

Novel Technique to Investigate the Effect of Different Drugs of Chemotherapy to Glioblastoma Tumor Cells Growth and Invasion in Homogeneous Medium

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ABSTRACT

Cancerous tumors from the mutations of one or more cells which usually undergo rapid uncontrolled growth thereby are impairing the functioning of normal tissues. There are many different types of cancers, we concerned with the brain tumor predominantly Glioblastoma, which make up about half of all primary brain tumors diagnosed. It is very nasty tumors with a depressingly dismal prognosis for recovery. We believe that model could be used to make predictions regarding the survival time of the patient following the various types of treatments including resection, radiation and chemotherapy. For clarification of the models we established the Kamal Variational Iteration Method (KVIM).

KVIM gives rapidly convergent successive approximations without any restrictive assumptions or transformations which may change the physical behaviour of the problem. Kamal Transform is a new arrival of an integral transform which is useful to apply to find the Lagrange Multiplier in VIM method. Generally it is difficult to crack out. Using the Kamal Transform it's become easier to find the same and one can hoard the time.

Aim of this research is to thrash out the current developments of the methods to crack the mathematical models of effect of different drugs used in chemotherapy treatment upon Glioblastoma tumor cells growth and invasion when early cancer cells are known and unknown when both appeared in Homogeneous medium. It might be unlock the doors for improvement in prediction of recovery of the patient.

Keywords:

Glioblastoma Tumor, White and Gray Matter, Homogeneous Medium, Chemotherapy, Kamal Variational Iteration Method.

Introduction

Glioblastoma also known as Glioblastoma multiform (GBM), is a fast growing tumor develops from glial cells that supports the health of the nerve cells within the brain. GBM is high grade IV, most invasive type of [glial tumors](#), rapidly growing and commonly spreading into nearby brain tissues. It mainly occurs in adults and rarely occurs in children. GBM is a devastating brain cancer that typically results in death in the first 15 months after diagnosis [W1].

Brain tumors, whether primary or metastatic, benign or malignant, usually are treated with surgery, radiation, and/or chemotherapy- alone or in various combinations. While it is true that radiation and chemotherapy are used more often for malignant, residual or recurrent tumors, decisions as to what treatment to use are made on a case-by-case basis and depends on a number of factors. There are risks and side effects associated with each type of therapy [W2]. So sometimes doctors are not prefer chemotherapy for some patients.

To solve the mathematical model of applying the different drugs of chemotherapy upon Glioblastoma Tumor cells growth and invasion in homogeneous medium when early cancer cells

are known and unknown, we will use the combination of the new arrival of transform – Kamal Transform and Variational Iteration Method- Kamal Variational Iteration Method.

Methodology:

Kamal transform was introduced by Abdelilah Kamal in 2016 [17], for soft growth of the process of solving ordinary and partial differential equations from time domain to frequency domain.

Kamal Transform is denoted and defined as, [17]

$$K[f(t)] = T(u) = \int_0^{\infty} e^{-\frac{t}{u}} f(t) dt ; t \geq 0, m_1 < u < m_2$$

Kamal Variational Iteration Method (New KVIM): [17, 20]

To exemplify the design of Kamal Variational Iteration Method, we consider the general differential equation, as

$$L[C(x, t)] + N[C(x, t)] = h(x, t) \quad (1)$$

Where L is a Linear partial differential operator given by $\frac{\partial^2}{\partial t^2}$, N is a nonlinear operator and $h(x, t)$ is an analytical continuous source function.

According to the Variational Iteration Method, we can construct a correctional function for (1),

$$C_{n+1}(x, t) = C_n(x, t) + \int_0^t \lambda(x, \tau) [L[C_n(x, \tau)] + N[C_n(x, \tau)] - h(x, \tau)] d\tau, n \geq 0 \quad (2)$$

Where λ is a general Lagrange Multiplier, which can be identified optimally via variational theory. $\bar{C}_n$ is restricted variation. i.e. $\delta \bar{C}_n = 0$.

Take Kamal Transform on both the sides of (2), and doing integration by parts, we get the iteration formula for KVIM as;

$$\therefore K[C_{n+1}(x, t)] = K[C_n(x, t)] + K[-t] \cdot K[L[C_n(x, t)] + N[C_n(x, t)] - h(x, t)] \quad (3)$$

Mathematical Models of Effect of different drugs in Chemotherapy to Glioblastoma Tumor Cells Growth and Invasion in Homogeneous Medium: [10]

We have taken in account the model developed by Cruywagen et al. and Tracqui et al.[3,4] to quantify the effect of different drugs to chemotherapy on tumor growth. This model estimated with parameters to give the best fit of the simulated tumor area to the CT scan data. We carried out numerical simulations on one dimensional domain. Drugs, Combination of drugs in chemotherapy are decided regarding to many factors and also depends upon the patient's cell structure, where it has been placed, how much it spread out etc. In our data the value K_1 is taken for a course of six drugs- 6-thioguanine, procarbazine, dibromodulcitol, CCNN, 5-florouracil and hydroxyurea given over 15 days and repeated every six to eight weeks to allow for recovery of bone marrow [10], and the value K_2 is taken for Cisplatinum drug.

To quantify the effect of drugs of chemotherapy model taken in account is,

$$\frac{\partial C}{\partial t} = D \nabla^2 C + \rho C - K(t)C \quad (4)$$

Where C : Cell Density or No. of cells, t : Time , ρ : Growth Rate, D : Diffusion Coefficient, $K(t)C$: No. of cells killed through drugs.

As we are examine the Effect of different drugs of Chemotherapy to GlioblastomaTumor cells growth and Invasion in homogeneous medium that means tumors are detected in White matter or in Gray Matter only.

Case:1 When spread of early cancer cells are unknown:

$$\frac{\partial W}{\partial t} = D \frac{\partial^2 W}{\partial x^2} + \rho W - K_i W \quad ; \quad W(x,0) = W_0 \quad \text{(White Matter) [9,10] or} \quad (5)$$

$$\frac{\partial G}{\partial t} = D \frac{\partial^2 G}{\partial x^2} + \rho G - K_i G \quad ; \quad G(x,0) = G_0 \quad \text{(Gray Matter) [9,10]} \quad (6)$$

Applying KVIM method, we get the solution; $W(x,t) = W_0 e^{(\rho-K_i)t}$ & $G(x,t) = G_0 e^{(\rho-K_i)t}$

Numerical Parameters:

Parameters	White Matters	Gray Matters
No. of Cells	$W_0 = 20,000 \text{ cells} / \text{cm}^2$	$G_0 = 8000 \text{ cells} / \text{cm}^2$
High ρ	$\rho_w = 0.012 \text{ cm} / \text{day}$ [9,10]	$\rho_G = 0.012 \text{ cm} / \text{day}$ [9,10]
Low ρ	$\rho_w = 0.008 \text{ cm} / \text{day}$ [9,10]	$\rho_G = 0.008 \text{ cm} / \text{day}$ [9,10]
K_1 -Combination of Six Drugs	0.0391 [9,10]	0.0391 [9,10]
K_2 -Cisplatinium	0.0285 [9,10]	0.0285 [9,10]

Graphical Representation:

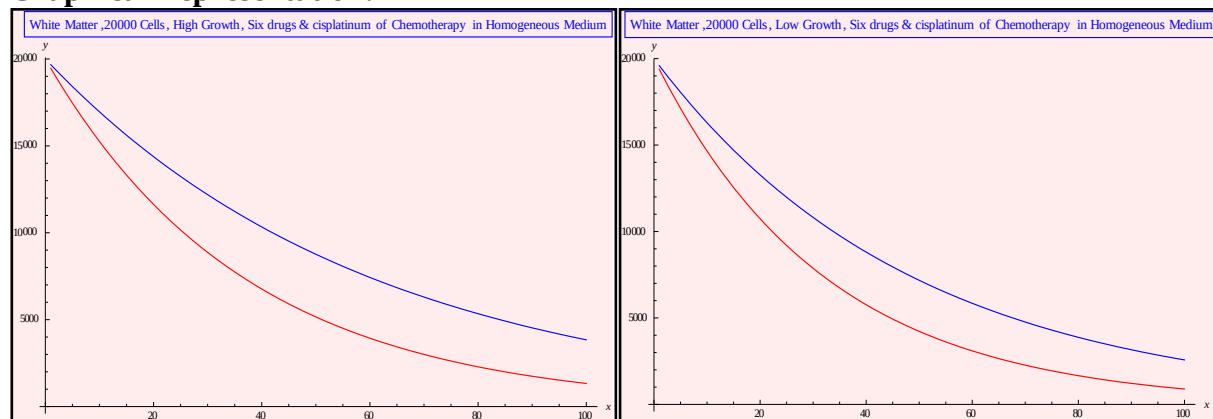


Fig.1 White Matter, High Growth, Six Drugs & Cisplatinium, 20000 Cells

Fig.2 White Matter, Low Growth, Six Drugs & Cisplatinium, 20000 Cells

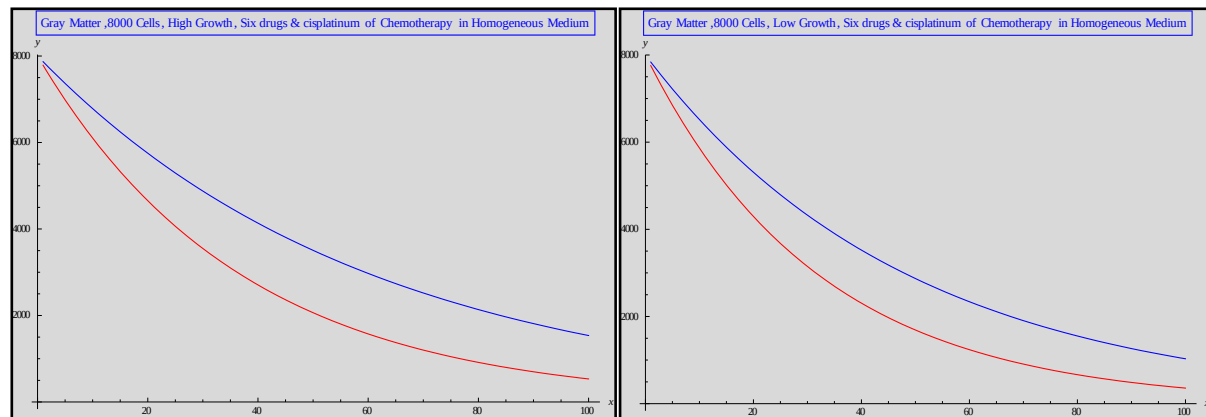


Fig.3 Gray Matter, High Growth, Six Drugs & Cisplatinum, 8000 Cells
Fig.4 Gray Matter, Low Growth, Six Drugs & Cisplatinum, 8000 Cells
 (Combination of Six Drugs-Red Line & Cisplatinum- Blue Line)

