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Mathematical modeling of Vitreous Fluid during Saccadic Eye

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Abstract:
We generate the mathematical model of motion of vitreous fluid during saccadic eye. To describe the present model we used Navier Stokes equation in spherical form and find the expression of velocity potential and velocity of vitreous fluid during the saccadic eye. In present model we established the relationship between velocity potential and saccadic amplitude, we have also discussed the effect of viscosity and stress corresponding to the depth of vitreous on motion of vitreous fluid.The main consideration in this research , the motion of fluid in vitreous towards to center.
Key words: Vitreous Fluid ,Saccadic amplitude,Navier-Stokes ,Velocity Potential, Viscosity .

Introduction:

In this research paper, we propose a parametric model for saccadic eye. During a saccade, the eye rotates from one angle to another angle as it moves its focus from one object of interest to another .Our proposed

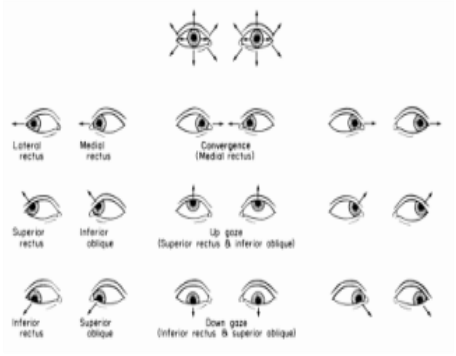

Fig:1(a) Sketch of saccadic eye[vision therapist Resources]

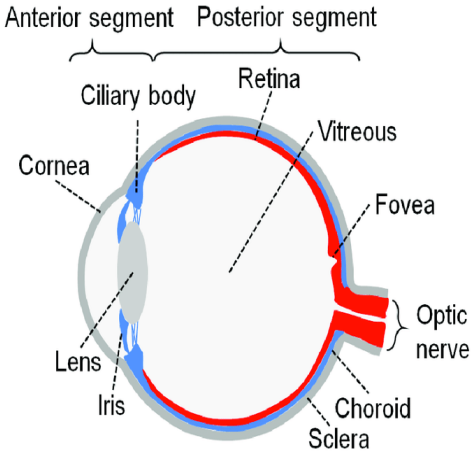

Fig:1(b) Diagram of anterior segment of human eye[19].

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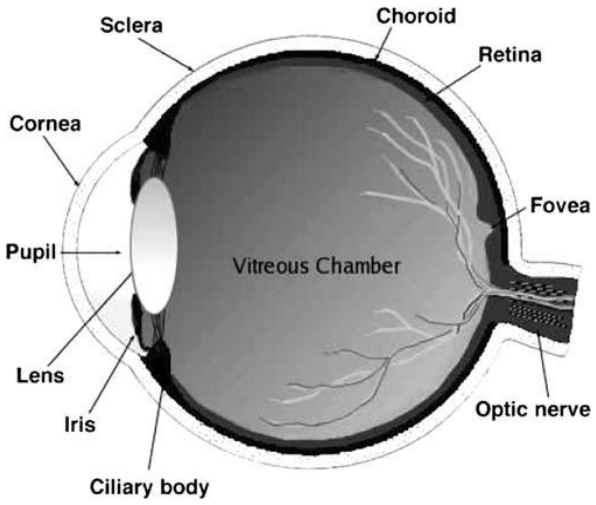

Fig:2 Diagram of the vertical cross section of a human eye[5].

the human eye divide in two segments anterior segment fig:2 ,the anterior segment bounded by cornea, lens, and vitreous body ,which space enclosed by lens and retina. The anterior chamber contains aqueous humor and physiological liquid whose properties are similar to water ,the vitreous body forge between 3/5 & 2/3 of toatal capacity of the eye globe , is ocupied by vitreous humor. Lee et al.[1] Describe the viscoelastic proprties of vitreous body, they told that vitreous is a clear gel –like substances . The vitreous body contains mechanical properties to securing retina obedience to outer layers of the eye. As the advancing age , generally the vitreous pass off a procees of liquefaction, in these conditions , the creation of liquid cavities within the vitreous body is normally observed.

