

## Computation of Domination Sets in Koch Snowflake

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### ABSTRACT

The goal of this paper is to calculate the Maximal dominating set and Minimal Dominating set of the famous Fractal Graph Koch Snowflake. In this paper implements a new study about advanced Graph theory Application through the Fractal Graphs. Nature looks so many self-similarity pictures and things all around the world. Those self-similarity objects has classified into four categories. They are exact self-similarity that means this type of object are accurate self-similar measurement itself, statistical self-similarity that means, Domination is the newest developed area of Graph Theory. It plays a vital role in many areas such as Engineering, Chemical Bonding, Purchase and Sales, Ordering through online and so on. Fractal Graph is the new trend of Mathematics which involves more connection with computer applications, clouding and modelling. This paper shows the shortest way of calculating Maximum dominating set and Maximal dominating sets and their corresponding cardinality value in Famous Fractal Graph Koch Snowflake. It introduces a new formula for the calculation of dominating cardinality by an Iterative method. This formula is common for all the Iteration of Koch Snowflake. Also this paper shows the history, structure, characteristics, implementation of vertices and edges of Koch Snowflake.

**Keywords:** Graph, Fractals, Domination

**AMS Classification key:** 05C30, 05C70, 05C07

### 1. Introduction

Graph Theory [14] is the main area of mathematics which is given the detail about graphs. Graphs can be made by the set of vertices and edges [2]. Nowadays Advanced level of Graph Theory plays a vital role of all over the world. The most developed Advanced level of Graph theory is labeling, matching, dominating and coloring [15]. Its application and model involve connection with real world and expressed as the graphical terms [18]. In this paper applied the concept of Domination in graphs in the Fractal Graph. Fractal Graph has the special characteristics of Graphs like self-similarity, recursive function and fine structure at arbitrarily at small scales.

### 2. Domination of Graphs

Domination [1] in Graphs is the advanced level of Graph Theory. Domination in Graph Theory applied in discrete optimization, combinatorial problems and classical algebraic problem [3]. It has studied form 1950 onwards, but the rate of research of domination significantly increased in 1970. Richard Karp proved NP complete cover problem [4]. This had implemented for the dominating set problem.

### 3. Fractal Graph

People are exclaimed by the beautiful images called as fractals. Well defined self-similarity objects are considered as fractals. Fractal was found by Hedge Von Koch in 1904. In later Fractals were originated by Benoit Mandelbrot in 1975. Fractals are classified according to their self-similarity. There are three types of self-similarity founds in fractals such as Exact

self-similarity, Quasi Self-similarity and Statistical Self similarity. It has Hausdroff dimension. It is better than topological dimension that means one dimensional object. It is very simple and recursive definition. Fractals are the mathematical defined structure. Normally sets and functions are not enough to describe smooth irregular shape objects. These objects has been divided or splitted, each of the objects has same properties and mathematically defined structure

#### 4. Koch Snowflake

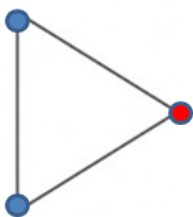
Koch Snowflake[7] is the most famous earlier Fractals. It was invented by Hedge Von Koch. He was Swedish mathematician. Karl Weierstrass's abstract of his paper is opponent of self-similarity object[8]. Hedge disproved Karl Weierstrass's abstract and gives more definitions about geometric objects with self-similarity. Koch Snowflake is called as Koch Star or Koch Island[9]. It has been built up iteratively at sequence of stages. The first stage is an equilateral triangle and each successive stage is formed from adding outward bends to each side, making smaller triangle. The total length of the curve increased by  $\frac{4}{3}$  in each iteration[11]. Each Iteration increases four time of line segment in the previous Iteration. The perimeter of Koch Snowflake is  $3*s*(\frac{4}{3})^n$  where s is the length of original equilateral triangle[10]. Number of sides of Koch Snowflake in  $n^{\text{th}}$  iteration is  $3*(4^n)$

#### 5. Calculation of Maximum Dominating Set and Maximal Dominating Set in Koch Snowflake

The set of collection of non-adjacent vertices in the graph is called as Maximum Dominating set[12]. The number of vertices in this set is called Maximum Domination Cardinality[6]. It is denoted by  $D(G)$ . These set of vertices must be adjacent with other vertices in the given graph[13]. Maximal Dominating set is the set of minimum number of collection of non-adjacent vertices in the given graph. The count of the vertices in the Maximal Dominating set is called as Maximal Domination Cardinality[5]. In this paper calculated Maximum and Maximal Domination Cardinality in Koch Snowflake for all iteration. Red nodes of the following figures 1,2,3of Koch Snowflake are the nodes of dominating set. Koch Snowflake has equal number of vertices and edges for all iteration. The following formula gives the cardinality of Maximal dominating set and Maximum Dominating set[16]. Number of vertices and edges of Koch Snowflake is  $3*(4^n)$  where n is started at zero and it is increased by 1[17]. Maximal dominating Cardinality =Maximum Dominating cardinality  $= \lfloor \frac{E(G)}{2} \rfloor = \lfloor \frac{V(G)}{2} \rfloor$ . Check whether the formula is applied for all iteration of Koch Snowflake.