Conclusion of case-1:

We can conclude from the graph that, in white matter for high growth rate, no. of cells are decreasing from 20,000 to 100 cells using six drugs combination, while using Cisplatinum no. of cells are reducing to 4000 cells. While in Gray Matter for high growth rate, no. of cells are decreasing from 8,000 to 500 cells using six drugs combination, while using Cisplatinum no. of cells are reducing to 1800 cells.

Case:2 When spread of early cancer cells are known:

$$\frac{\partial W}{\partial t} = D \frac{\partial^2 W}{\partial x^2} + \rho W - K_i W \quad ; \quad W(x, 0) = W_0 e^{-\frac{|x-x_0|^2}{b}} \quad [9,10] \quad (7)$$

$$\frac{\partial G}{\partial t} = D \frac{\partial^2 G}{\partial x^2} + \rho G - K_i G \quad ; \quad G(x, 0) = G_0 e^{-\frac{|x-x_0|^2}{d}} \quad [9,10] \quad (8)$$

Using KVIM, we get the iteration formulas in White and Gray Matter as,

$$W_1(x, t) = W_0 e^{-\frac{|x-x_0|^2}{b}} \left[1 - \frac{2Dt}{b} + \frac{4Dt|x-x_0|^2}{b^2} + \rho t - K_i t \right]$$

$$G_1(x, t) = G_0 e^{-\frac{|x-x_0|^2}{d}} \left[1 - \frac{2Dt}{d} + \frac{4Dt|x-x_0|^2}{d^2} + \rho t - K_i t \right]$$

$$W_2(x, t) = W_0 e^{-\frac{|x-x_0|^2}{b}} \left[1 - \frac{2Dt}{b} + \rho t - K_i t + \frac{4Dt|x-x_0|^2}{b^2} + \frac{6D^2t^2}{b^2} - \frac{2D\rho t^2}{b} + \frac{2K_i Dt^2}{b} + \frac{4\rho Dt^2|x-x_0|^2}{b^2} \right. \\ \left. - \frac{4K_i Dt^2|x-x_0|^2}{b^2} + \frac{\rho^2 t^2}{2} + \frac{K_i^2 t^2}{2} - \rho K_i t^2 - \frac{24D^2t^2|x-x_0|^2}{b^3} + \frac{8D^2t^2|x-x_0|^4}{b^4} \right]$$

$$G_2(x, t) = G_0 e^{-\frac{|x-x_0|^2}{d}} \left[1 - \frac{2Dt}{d} + \rho t - K_i t + \frac{4Dt|x-x_0|^2}{d^2} + \frac{6D^2t^2}{d^2} - \frac{2D\rho t^2}{d} + \frac{2K_i Dt^2}{d} + \frac{4\rho Dt^2|x-x_0|^2}{d^2} \right. \\ \left. - \frac{4K_i Dt^2|x-x_0|^2}{d^2} + \frac{\rho^2 t^2}{2} + \frac{K_i^2 t^2}{2} - \rho K_i t^2 - \frac{24D^2t^2|x-x_0|^2}{d^3} + \frac{8D^2t^2|x-x_0|^4}{d^4} \right]$$

And so on..

For calculation we considered second iteration. No. of cells are taken highest as 40000 and lowest as 500. Tumors with more than 40000 cells, it's difficult to survive for patients, while Tumors less than 500 cells are generally not detected in CT scan.

Numerical Parameters: For HH Grade (High ρ , High D)

Parameters	White Matter	Gray Matter
Velocity of Cells	$v_w = 0.018 \text{ cm / day}$ [9,10]	$v_G = 0.008 \text{ cm / day}$ [9,10]
Diffusion Rate	$D_w = 0.0065 \text{ cm}^2 / \text{day}$ [9,10]	$D_G = 0.0013 \text{ cm}^2 / \text{day}$ [9,10]
Growth Rate	$\rho_w = 0.012 \text{ cm / day}$ [5,6]	$\rho_G = 0.012 \text{ cm / day}$ [9,10]
Number of Cells	$W_0 = a = 40000 \text{ \& } 500$	$G_0 = c = 40000 \text{ \& } 500$
Measure of spread of tumor cells	$b = 0.8 \text{ cm}$	$d = 0.4 \text{ cm}$
Centre	$x_0 = 0$	$x_0 = 0$
K_1 -Combination of Six Drugs	0.0391 [9,10]	0.0391 [9,10]
K_2 -Cisplatinium	0.0285 [9,10]	0.0285 [9,10]

Table: 1 Case-2 HH Grade, White Matter, 40000 cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39503.1	38049.2	35743.9	32749.2	29264.6	25505.1	21679.8	17973.2	14532.4	11460.2
10	27256.8	26443.8	24984.8	23154.	20959.	18925.2	16433.3	14036.8	11901.6	9843.9
20	24490.5	23208.4	20873.7	18340.8	15415.4	13519.3	10709.8	8374.36	6944.78	5705.6
30	31204.2	28343.	23410.6	18309.6	12633.8	9287.43	4509.27	985.96	-338.02	-954.7
40	47397.9	41847.6	32595.5	23060.4	12614.2	6229.53	-2168.23	-8128.44	-9946.82	-10137.
50	73071.6	63722.2	48428.4	32593.2	15356.6	4345.63	-9322.73	-18968.8	-21881.6	-21841.3
60	108225.	93966.8	70909.3	46908.	20861.	3635.73	-16954.2	-31535.2	-36142.4	-36067.6
70	152859.	132581.	100038.	66004.8	29127.4	4099.83	-25062.	-45827.	-52729.	-52815.

							7	6	2	9
80	20697 3.	17956 6.	13581 5.	89883. 6	40155. 8	5737.9 3	- 33648. 2	- 61846.	- 71642.	- 72086. 2
90	27056 6.	23492 1.	17824 0.	11854 4.	53946. 2	8550.0 3	- 42710. 7	- 79590. 4	- 92880. 8	- 93878. 5
100	34364 0.	29864 5.	22731 3.	15198 7.	70498. 6	12536. 1	- 52250. 2	- 99060. 8	- 11644 6.	- 11819 3.

Table: 1a Case-2 HH Grade, White Matter, 40000 cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39503. 1	38049. 2	35743. 9	32749. 2	29264. 6	25505. 1	21679. 8	17973. 2	14532. 4	11460. 2
10	29904. 1	28917.	27415. 2	25511. 4	23213. 1	18491. 2	18319. 8	15744. 9	13442. 1	11173. 3
20	26389. 1	25110. 8	23160. 5	20827. 6	18227. 6	6631.3 3	13659. 8	11396. 6	10025. 8	8364.4
30	28958. 1	26630. 6	22979. 8	18697. 8	14308. 1	- 10074. 6	7699.8 6	4928.2 6	4283.4 8	3033.5
40	37611. 1	33476. 4	26873. 1	19122.	11454. 6	- 31626. 5	439.89	- 3660.0 4	- 3784.8 2	- 4819.4
50	52348. 1	45648. 2	34840. 4	22100. 2	9667.1 3	- 58024. 4	- 8120.0 8	- 14368. 3	- 14179. 1	- 15194. 3
60	73169. 1	63146.	46881. 7	27632. 4	8945.6 3	- 89268. 3	- 17980. 1	- 27196. 6	- 26899. 4	- 28091. 2
70	10007 4.	85969. 8	62997.	35718. 6	9290.1 3	- 12535 8.	- 29140.	- 42144. 9	- 41945. 7	- 43510. 1
80	13306 3.	11412 0.	83186. 3	46358. 8	10700. 6	- 16629 4.	- 41600.	- 59213. 2	- 59318.	- 61451.
90	17213 6.	14759 5.	10745 0.	59553.	13177. 1	- 21207 6.	- 55360.	- 78401. 5	- 79016. 3	- 81913. 9
100	21729 3.	18639 7.	13578 7.	75301. 2	16719. 6	- 26270 4.	- 70419. 9	- 99709. 8	- 10104 1.	- 10489 9.