The liquefaction of vitreous may involve in whole vitreous body and it is more frequent in short sighted disorder. In case of other liquids like silicon oils, which can be contins in the chamber for limited time then after it removed naturally by aqueous humor. The shape of eye globe volume obtined with the help of fluids ,therefore many problems solved by fluid dynamic properties. The study of fluid in the vitreous body is limited or not effectively studied yet. Generally it is assumed that the motion of vitreous body plays an important role in the pathogenes of retinal detachment. It is well known that if retina has been tore then the infiltration of liquified vitreous through aperture ,between retina and pigmented epithelium may be retinal detachment. This process is known as rhegmatogeneous retinal detachment. Repetto et al.[2] proposed a model to produce the possible mechanism where the tensile stress may be created on the retina most probable the break of retina. Linder[3] describe the dynamics of fluid in vitreous body experimentally induced by movement of eye.

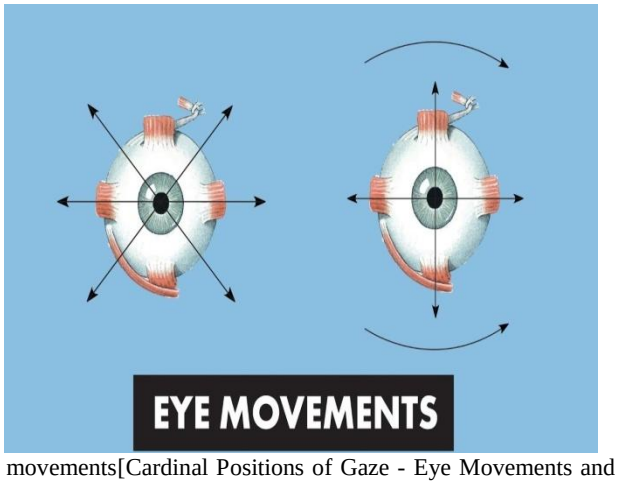

Fig:3(a) Schematic diagram of eye movements[Cardinal Positions of Gaze - Eye Movements and Eye Muscles - Cranial Nerves – MEDZCOOL]

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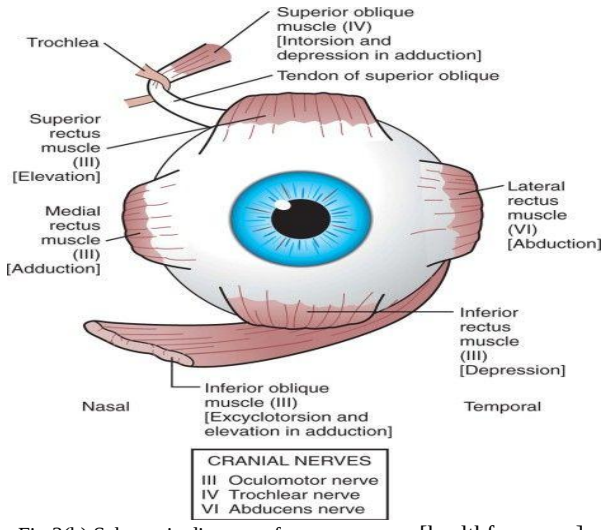

Fig:3(b) Schematic diagram of eye movements[healthfavo.com]

The authors work is important to determine the description of flow of fluid in vitreous humor, in order to know the physical mechanism of retinal detachment. They also produce the importance of lens to confer the irregular shape to the chamber and it may significantly improve the characteristics of flow field.

Devid et al.[4] again considered the problem of vitreous motion induced by eye movements. Devid et al. present the analytical model of vitreous motion according to the periodic rotation of eye globe. In this paper the shape of vitreous body considered in spherical form, in fact the motion of may be drive motion of fluid but due to viscosity it may be different. It can be determine if the viscous stress applicable at the wall. In really the anterior part vitreous body can not be spherically due to presence of lens. According to some researchers did a work to complete current work, the authors review some literature which divides in two sections. First ,the majority is focused on the motion of vitreous during saccadic eye movement latest study done by David et al.[4] and many later studies by Repetto[5] , Repitto et al.[6], Meskauskas et al. [10] , Bonglio et al. [11] , Abouali et al. [8] ,Modarreszadeh et al. [9],Modarreszadeh and Abouali[7]. All above authors did a work related to the geometry of cavity filled by the vitreous body is assumed to be constant. In general the shape of cavity is either spherical or perturbed spherical shape.