##### 5.1 Initial Iteration

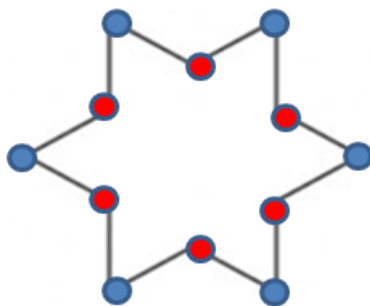
In the Initial Iteration of Koch Snowflake has 3 ( ie  $3*4^0$ , here  $n=0$ ) vertices and 3 edges. Maximum Domination Cardinality =1= Maximal Domination Cardinality



**Figure 1: Domination set of Initial Iteration of Koch Snowflake**

##### 5.2 First Iteration

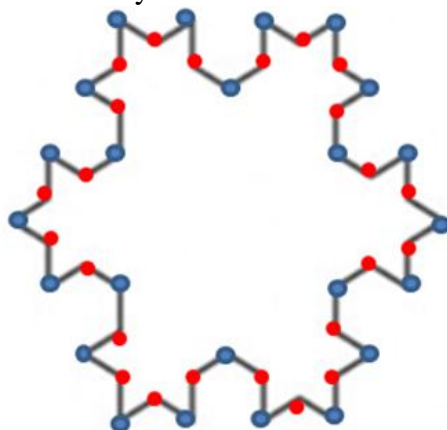
In the first Iteration of Koch Snowflake has 12( ie  $3*4^1$ , here  $n=1$ ) vertices and 12 edges. Maximum Domination Cardinality =6 =Maximal Domination Cardinality



**Figure 2: Domination set of First Iteration of Koch Snowflake**

**5.3 Second Iteration**

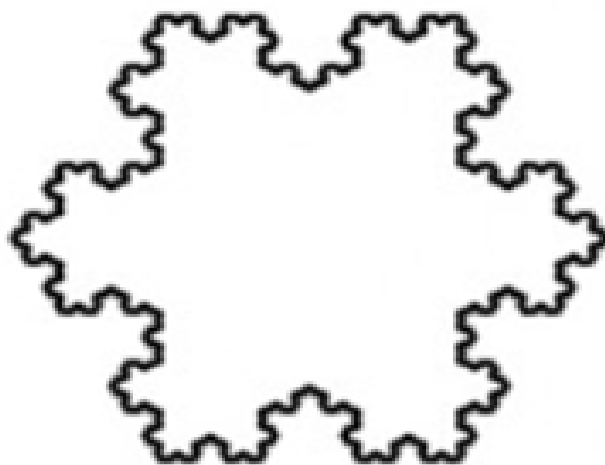
In the second Iteration of Koch Snowflake has 48 (ie  $3 \cdot 4^2$ , here  $n = 2$ ) vertices and 48 edges. Maximum Domination Cardinality = 24 =Maximal Domination Cardinality.



**Figure 3: Domination set of Second Iteration of Koch Snowflake**

**5.4  $n^{\text{th}}$  Iteration**

In the  $n^{\text{th}}$  Iteration of Koch Snowflake has  $3 \cdot 4^n$  vertices and edges. Maximum Domination Cardinality =  $(3 \cdot 4^n) / 2$  =Maximal Domination Cardinality.



**Figure 4: Domination set of Second  $n^{\text{th}}$  Iteration of Koch Snowflake**

**6. Conclusion**

Domination in Fractal Graphs has much real life application in world wide. By this concept, it is liked how decided to use many different distances to prove that the Koch snowflake works. It is wondered if there is a further longest distance that can be measured and applied to

the problem. It is 2-colorable problem. Only two coloring are enough to put to the non-adjacent vertices. The highest iteration of Koch Snowflake is not measureable easily. The simple formulae of calculation of vertices, edges and domination cardinality are introduced in this paper. It is used to give domination cardinality of the highest Iteration in easy manner.

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