Table: 2 Case-2 HH Grade, White Matter, 500 cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	493.79	475.61	446.8	409.37	365.81	318.81	270.99	224.66	181.65	143.25
10	340.49	330.21	312.8	289.57	261.71	236.51	205.39	175.26	148.35	122.85
20	305.19	288.81	262.8	229.77	191.61	168.81	133.79	103.86	85.05	70.45
30	387.89	351.41	296.8	229.97	155.51	115.71	56.19	10.46	-8.25	-13.95
40	588.59	518.01	414.8	290.17	153.41	77.21	-27.41	-104.94	-131.55	-130.35
50	907.29	788.61	616.8	410.37	185.31	53.31	-117.01	-242.34	-284.85	-278.75
60	1343.99	1163.21	902.8	590.57	251.21	44.01	-212.61	-401.74	-468.15	-459.15
70	1898.69	1641.81	1272.8	830.77	351.11	49.31	-314.21	-583.14	-681.45	-671.55
80	2571.39	2224.41	1726.8	1130.97	485.01	69.21	-421.81	-786.54	-924.75	-915.95
90	3362.09	2911.01	2264.8	1491.17	652.91	103.71	-535.41	-1011.94	-1198.05	-1192.35
100	4270.79	3701.61	2886.8	1911.37	854.81	152.81	-655.01	-1259.34	-1501.35	-1500.75

Table: 2a Case-2 HH Grade, White Matter, 500 cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	493.79	475.61	446.8	409.37	365.81	318.81	270.99	224.66	181.65	143.25
10	373.79	361.21	343.2	318.97	290.21	231.11	229.09	197.06	167.56	139.45
20	329.79	312.81	291.6	260.57	228.01	82.81	171.19	143.46	123.47	103.65
30	361.79	330.41	292.	234.17	179.21	-126.09	97.29	63.86	49.38	35.85
40	469.79	414.01	344.4	239.77	143.81	-395.59	7.39	-41.74	-54.71	-63.95
50	653.79	563.61	448.8	277.3	121.8	-	-98.51	-	-188.8	-

				7	1	725.69		173.34		195.75
60	913.79	779.21	605.2	346.9 7	113.2 1	- 1116.3 9	- 220.4 1	- 330.94	- 352.89	- 359.55
70	1249.7 9	1060.8 1	813.6	448.5 7	118.0 1	- 1567.6 9	- 358.3 1	- 514.54	- 546.98	- 555.35
80	1661.7 9	1408.4 1	1074.	582.1 7	136.2 1	- 2079.5 9	- 512.2 1	- 724.14	- 771.07	- 783.15
90	2149.7 9	1822.0 1	1386. 4	747.7 7	167.8 1	- 2652.0 9	- 682.1 1	- 959.74	- 1025.1 6	- 1042.9 5
100	2713.7 9	2301.6 1	1750. 8	945.3 7	212.8 1	- 3285.1 9	- 868.0 1	- 1221.3 4	- 1309.2 5	- 1334.7 5

Table: 3 Case-2 HH Grade, Gray Matter, 40000 cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39012. 4	36193. 5	31940. 7	26812. 8	21410. 5	16262. 8	11750. 3	8075.8 6	5279.7 5	3283.4
10	28401. 3	26457.	23635. 9	20217.	16485. 8	12945.	9670.2 1	6937.2 6	4751.4 5	3128.7
20	22316. 2	20628. 5	18333. 1	15659. 2	12631. 1	10017. 2	7566.1 1	5556.6 6	3969.1 5	2704.
30	20757. 1	18708.	16032. 3	13139. 4	9846.3 6	7479.3 9	5438.0 1	3934.0 6	2932.8 5	2009.3
40	23724.	20695. 5	16733. 4	12657. 6	8131.6 6	5331.5 9	3285.9 1	2069.4 6	1642.5 5	1044.6
50	31216. 9	26591.	20436. 7	14213. 8	7486.9 6	3573.7 9	1109.8 1	-37.14	98.25	-190.1
60	43235. 8	36394. 5	27141. 9	17808.	7912.2 6	2205.9 9	- 1090.2 9	- 2385.7 4	- 1700.0 5	- 1694.8
70	59780. 7	50106.	36849.	23440. 2	9407.5 6	1228.1 9	- 3314.3 9	- 4976.3 4	- 3752.3 5	- 3469.5
80	80851. 6	67725. 5	49558. 2	31110. 4	11972. 9	640.39	- 5562.4 9	- 7808.9 4	- 6058.6 5	- 5514.2
90	10644 9.	89253.	65269. 5	40818. 6	15608. 2	442.59	- 7834.5 9	- 10883. 5	- 8618.9 5	- 7828.9
100	13657 1.	11468 9.	83982. 7	52564. 8	20313. 5	634.79	- 10130.	- 14200.	- 11433.	- 10413.

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Table: 3a Case-2 HH Grade, Gray Matter, 40000 cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39012.4	36193.5	31940.7	26812.8	21410.5	16262.8	11750.3	8075.86	5279.75	3283.4
10	31248.5	29207.9	26223.6	22388.5	21219.9	14421.9	10786.8	7744.36	5295.15	3490.2
20	25902.6	24176.3	21912.5	18768.2	21291.3	12477.	9541.31	7072.86	5024.55	3486.8
30	22974.7	21098.7	19007.4	15951.9	21624.7	10428.1	8013.81	6061.36	4467.95	3273.2
40	22464.8	19975.1	17508.3	13939.6	22220.1	8275.19	6204.31	4709.86	3625.35	2849.4
50	24372.9	20805.5	17415.2	12731.3	23077.5	6018.29	4112.81	3018.36	2496.75	2215.4
60	28699.	23589.9	18728.1	12327.	24196.9	3657.39	1739.31	986.86	1082.15	1371.2
70	35443.1	28328.3	21447.	12726.7	25578.3	1192.49	-916.19	-1384.64	-618.45	316.8
80	44605.2	35020.7	25571.9	13930.4	27221.7	-1376.41	-3853.69	-4096.14	-2605.05	-947.8
90	56185.3	43667.1	31102.8	15938.1	29127.1	-4049.31	-7073.19	-7147.64	-4877.65	-2422.6
100	70183.4	54267.5	38039.7	18749.8	31294.5	-6826.21	-10574.7	-10539.1	-7436.25	-4107.6

Table: 4 Case-2 HH Grade, Gray Matter, 500 cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	487.65	452.42	399.26	335.16	267.63	203.28	146.88	100.95	65.997	41.04
10	354.75	330.32	295.66	252.96	206.43	161.38	120.83	86.75	59.397	39.09
20	277.85	256.22	230.06	196.76	159.23	123.48	94.48	69.55	49.597	33.74
30	256.95	230.12	202.46	166.56	126.03	89.58	67.83	49.35	36.597	24.99
40	292.05	252.02	212.86	162.36	106.83	59.68	40.88	26.15	20.397	12.84
50	383.15	321.92	261.26	184.1	101.6	33.78	13.63	-0.05	0.997	-2.71

				6	3					
60	530.25	439.82	347.66	231.9 6	110.4 3	11.88	-13.92	-29.25	-21.603	-21.66
70	733.35	605.72	472.06	305.7 6	133.2 3	-6.02	-41.77	-61.45	-47.403	-44.01
80	992.45	819.62	634.46	405.5 6	170.0 3	-19.92	-69.92	-96.65	-76.403	-69.76
90	1307.5 5	1081.5 2	834.86	531.3 6	220.8 3	-29.82	-98.37	- 134.8 5	- 108.60 3	-98.91
100	1678.6 5	1391.4 2	1073.2 6	683.1 6	285.6 3	-35.72	- 127.1 2	- 176.0 5	- 144.00 3	- 131.4 6

Table: 4a Case-2 HH Grade, Gray Matter, 500 cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	487.65	452.42	399.26	335.16	267.63	203.28	146.88	100.95	65.997	41.04
10	390.45	364.92	327.96	279.86	229.53	180.23	134.78	96.831	66.177	43.64
20	323.25	301.42	274.66	234.56	195.43	155.88	119.08	88.512	62.757	43.64
30	286.05	261.92	239.36	199.26	165.33	130.23	99.78	75.993	55.737	41.04
40	278.85	246.42	222.06	173.96	139.23	103.28	76.88	59.274	45.117	35.84
50	301.65	254.92	222.76	158.66	117.13	75.03	50.38	38.355	30.897	28.04
60	354.45	287.42	241.46	153.36	99.03	45.48	20.28	13.236	13.077	17.64
70	437.25	343.92	278.16	158.06	84.93	14.63	-13.42	- 16.083	-8.343	4.64
80	550.05	424.42	332.86	172.76	74.83	-17.52	-50.72	- 49.602	- 33.363	- 10.96
90	692.85	528.92	405.56	197.46	68.73	-50.97	-91.62	- 87.321	- 61.983	- 29.16
100	865.65	657.42	496.26	232.16	66.63	-85.72	- 136.12	- 129.24	- 94.203	- 49.96

Graphical Representation: HH Grade: High ρ , High D

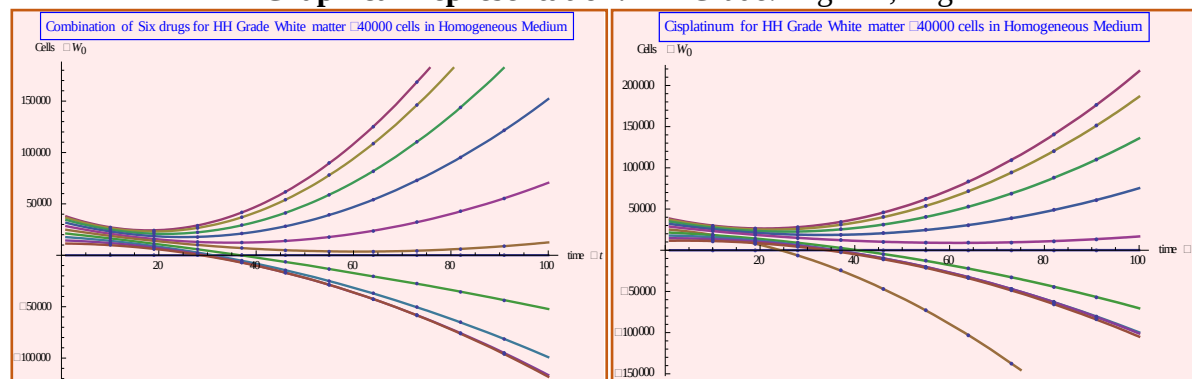


Fig. 5 & 5a: HH Grade, White Matter, 40000 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