Secondly, the research work was focused on the motion of vitreous body mainly assumed; the vitreous body is an incompressible Navier-Stokes. Some authors did an excellent work to express above assumptions , in recently Lee et al. [1] and more recent study by Sharif-Kashani et al. [13].In this queue the effect of complex viscoelastic (Non-Newtonian fluid) . some related models discussed by Modarreszadeh and Abouali [10], Giesekus [14],

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Secondly, the research work was focused on the motion of vitreous body and it is assumed that the vitreous body is an incompressible Navier-Stokes. Some authors did an excellent work to express above assumptions , in recently Lee et al. [1] and more recent study by Sharif-Kashani et al. [12].In this queue the effect of complex viscoelastic (Non-Newtonian fluid)[14] . some related models discussed by Modarreszadeh and Abouali [13], Giesekus [1], Sharif-Kashani et al.

Mathematical Model & Solution:

Consider the shape of vitreous is spherical and let the center of vitreous at origin and the motion in the z direction and we determine the velocity potential ∅ that will satisfy the given boundary conditions ∅ satisfies the equation of Laplace ,which may be written as follows

$$\frac{\partial^2 \emptyset}{\partial r^2}+\frac{2}{r}\frac{\partial \emptyset}{\partial r}+\frac{1}{r}\frac{\partial^2 \emptyset}{\partial \theta^2}+\frac{cot\theta}{r^2}\frac{\partial \emptyset}{\partial \theta}=0--(1)$$

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Flow about z axis

Normal velocity at any point of sphere =velocity of liquid at that point in that direction

$$= -\frac{\partial \emptyset}{\partial r} = V\cos\theta|_{r=a} \quad ---------(2)$$

Let fluid is at rest at∞,$-\frac{\partial \emptyset}{\partial r} = 0 \quad at\ r = \infty \quad --(3)$

$\emptyset$ must be in the form $\emptyset = f(r)\cos\theta$— (4) here $\emptyset$ is velocity potential then , by using (4) equation 1 become

$$\left(\frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r}\right)\cos\theta - \frac{f(r)}{r^2}\cos\theta - \frac{f(r)\cos\theta}{r^2} = 0 \quad ----------- (5)$$

$$\frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r} - \frac{2f}{r^2} = 0 \quad ----------- (6)$$

Equation (5) is homogeneous diffrential equation so

Solution of equation (6) is

$f(r) = \left[Ar + \frac{B}{r^2}\right]$

Then solution of (1) will ,by usng (4)

