component of the mixture, v_i the velocity of i -th component of the mixture, ρ_i – the density of i -th component of the mixture, g – free fall acceleration, - J_{21}, J_{31}, intensities of gas and liquid inflow through permeable pipe walls, J_{32}, J_{23}, - intensities of droplet carry-over from the film surface and droplet settling, F_{13}, F_w - force interaction between the film and droplet and the channel wall.
Dispersed flow parameters were calculated according to the method [6].

The characteristic time of establishing the stationary profiles of velocities and pressures is much less than establishing the stationary profiles of volume concentrations, which are equal (by order of magnitude) to the characteristic time of practically important problems. Consideration of this circumstance allows us to bring the initial system of differential equations to a quasi-stationary form [1, 7].

Ratios for determining the intensities of entrainment J_{32} and deposition of droplets J_{23} on the film's surface, force interaction between the liquid film and droplets and the channel wall F_{13}, F_w were calculated using the formulas given in [1].

A mathematical model is proposed to describe gas occurrences during drilling, which will allow a qualitative and quantitative assessment of process parameters. It is assumed that at depth L the well has penetrated a thick gas reservoir with initial pressure p_o, permeability k, and porosity m. Gas in formation conditions has viscosity μ. Filtration in the gas reservoir to the well will be considered radial-spherical. Pressure distribution in gas reservoir follows filtration equation [1].

The filtration equation's boundary condition will be the bottomhole pressure at the remote supply contour at the boundary of the well and the reservoir and the volume flow rate of gas entering the well from the reservoir.

As a rule, the distribution of flow parameters corresponding to stationary conditions was used as initial conditions for equations. To solve equations (1), explicit finite-difference schemes were used [10].

The gas-perforation model consisted of two elements: the well and the gas reservoir. These elements are conjugated due to internal boundary conditions at the bottomhole. And gas flow rate

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and bottomhole pressure can be obtained by joint solution of differential equations (1) and filtration.

In modeling of gaslift oil production, it is assumed that vertical well of radius **R** with open bottom hole completely penetrates a flat oil layer of thickness **h**, the roof of which is at depth **L**. Initial pressure at the top of the reservoir is equal to p_c, permeability k, piezo conductivity χ. Filtration in the well will be considered single-phase and flat-radial, then pressure distribution at the top of the reservoir p will follow the piezo conductivity equation [1, 7-8, 11].

The gas-lift model considers the combined dynamics of three elements: the well, the oil reservoir, and the annulus' gas. The reservoir's boundary conditions will be the pressure at the remote supply circuit and the pressure at the bottom of the well. The volumetric flow rate of oil from the reservoir is determined by the formula [1, 8, 12]. This value will be the boundary condition at the bottom of the well. At the wellhead, constant pressure and internal gas source are set.

In the considered hydraulic system, well with the gas-liquid mixture and oil reservoir are systems with distributed parameters, gas in annular space is a system with concentrated parameters.

With the constructed mathematical model's help, the unsteady flows of liquid and gas arising during gas leakage and start-up of the gas-lift well are investigated.

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