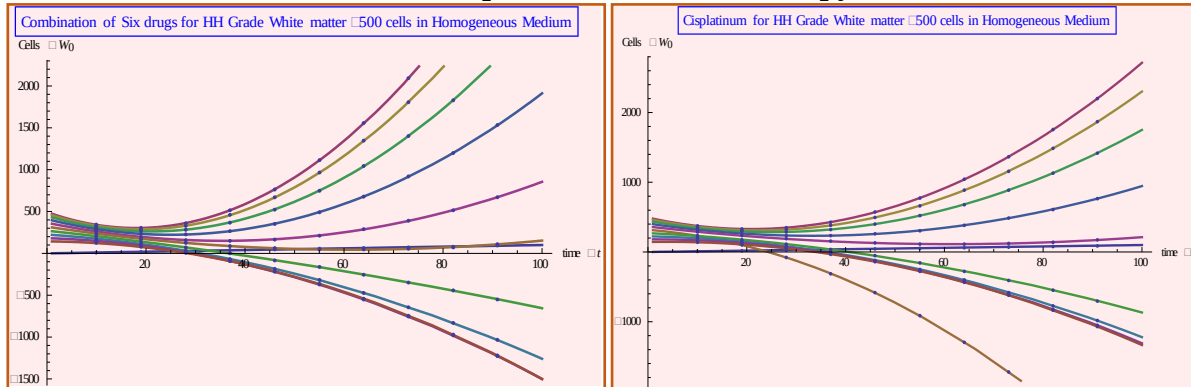


Fig. 6 & 6a: HH Grade, White Matter, 500 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

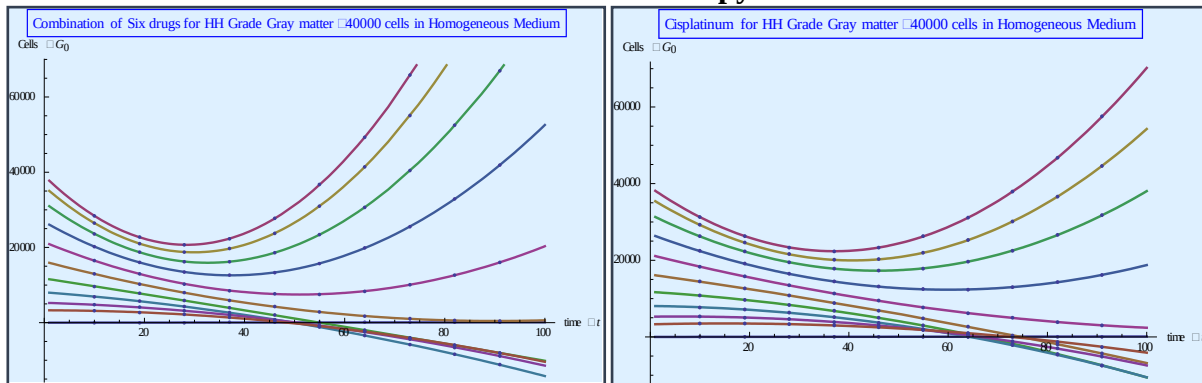


Fig. 7 & 7a: HH Grade, Gray Matter, 40000 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

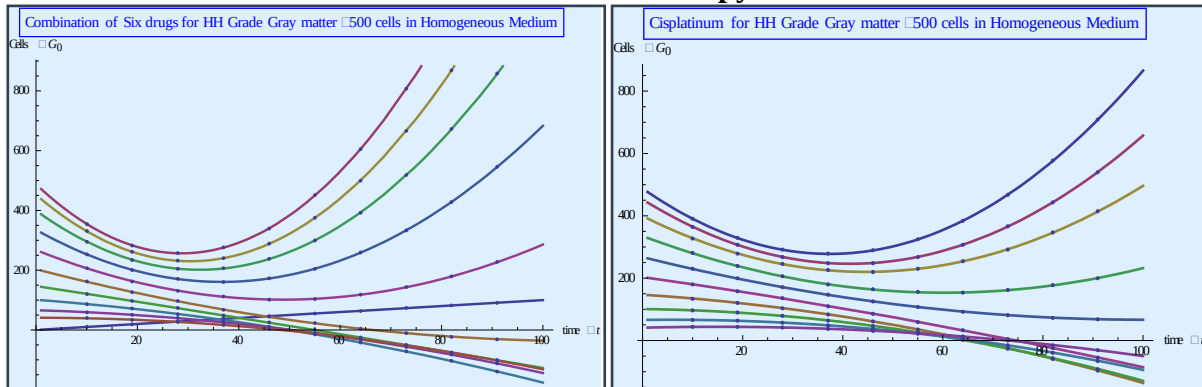


Fig. 8 & 8a: HH Grade, Gray Matter, 500 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

Numerical Parameters: For HL Grade (High ρ , Low D)

Parameters	White Matter	Gray Matter
Velocity of Cells	$v_w = 0.00088 \text{ cm / day}$	$v_g = 0.0004 \text{ cm / day}$
Diffusion Rate	$D_w = 0.000016 \text{ cm}^2 / \text{day}$	$D_g = 0.0000033 \text{ cm}^2 / \text{day}$
Growth Rate	$\rho_w = 0.012 \text{ cm / day}$ [9,10]	$\rho_g = 0.012 \text{ cm / day}$ [9,10]

Number of Cells	$W_0 = a = 40000 \text{ \& } 500$	$G_0 = c = 40000 \text{ \& } 500$
Measure of spread of tumor cells	$b = 0.8 \text{ cm}$	$d = 0.4 \text{ cm}$
Centre	$x_0 = 0$	$x_0 = 0$
K_1 - Combination of Six Drugs	0.0391 [9,10]	0.0391 [9,10]
K_2 -Cisplatinium	0.0285 [9,10]	0.0285 [9,10]

Table: 5 Case-2 HL Grade, White Matter, 40000 cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39503. 1	38049. 2	35743. 9	32749. 2	29264. 6	25505. 1	21679. 8	17973. 2	14532. 4	11460. 2
10	30259. 3	29145. 9	27380. 9	25085. 9	22417. 1	19536. 7	16606. 3	13767. 4	11131. 7	8778.9
20	23859. 5	22982. 6	21590. 1	19780. 6	17677. 6	15404. 3	13092. 8	10855. 6	8776.9 8	6923.6
30	20303. 7	19559. 3	18374. 2	16833. 3	15046. 1	13107. 9	11139. 3	9237.7 6	7468.2 8	5894.3
40	19591. 9	18876. 3	17732. 3	16244. 3	14522. 6	12647. 5	10745. 8	8913.9 6	7205.5 8	5691.
50	21724. 1	20932. 7	19664. 4	18012. 7	16107. 1	14023. 1	11912. 3	9884.1 6	7988.8 8	6313.7
60	26700. 3	25729. 4	24170. 5	22139. 4	19799. 6	17234. 7	14638. 8	12148. 4	9818.1 8	7762.4
70	34520. 5	33266. 1	31250. 6	28624. 1	25600. 1	22282. 3	18925. 3	15706. 6	12693. 5	10037. 1
80	45184. 7	43542. 8	40904. 7	37466. 8	33508. 6	29165. 9	24771. 8	20558. 8	16614. 8	13137. 8
90	58692. 9	56559. 5	53132. 8	48667. 5	43525. 1	37885. 5	32178. 3	26705. 3	21582. 1	17064. 5
100	75045. 1	72316. 2	67934. 9	62226. 2	55649. 6	48441. 1	41144. 8	34145. 2	27595. 4	21817. 2

Table: 5a Case-2 HL Grade, White Matter, 40000 Cells, Cisplatinium in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39503. 1	38049. 2	35743. 9	32749. 2	29264. 6	25505. 1	21679. 8	17973. 2	14532. 4	11460. 2
10	33340. 6	32113. 8	30167. 4	27639. 8	24699. 6	21526. 2	18298. 2	15169. 8	12264. 9	9672.
20	28284. 1	27244. 4	25590. 9	23446. 4	20954. 6	18261. 3	15524. 6	12870. 4	10403. 4	8203.8
30	24333. 6	23441. 4	22014. 4	20169. 4	18029. 6	15710. 4	13359. 4	11075. 8	8947.8 8	7055.6
40	21489.	20703.	19437.	17807.	15924.	13873.	11801.	9783.5	7898.3	6227.4

	1	6	9	6	6	5	4	6	8	
50	19750. 6	19032. 2	17861. 4	16362. 2	14639. 6	12750. 6	10851. 8	8996.1 6	7254.8 8	5719.2
60	19118. 1	18426. 8	17284. 9	15832. 8	14174. 6	12341. 7	10510. 2	8712.7 6	7017.3 8	5531.
70	19591. 6	18887. 4	17708. 4	16219. 4	14529. 6	12646. 8	10776. 6	8933.3 6	7185.8 8	5662.8
80	21171. 1	20414. 9	19131. 9	17522. 6	15704. 6	13665. 9	11651. 6	9657.9 6	7760.3 8	6114.6
90	23856. 6	23006. 6	21555. 4	19740. 6	17699. 6	15399. 4	13133. 4	10886. 6	8740.8 8	6886.4
100	27648. 1	26665. 2	24978. 9	22875. 2	20514. 6	17846. 1	15223. 8	12619. 2	10127. 4	7978.2