$$\emptyset(r) = \left[Ar + \frac{B}{r^2}\right]\cos\theta \quad -------- (7)$$

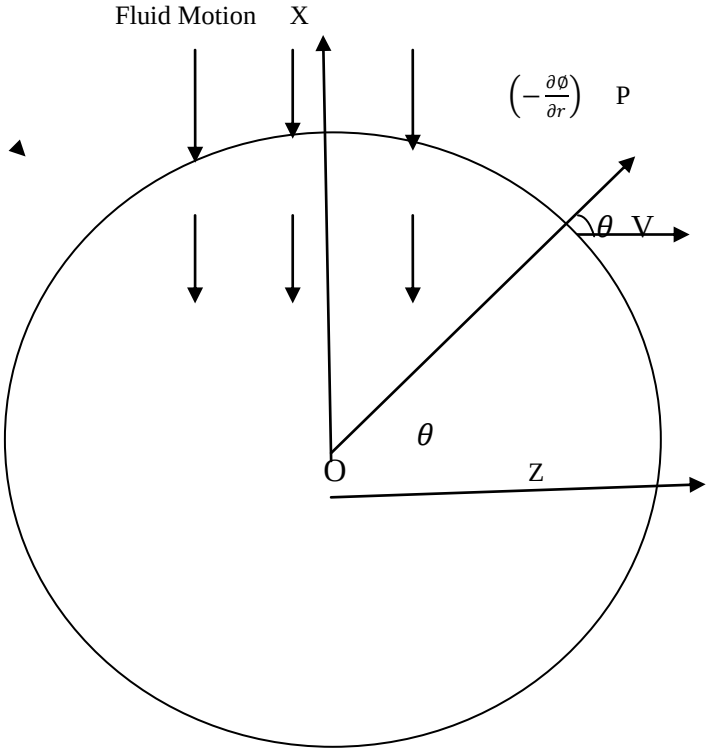

Fig:4 Schematic diagram of human vitreous in spherical form

Now by using boundary condition 2 and 3 for 7, we get

to obtained the solution of equation (4) will be

$$\emptyset(r) = \frac{1}{2}Va^3\frac{\cos\theta}{r^2} \quad -------- (8)$$

$\emptyset$ is the velocity potential and θ is the saccade amplitude it depends on time it is given by [15]

$$\theta(t) = \frac{\Delta\theta}{2}\left[1 + \tanh\frac{2\alpha_m(t - t_1)}{\Delta\theta}\right] --(9)$$

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Velocity of vitreous fluid

$$-\frac{\partial \emptyset}{\partial r} = \frac{1}{2} Va^3 \frac{\cos\theta}{r^3} - - - - - - - - - - (10)$$

V – Velocity of Eye movement

$$\tau = \mu \frac{u_{max}}{\gamma} - - - - - - - - - - - - - - (11)$$

$$\gamma = \sqrt{\frac{\mu}{\rho\omega}}$$

$$\tau = \mu \frac{\frac{1}{2r^3} Va^3}{\sqrt{\frac{\mu}{\rho\omega}}} \text{ because Maximum value of } \cos\theta = 1$$

$$\tau = \sqrt{\mu\rho\omega} \frac{1}{2r^3} Va^3 - - - - - - - - (12)$$

Where

$\tau -$ Max shear stress.

$\mu -$ Viscosity of vitreous fluid.

$\gamma -$ boundary layer thickness of vitreous body .

$\rho -$ density of vitreous fluid.

$\omega -$ angular velocity of vitreous fluid.

$\alpha_m -$ maximum angular velocity vitreous fluid.

$\Delta\theta -$ angular displacement

$t_1 -$ meadian time

Now we determine the equation of flow in the vitreous fluid ,the differential equation of lines of flow at any instant ,at that instant centre of vitreous passes through origin given by

$$\frac{dr}{\frac{\partial \emptyset}{\partial r}} = \frac{rd\theta}{r\frac{\partial \emptyset}{\partial \theta}} - - - - - - - - - - - (13)$$

Using (5) ,(6) become

$$\frac{dr}{Va^3 \frac{\cos\theta}{r^3}} = \frac{d\theta}{Va^3 \frac{\sin\theta}{2r^2}}$$

$$\frac{dr}{r} = \frac{2cos\theta}{\sin\theta} d\theta - - - - - - - (14)$$

$$logr = 2log \sin\theta + \log c \ \ or\ r = c \sin^2\theta - -(15)$$

Which is equation of lines of fluid flow in the vitreous and flow towards to centre.

Discussion:

In this paper ,the assumption that the shape of vitreous is spherical and we solve the an equation in spherical form. Almost authors discussed the motion of vitreous body but very few authors discussed about the motion of fluid in the vitreous humor.

Recently **R. Repetto** etal. (2010) used a method to describe the rotation of saccadic eye induced a flow in vitreous body,also in this queue **cantrill and pederson,1984 , moses,1987** did a work in this direction. In present research paper the authors found an expression to determine the velocity/ vibration in the vitreous fluid. We used MATLAB software to complete this task.

From fig 5 , the concluding remark is that the velocity potential (at y axis) profile attains the maximum near the wall of vitreous body as the saccade amplitude increase then the potential of velocity.

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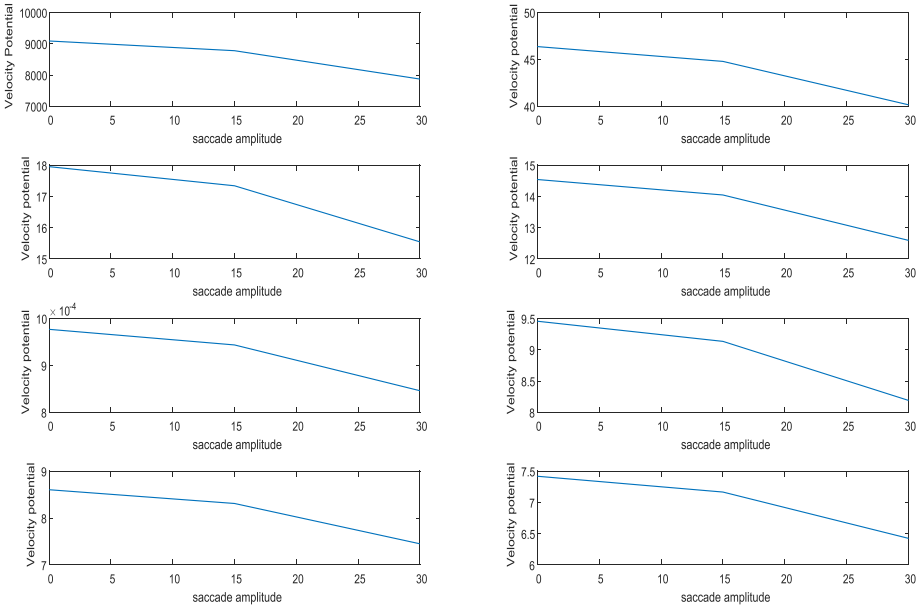

Fig:5 Effect of saccade amplitude on velocity potential

Fig 6 describe the change in saccade amplitude with respect time , from above graph the authors conclude that according to time the fluctuations in amplitude increase and at critical stage($t = .5$ to 1sec it become constant.