Table: 6 Case-2 HL Grade, White Matter, 500 Cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	493.79	475.61	446.8	409.37	365.81	318.81	270.99	224.66	181.65	143.25
10	378.49	364.21	342.2	313.87	280.01	243.71	207.79	171.96	139.65	109.55
20	299.19	286.81	269.6	248.37	220.21	190.61	164.59	135.26	111.65	85.85
30	255.89	243.41	229.	212.87	186.41	159.51	141.39	114.56	97.65	72.15
40	248.59	234.01	220.4	207.37	178.61	150.41	138.19	109.86	97.65	68.45
50	277.29	258.61	243.8	231.87	196.81	163.31	154.99	121.16	111.65	74.75
60	341.99	317.21	299.2	286.37	241.01	198.21	191.79	148.46	139.65	91.05
70	442.69	409.81	386.6	370.87	311.21	255.11	248.59	191.76	181.65	117.35
80	579.39	536.41	506.	485.37	407.41	334.01	325.39	251.06	237.65	153.65
90	752.09	697.01	657.4	629.87	529.61	434.91	422.19	326.36	307.65	199.95
100	960.79	891.61	840.8	804.37	677.81	557.81	538.99	417.66	391.65	256.25

Table: 6a Case-2 HL Grade, White Matter, 500 Cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	493.79	475.61	446.8	409.37	365.81	318.81	270.99	224.66	181.65	143.25
10	416.79	401.41	377.1	345.47	308.71	269.11	228.69	189.56	153.25	120.85
20	353.59	340.61	320.	292.97	261.81	228.41	193.99	160.66	129.85	102.45
30	304.19	293.21	275.5	251.87	225.11	196.71	166.89	137.96	111.45	88.05
40	268.59	259.21	243.6	222.17	198.61	174.01	147.39	121.46	98.05	77.65
50	246.79	238.61	224.3	203.87	182.31	160.31	135.49	111.16	89.65	71.25
60	238.79	231.41	217.6	196.97	176.21	155.61	131.19	107.06	86.25	68.85
70	244.59	237.61	223.5	201.47	180.31	159.91	134.49	109.16	87.85	70.45
80	264.19	257.21	242.	217.37	194.61	173.21	145.39	117.46	94.45	76.05
90	297.59	290.21	273.1	244.67	219.11	195.51	163.89	131.96	106.05	85.65
100	344.79	336.61	316.8	283.37	253.81	226.81	189.99	152.66	122.65	99.25

Table: 7 Case-2 HL Grade, Gray Matter, 40000 Cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39012.4	36193.5	31940.7	26812.8	21410.5	16262.8	11750.3	8075.86	5279.75	3283.4
10	29883.1	27724.3	24466.7	20538.3	16400.7	12456.8	9000.71	6186.36	4044.25	2514.9
20	23561.8	21861.1	19292.7	16193.8	12932.9	9820.79	7097.11	4878.86	3188.75	1982.4
30	20048.5	18603.9	16418.7	13779.3	11007.1	8354.79	6039.51	4153.36	2713.25	1685.9
40	19343.2	17952.7	15844.7	13294.8	10623.3	8058.79	5827.91	4009.86	2617.75	1625.4
50	21445.9	19907.5	17570.7	14740.3	11781.5	8932.79	6462.31	4448.36	2902.25	1800.9
60	26356.6	24468.3	21596.7	18115.8	14481.7	10976.8	7942.71	5468.86	3566.75	2212.4
70	34075.3	31635.1	27922.7	23421.3	18723.9	14190.8	10269.1	7071.36	4611.25	2859.9
80	44602.	41407.9	36548.7	30656.8	24508.1	18574.8	13441.5	9255.86	6035.75	3743.4
90	57936.7	53786.7	47474.7	39822.3	31834.3	24128.8	17459.9	12022.4	7840.25	4862.9
100	74079.4	68771.5	60700.7	50917.8	40702.5	30852.8	22324.3	15370.9	10024.7	6218.4

Table: 7a Case-2 HL Grade, Gray Matter, 40000 Cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39012.4	36193.5	31940.7	26812.8	21410.5	16262.8	11750.3	8075.86	5279.75	3283.4
10	32926.3	30547.6	26957.8	22629.6	18070.4	13726.1	9917.71	6815.96	4456.15	2771.2
20	27932.2	25915.7	22868.9	19196.4	15329.7	11645.4	8415.11	5782.06	3780.55	2351.
30	24030.1	22297.8	19674.	16513.2	13188.4	10020.7	7242.51	4974.16	3252.95	2022.8
40	21220.	19693.9	17373.1	14580.	11646.5	8851.99	6399.91	4392.26	2873.35	1786.6
50	19501.9	18104.	15966.2	13396.8	10704.	8139.29	5887.31	4036.36	2641.75	1642.4
60	18875.8	17528.1	15453.3	12963.6	10360.9	7882.59	5704.71	3906.46	2558.15	1590.2
70	19341.7	17966.2	15834.3	13280.4	10617.2	8081.89	5852.11	4002.56	2622.55	1630.
80	20899.	19418.	17109.	14347.	11472.	8737.1	6329.5	4324.6	2834.9	1761.

	6	3	5	2	9	9	1	6	5	8
90	23549.5	21884.4	19278.6	16164.	12928.	9848.49	7136.91	4872.76	3195.35	1985.6
100	27291.4	25364.5	22341.7	18730.8	14982.5	11415.8	8274.31	5646.86	3703.75	2301.4

Table: 8 Case-2 HL Grade, Gray Matter, 500 Cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	487.65	452.42	399.26	335.16	267.63	203.28	146.88	100.95	65.997	41.04
10	373.95	346.22	305.46	256.66	205.33	155.38	112.18	77.65	50.197	30.94
20	296.25	272.02	239.66	202.16	163.03	121.48	87.48	62.35	38.397	22.84
30	254.55	229.82	201.86	171.66	140.73	101.58	72.78	55.05	30.597	16.74
40	248.85	219.62	192.06	165.16	138.43	95.68	68.08	55.75	26.797	12.64
50	279.15	241.42	210.26	182.66	156.13	103.78	73.38	64.45	26.997	10.54
60	345.45	295.22	256.46	224.16	193.83	125.88	88.68	81.15	31.197	10.44
70	447.75	381.02	330.66	289.66	251.53	161.98	113.98	105.85	39.397	12.34
80	586.05	498.82	432.86	379.16	329.23	212.08	149.28	138.55	51.597	16.24
90	760.35	648.62	563.06	492.66	426.93	276.18	194.58	179.25	67.797	22.14
100	970.65	830.42	721.26	630.16	544.63	354.28	249.88	227.95	87.997	30.04

Table: 8a Case-2 HL Grade, Gray Matter, 500 Cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	487.65	452.42	399.26	335.16	267.63	203.28	146.88	100.95	65.997	41.04
10	411.55	381.82	336.96	282.86	225.83	171.48	124.01	85.15	55.697	34.66
20	349.05	323.82	285.86	239.96	191.43	145.28	105.34	72.15	47.197	29.48
30	300.15	278.42	245.96	206.46	164.43	124.68	90.87	61.95	40.497	25.5
40	264.85	245.62	217.26	182.36	144.83	109.68	80.6	54.55	35.597	22.72
50	243.15	225.42	199.76	167.66	132.63	100.28	74.53	49.95	32.497	21.14
60	235.05	217.82	193.46	162.36	127.83	96.48	72.66	48.15	31.197	20.76
70	240.55	222.82	198.36	166.46	130.43	98.28	74.99	49.15	31.697	21.58
80	259.65	240.42	214.46	179.96	140.43	105.68	81.52	52.95	33.997	23.6
90	292.35	270.62	241.76	202.86	157.83	118.68	92.25	59.55	38.097	26.82
100	338.65	313.42	280.26	235.16	182.63	137.28	107.18	68.95	43.997	31.24

Graphical Representation: HL Grade: High ρ , Low D

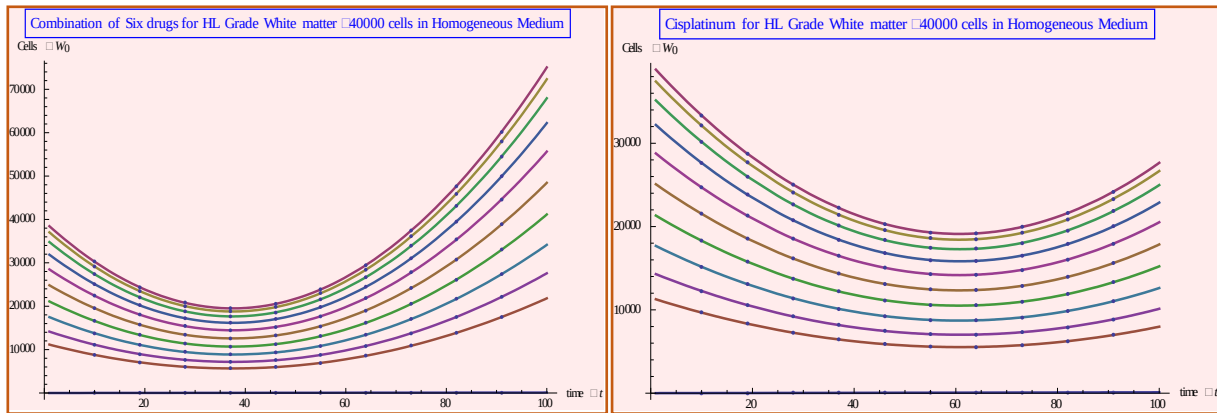


Fig. 9 & 9a: HL Grade, White Matter, 40000 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

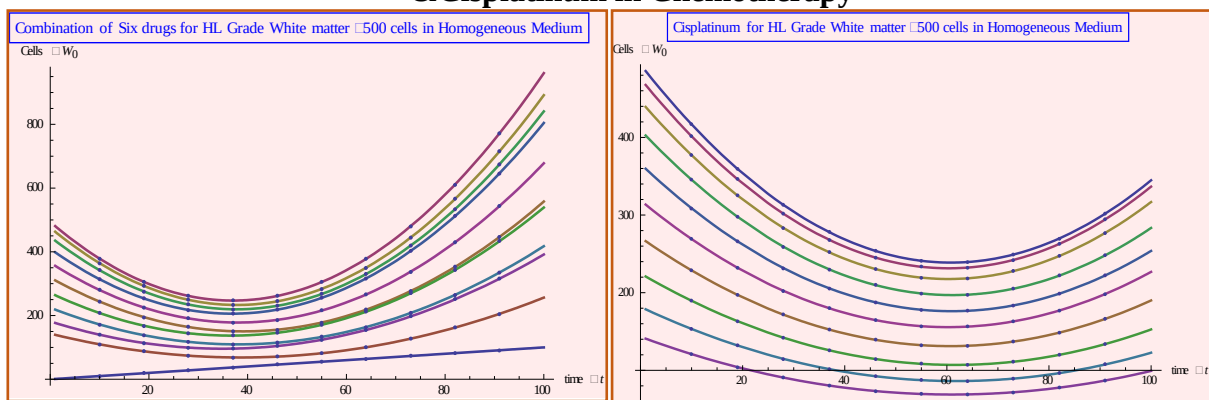


Fig. 10 & 10a: HL Grade, White Matter, 500 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

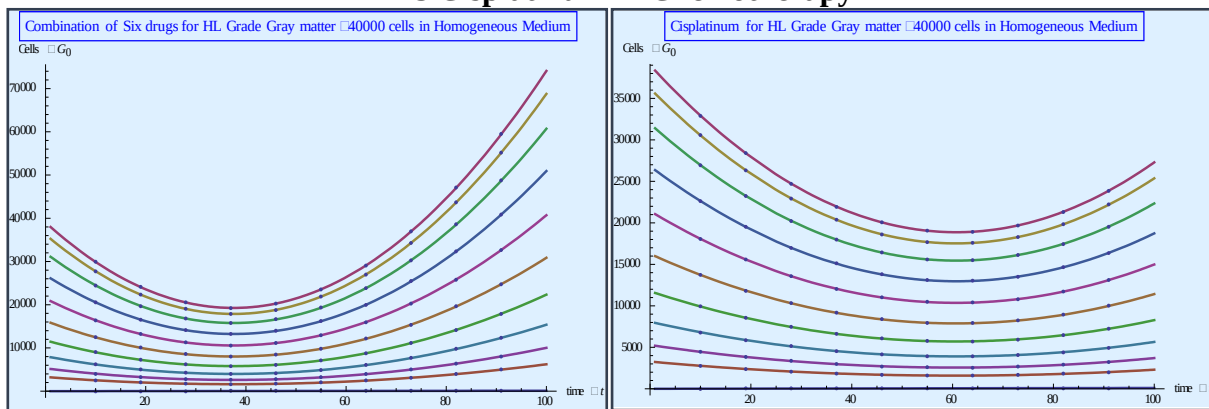


Fig. 11 & 11a: HL Grade, Gray Matter, 40000 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

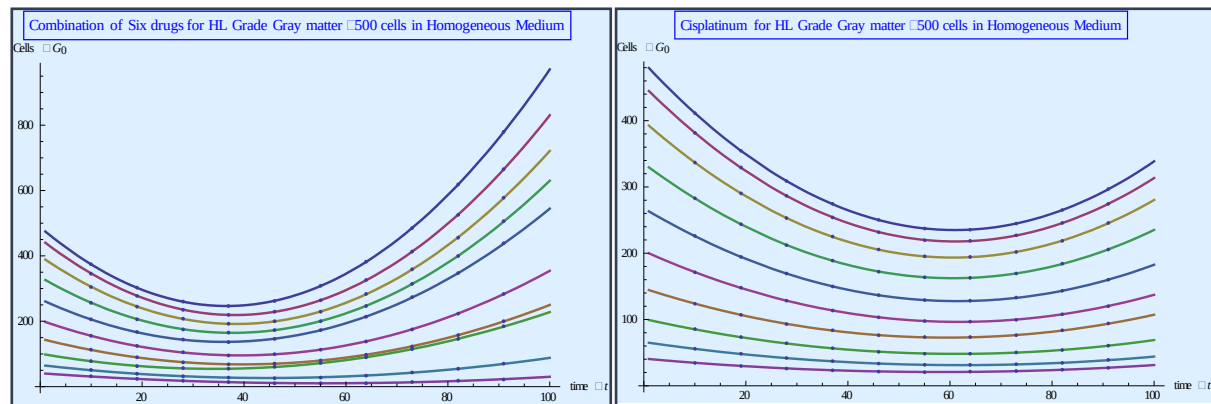


Fig. 12 & 12a: HL Grade, Gray Matter, 500 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

Numerical Parameters: For LH Grade (Low ρ , High D)

Parameters	White Matter	Gray Matter
Velocity of Cells	$v_w = 0.018 \text{ cm / day}$ [9,10]	$v_G = 0.008 \text{ cm / day}$ [9,10]
Diffusion Rate	$D_w = 0.0065 \text{ cm}^2 / \text{day}$ [9,10]	$D_G = 0.0013 \text{ cm}^2 / \text{day}$ [9,10]
Growth Rate	$\rho_w = 0.008 \text{ cm / day}$ [9,10]	$\rho_G = 0.008 \text{ cm / day}$ [9,10]
Number of Cells	$W_0 = a = 40000 \text{ \& } 500$	$G_0 = c = 40000 \text{ \& } 500$
Measure of spread of tumor cells	$b = 0.8 \text{ cm}$	$d = 0.4 \text{ cm}$
Centre	$x_0 = 0$	$x_0 = 0$
K_1	0.0391 [9,10]	0.0391 [9,10]

Table: 9 Case-2 LH Grade, White Matter, 40000 Cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39503.1	38049.2	35743.9	32749.2	29264.6	25505.1	21679.8	17973.2	14532.4	11460.2
10	10389.4	10083.0	95436.9	88095.7	76965.7	68353.5	59186.3	50144.8	40109.9	32432.9
20	33419.8	32341.7	30525.2	28098.7	24172.5	21322.2	18341.3	15420.8	12091.1	96951.8
30	73041.7	70581.0	66519.2	61142.5	52354.2	46011.0	39435.9	33016.4	25693.7	20502.0
40	1.29255*10 ⁶	1.24801*10 ⁶	1.17526*10 ⁶	1.07941*10 ⁶	92241.7	80901.9	69202.6	57801.2	44818.6	35663.5
50	2.0206*10 ⁶	1.95001*10 ⁶	1.83544*10 ⁶	1.68494*10 ⁶	1.43835*10 ⁶	1.25995*10 ⁶	1.07641*10 ⁶	89775.1	69466.0	55179.9
60	2.91456*10 ⁶	2.81182*10 ⁶	2.64576*10 ⁶	2.42801*10 ⁶	2.07134*10 ⁶	1.8129*10 ⁶	1.54752*10 ⁶	1.28938*10 ⁶	99635.7	79051.1

70	3.9744 3*10 ⁶	3.8334 4*10 ⁶	3.6061 9*10 ⁶	3.3086 4*10 ⁶	2.8213 9*10 ⁶	2.4678 6*10 ⁶	2.1053 5*10 ⁶	1.7529 1*10 ⁶	1.3532 8*10 ⁶	1.0727 7*10 ⁶
80	5.2002 2*10 ⁶	5.0148 6*10 ⁶	4.7167 5*10 ⁶	4.3268 *10 ⁶	3.6885 *10 ⁶	3.2248 5*10 ⁶	2.7498 9*10 ⁶	2.2883 2*10 ⁶	1.7654 2*10 ⁶	1.3985 8*10 ⁶
90	6.5919 2*10 ⁶	6.3560 9*10 ⁶	5.9774 4*10 ⁶	5.4825 2*10 ⁶	4.6726 6*10 ⁶	4.0838 6*10 ⁶	3.4811 6*10 ⁶	2.8956 3*10 ⁶	2.2327 9*10 ⁶	1.7679 3*10 ⁶
100	8.1495 4*10 ⁶	7.8571 3*10 ⁶	7.3882 4*10 ⁶	6.7757 8*10 ⁶	5.7738 9*10 ⁶	5.0448 9*10 ⁶	4.2991 4*10 ⁶	3.5748 3*10 ⁶	2.7553 9*10 ⁶	2.1808 4*10 ⁶

Table: 9a Case-2 LH Grade, White Matter, 40000 Cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39503.1	38049.2	35743.9	32749.2	29264.6	25505.1	21679.8	17973.2	14532.4	11460.2
10	28481.9	27890.	26414.4	24233.9	22182.1	20098.	17474.2	15007.2	12788.1	10657.6
20	24650.7	24046.8	22016.9	18992.6	16737.6	15098.9	12228.6	10063.2	8717.7	7333.
30	28009.5	26519.6	22551.4	17025.3	12931.1	10507.8	5942.9	3141.1	2321.4	1486.4
40	38558.3	35308.4	28017.9	18332.	10762.6	6324.7	-	-	-	-
							1382.6	5758.8	6400.8	6882.2
							3	4	2	
50	56297.1	50413.2	38416.4	22912.7	10232.1	2549.6	-	-	-	-
							9748.2	16636.8	17449.1	17772.8
							3	8	1	8
60	81225.9	71834.	53746.9	30767.4	11339.6	-	-	-	-	-
							817.47	19153.8	29492.8	30823.4
70	11334.5.	99570.8	74009.4	41896.1	14085.1	-	-	-	-	-
							3776.5	29599.4	44326.8	46523.7
							7	4	8	
80	15265.4.	13362.4.	99203.9	56298.8	18468.6	-	-	-	-	-
							6327.6	41085.	61138.8	64550.
							7			
90	19915.2.	17399.2.	12933.0.	73975.5	24490.1	-	-	-	-	-
							8470.7	53610.6	79928.8	84902.3
							7			
100	25284.1.	22067.7.	16438.9.	94926.2	32149.6	-	-	-	-	-
							10205.9	67176.2	10069.7.	10758.1.