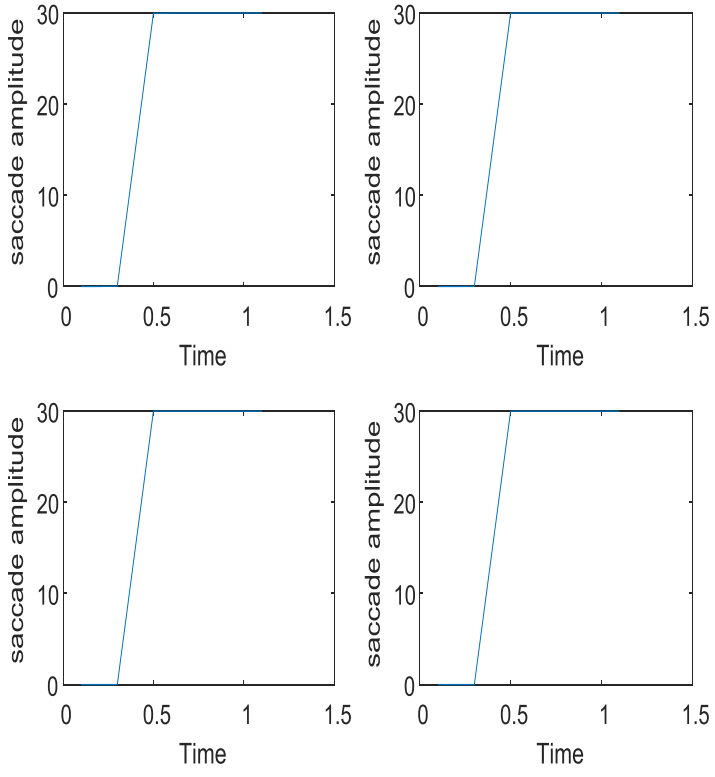

Fig:6: effect of time on saccade amplitude

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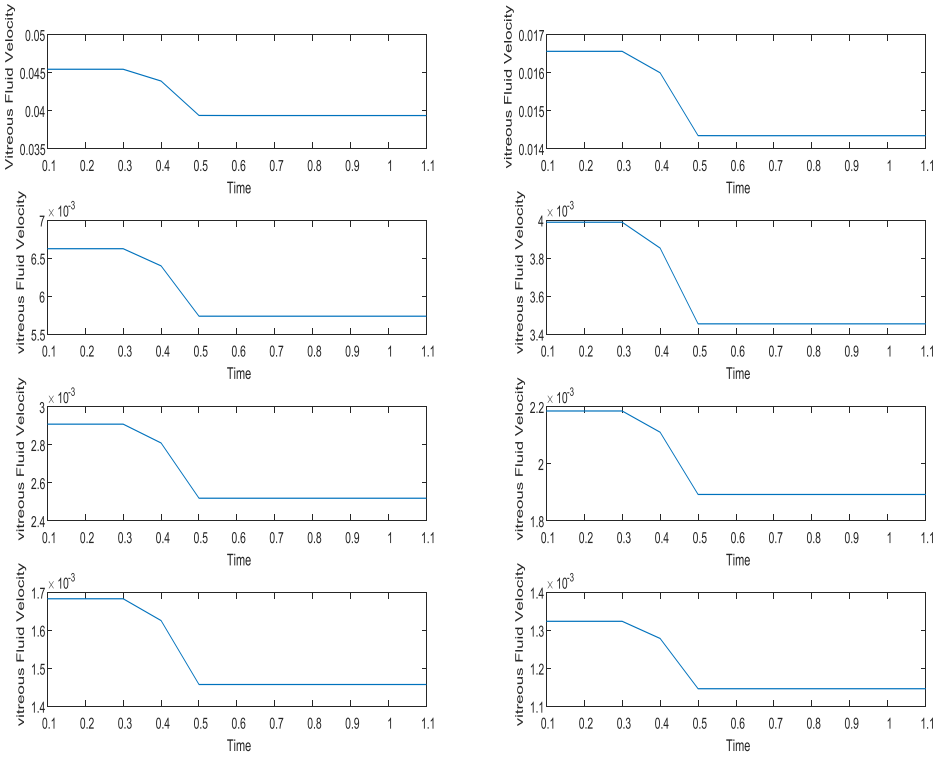

Fig:7: Evidence graphs of motion of vitreous fluid.

From the diagram 7 , the authors conclude that the velocity of vitreous fluid(at y axis) is maximum near vitreous wall and as saccade time increase then the velocity of vitreous fluid decrease and after few seconds it become constant. We observe some fluctuations in the vitreous fluid, we are not saying forever but it happens for small time duration.

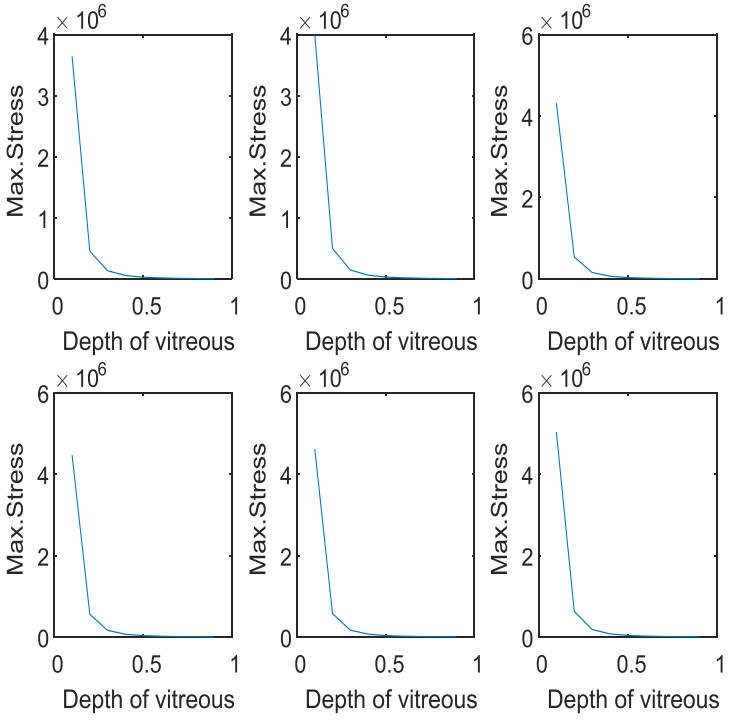

Fig:8: Effect of depth of vitreous on stress.

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The concluding remark from above graphical representation , the stress(at y axis) be maximum near vitreous humor wall and as the depth of vitreous increase the stress decrease and at centre it become or tends to zero.
Also discussed the effect of viscosity on stress, from graph 9, shows the sufficient effect of viscosity on shear stress ,we conclude from the graphs as viscosity deflect as shear stress also deflects it mean the shear stress is directly proportional to viscosity. It mean if viscosity of vitreous fluid is low due to any disease then the shear stress will be low. It is not obvious that if stress will increase or decrease with viscosity, it comes from fig 8,9.