Table: 10 Case-2 LH Grade, White Matter, 500 Cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	493.79	475.61	446.8	409.37	365.81	318.81	270.99	224.66	181.65	143.25
10	1298.69	1260.61	1192.7	1101.57	962.51	854.81	739.79	626.46	501.25	405.25
20	4177.59	4043.61	3814.6	3513.77	3023.21	2666.81	2292.59	1926.26	1510.85	1211.25
30	9130.49	8824.61	8312.5	7645.97	6547.91	5754.81	4929.39	4124.06	3210.45	2561.25
40	16157.4	15603.6	14686.4	13498.2	11536.6	10118.8	8650.19	7219.86	5600.05	4455.25
50	25258.3	24380.6	22936.3	21070.4	17989.3	15758.8	13455.	11213.7	8679.65	6893.25
60	36433.2	35155.6	33062.2	30362.6	25906.	22674.8	19343.8	16105.5	12449.3	9875.25
70	49682.1	47928.6	45064.1	41374.8	35286.7	30866.8	26316.6	21895.3	16908.9	13401.3
80	65005.	62699.6	58942.	54107.	46131.4	40334.8	34373.4	28583.1	22058.5	17471.3
90	82401.9	79468.6	74695.9	68559.2	58440.1	51078.8	43514.2	36168.9	27898.1	22085.3
100	101873.	98235.6	92325.8	84731.4	72212.8	63098.8	53739.	44652.7	34427.7	27243.3

Table: 10a Case-2 LH Grade, White Matter, 500 Cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	493.79	475.61	446.8	409.37	365.81	318.81	270.99	224.66	181.65	143.25
10	356.09	348.11	330.4	303.47	277.25	251.31	218.39	187.56	159.35	132.95
20	308.39	298.61	276.	239.57	209.17	189.01	152.79	125.66	107.05	90.65
30	350.69	327.11	283.6	217.67	161.57	131.91	74.19	38.96	24.75	16.35
40	482.99	433.61	353.2	237.77	134.45	80.01	-17.41	-72.54	-87.55	-89.95
50	705.29	618.11	484.8	299.87	127.81	33.31	-122.01	-208.84	-229.85	-228.25
60	1017.59	880.61	678.4	403.97	141.65	-8.19	-239.61	-369.94	-402.15	-398.55
70	1419.89	1221.11	934.	550.07	175.97	-44.49	-370.2	-555.84	-604.45	-600.85

							1			
80	1912.1 9	1639.6 1	1251. 6	738.17	230.7 7	-75.59	- 513.8 1	- 766.54	- 836.75	- 835.15
90	2494.4 9	2136.1 1	1631. 2	968.27	306.0 5	- 101.4 9	- 670.4 1	- 1002.0 4	- 1099.0 5	- 1101.4 5
100	3166.7 9	2710.6 1	2072. 8	1240.3 7	401.8 1	- 122.1 9	- 840.0 1	- 1262.3 4	- 1391.3 5	- 1399.7 5

Table: 11 Case-2 LH Grade, Gray Matter, 40000 Cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39012. 4	36193. 5	31940. 7	26812. 8	21410. 5	16262. 8	11750. 3	8075.8 6	5279.7 5	3283.4
10	26996. 7	25226. 9	22677. 5	19305. 6	15758. 4	12376. 5	9247.2 1	6629.9 6	4535.1 5	2988.2
20	20599.	19110. 3	17246. 3	14480. 4	11648. 3	9174.1 9	6908.1 1	5038.0 6	3568.5 5	2431.
30	19819. 3	17843. 7	15647.	12337. 2	9080.1 6	6655.8 9	4733.0 1	3300.1 6	2379.9 5	1611.8
40	24657. 6	21427. 1	17879. 9	12876.	8054.0 6	4821.5 9	2721.9 1	1416.2 6	969.35	530.6
50	35113. 9	29860. 5	23944. 7	16096. 8	8569.9 6	3671.2 9	874.81	- 613.64	- 663.25	-812.6
60	51188. 2	43143. 9	33841. 4	21999. 6	10627. 9	3204.9 9	- 808.29	- 2789.5 4	- 2517.8 5	- 2417.8
70	72880. 5	61277. 3	47570. 2	30584. 4	14227. 8	3422.6 9	- 2327.3 9	- 5111.4 4	- 4594.4 5	-4285.
80	10019 1.	84260. 7	65131.	41851. 2	19369. 7	4324.3 9	- 3682.4 9	- 7579.3 4	- 6893.0 5	- 6414.2
90	13311 9.	11209 4.	86523. 9	55800.	26053. 6	5910.0 9	- 4873.5 9	- 10193. 2	- 9413.6 5	- 8805.4
100	17166 5.	14477 8.	11174 9.	72430. 8	34279. 5	8179.7 9	- 5900.6 9	- 12953. 1	- 12156. 3	- 11458. 6

Table: 11a Case-2 LH Grade, Gray Matter, 40000 Cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	39012.	36193.	31940.	26812.	21410.	16262.	11750.	8075.8	5279.7	3283.4
			7	8	5	8	3	6	5	
10	31209.	29135.	26414.	22281.	18156.	14314.	10692.	7664.2	5223.5	3457.4
	7	7	9		5	4	8	6	5	2
20	28869.	26784.	25169.	20483.	16530.	13348.	10105.	7398.6	5160.5	3585.4
	4	4	1	2	5		3	6	5	8
30	31991.	29139.	28203.	21419.	16532.	13363.	9987.8	7279.0	5090.7	3667.5
	1	1	3	4	5	6	1	6	5	8
40	40574.	36199.	35517.	25089.	18162.	14361.	10340.	7305.4	5014.1	3703.7
	8	8	5	6	5	2	3	6	5	2
50	54620.	47966.	47111.	31493.	21420.	16340.	11162.	7477.8	4930.7	3693.9
	5	5	7	8	5	8	8	6	5	
60	74128.	64439.	62985.	40632.	26306.	19302.	12455.	7796.2	4840.5	3638.1
	2	2	9		5	4	3	6	5	2
70	99097.	85617.	83140.	52504.	32820.	23246.	14217.	8260.6	4743.5	3536.3
	9	9		2	5		8	6	5	8
80	12953	11150	10757	67110.	40962.	28171.	16450.	8871.0	4639.7	3388.6
	0.	3.	4.	4	5	6	3	6	5	8
90	16542	14209	13628	84450.	50732.	34079.	19152.	9627.4	4529.1	3195.0
	3.	3.	8.	6	5	2	8	6	5	2
100	20677	17739	16928	10452	62130.	40968.	22325.	10529.	4411.7	2955.4
	9.	0.	3.	5.	5	8	3	9	5	

Table: 12 Case-2 LH Grade, Gray Matter, 500 Cells, Combination of Six Drugs in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	487.65	452.42	399.26	335.1	267.6	203.2	146.8	100.9	65.997	41.04
				6	3	8	8	5		
10	337.35	315.02	283.56	241.5	197.3	154.3	115.5	82.84	56.797	37.34
				6	3	8	8			
20	257.05	237.62	215.86	181.9	147.0	113.4	86.28	62.91	44.997	30.44
				6	3	8				
30	246.75	220.22	196.16	156.3	116.7	80.58	58.98	41.16	30.597	20.34
				6	3					
40	306.45	262.82	224.46	164.7	106.4	55.68	33.68	17.59	13.597	7.04
				6	3					
50	436.15	365.42	300.76	207.1	116.1	38.78	10.38	-7.8	-6.003	-9.46
				6	3					
60	635.85	528.02	425.06	283.5	145.8	29.88	-10.92	-35.01	-28.203	-29.16
				6	3					
70	905.55	750.62	597.36	393.9	195.5	28.98	-30.22	-64.04	-53.003	-52.06
				6	3					
80	1245.2	1033.2	817.66	538.3	265.2	36.08	-47.52	-94.89	-80.403	-78.16

	5	2		6	3					
90	1654.9 5	1375.8 2	1085.9 6	716.7 6	354.9 3	51.18	-62.82	- 127.5 6	- 110.40 3	- 107.4 6
100	2134.6 5	1778.4 2	1402.2 6	929.1 6	464.6 3	74.28	-76.12	- 162.0 5	- 143.00 3	- 139.9 6