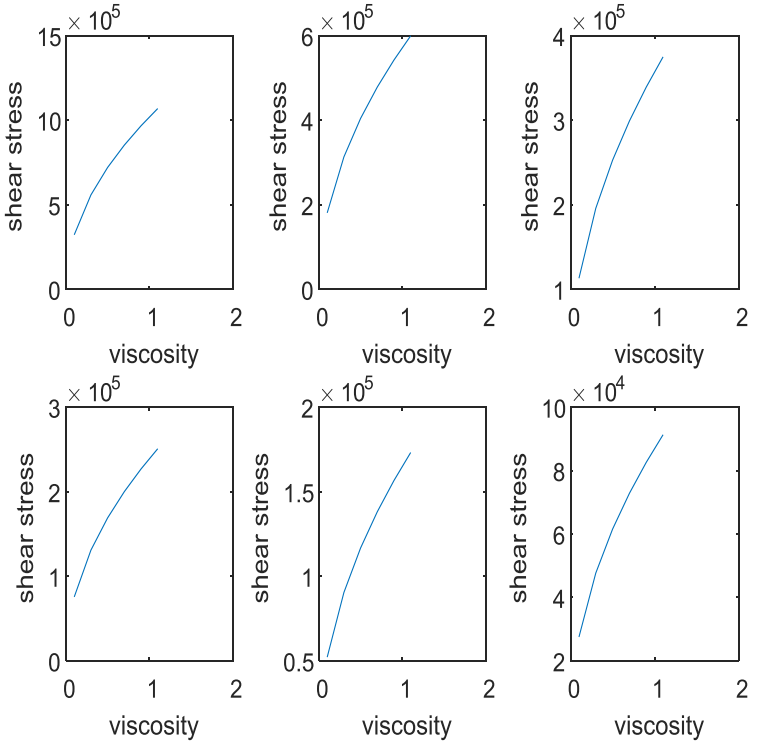

Fig:9: Graphs between shear stress & viscosity.

Table:

SR. No.	Notation	Meaning	Values	Unit	Source
1	θ	saccade amplitude	30	degree	[15]
2	α_m	Max. Angular velocity	500	$\frac{rad}{sec}$	[15]
3	M	Viscosity of vitreous fluid	.2- 1	Pascal-sec	[16]
4	ρ	Density of vitreous fluid	1.0053 - 1.0089	$\frac{gm}{cm^2}$	[17]
5	ω	Angular velocity	12.5	$\frac{rad}{sec}$	[17]

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6	V	linearVelocity of globe during eye moment.	1532-1550	m/s	[18]
7	R	Radius of vitreous body	1.2	Cm	[11]
8	R	Depth of vitreous	.1-1.2	Cm	[11]
9	$\Delta\theta$	Angular displacement	30	Degree	[15]

Constants table

Result:

The major finding in present research is formulae of velocity potential as well as velocity of vitreous fluid and also write an expression for shear stress and viscosity, from these expressions authors describe the effect of different parameters like diffusion coefficient ,saccade amplitude.In this research we found the relation betweem change saccadic amplitude and corresponding to time, also we check the effect of increase of depth of vitreous humor fluid motion, an another important finding is relation between shear stress and viscosity and many more, all computed results are presented by graphs.

Conclusion:

In present research paper we have discussed theoretical results by using mathematical model, it helps to understand the motion of vitreous fluid during saccade, also described the velocity potential and condition of stress and affectness of viscosity and it help to understand the basic characteristics of fluid mechanics of vitreous body. We have focused on flow genetrated in the vitreous body during saccadic eye globe , which is necessary to generate the fluid motion in the vitreous body. Main goal of this research paper, to get the full information about velocity profile of vitreous fluid , effect of viscosity on shear stress, change in velocity potential as well as velocity by saccade angle, another important effect on cerresponding to depth of vitreous body. This model is very useful to deep study of distribution of fluid (Drug) in cornea ,retina and vitreous body.

Conflict of interest :

The authors have no conflict of interest.

Acknowledgement:

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