Table: 12a Case-2 LH Grade, Gray Matter, 500 Cells, Cisplatinum in Chemotherapy, Homogeneous Medium

t	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=0.7	x=0.8	x=0.9	x=1
0	487.65	452.42	399.26	335.16	267.6 3	203.2 8	146.8 8	100.9 5	65.99 7	41.04
10	389.95	363.82	330.46	278.46	227.0 3	178.8 8	133.5 8	95.76	65.29 4	43.21 2
20	360.25	333.22	315.66	255.76	206.8 3	166.6 8	126.0 8	92.39	64.50 5	44.80 4
30	398.55	360.62	354.86	267.06	207.0 3	166.6 8	124.3 8	90.84	63.63	45.81 6
40	504.85	446.02	448.06	312.36	227.6 3	178.8 8	128.4 8	91.11	62.66 9	46.24 8
50	679.15	589.42	595.26	391.66	268.6 3	203.2 8	138.3 8	93.2	61.62 2	46.1
60	921.45	790.82	796.46	504.96	330.0 3	239.8 8	154.0 8	97.11	60.48 9	45.37 2
70	1231.7 5	1050.2 2	1051.6 6	652.26	411.8 3	288.6 8	175.5 8	102.8 4	59.27	44.06 4
80	1610.0 5	1367.6 2	1360.8 6	833.56	514.0 3	349.6 8	202.8 8	110.3 9	57.96 5	42.17 6
90	2056.3 5	1743.0 2	1724.0 6	1048.8 6	636.6 3	422.8 8	235.9 8	119.7 6	56.57 4	39.70 8
100	2570.6 5	2176.4 2	2141.2 6	1298.1 6	779.6 3	508.2 8	274.8 8	130.9 5	55.09 7	36.66

Graphical Representation: LH Grade: Low ρ , High D

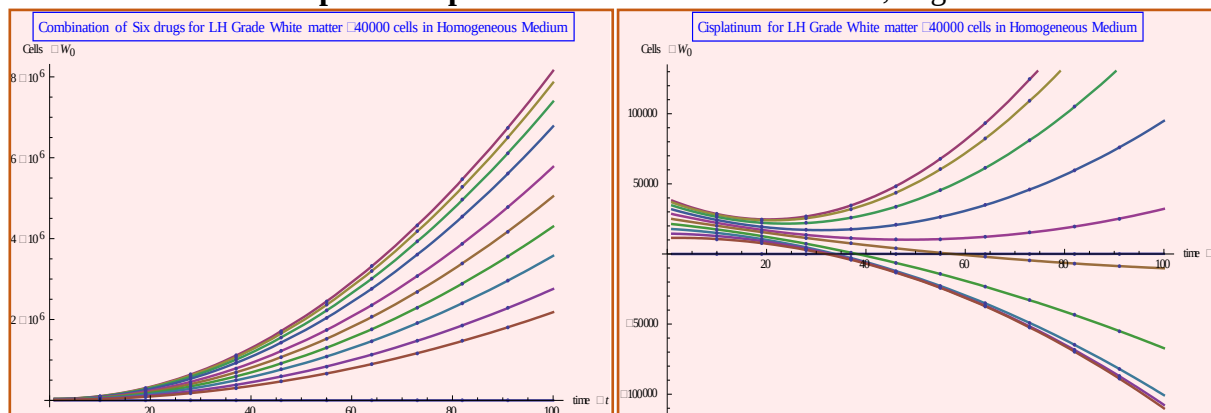


Fig. 13 & 13a: LH Grade, White Matter, 40000 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

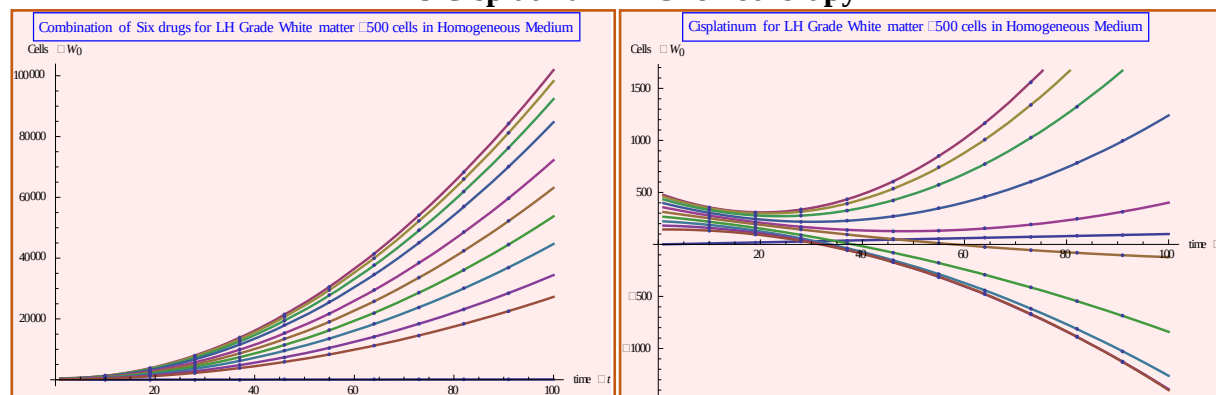


Fig. 14 & 14a: LH Grade, White Matter, 500 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

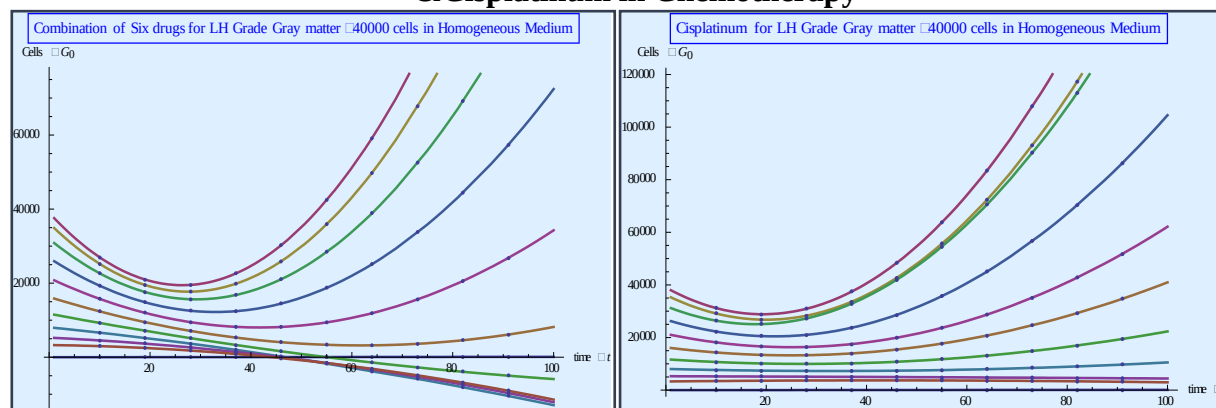


Fig. 15 & 15a: LH Grade, Gray Matter, 40000 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

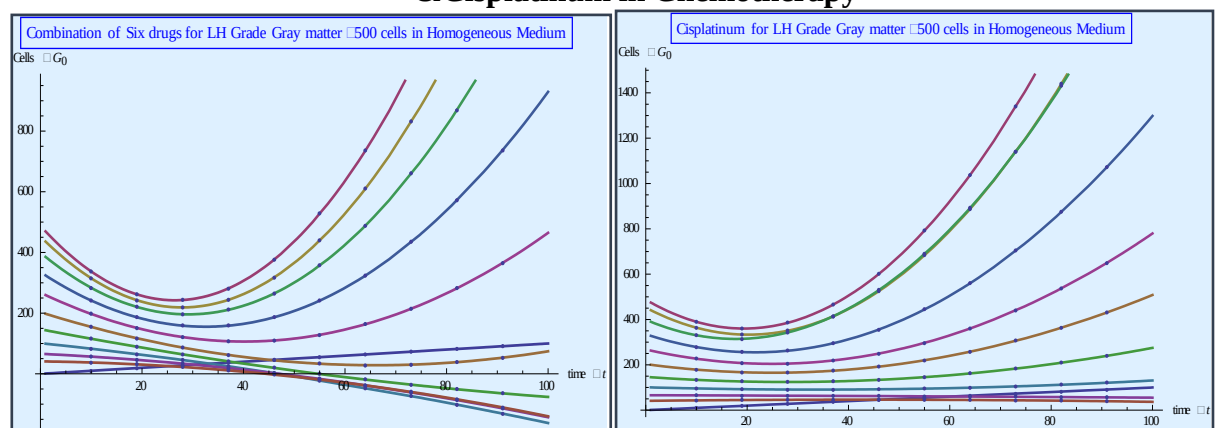


Fig. 16 & 16a: LH Grade, Gray Matter, 500 cells, Combination of Six Drugs & Cisplatin in Chemotherapy

Conclusion of case-2: From the graph we can predict that after how many days second dose of drugs can be given to patient.

	40000 Cell		500 Cells	
White Matter	Combination of Six Drugs	Cisplatin	Combination of Six Drugs	Cisplatin

HH Grade	After 20-22 days	After 30-32 days	After 20-22 days	After 30-32days
HL Grade	After 40-42 days	After 65-67 days	After 40-42 days	After 65-67days
LH Grade	After 10-12 days	After 20-22 days	After 10-12 days	After 20-22days
Gray Matter	Combination of Six Drugs	Cisplatin	Combination of Six Drugs	Cisplatin
HH Grade	After 25-27 days	After 40-42 days	After 28-30 days	After 40-42days
HL Grade	After 40-42 days	After 68-70 days	After 40-42days	After 68-70days
LH Grade	After 30-32 days	After 20-22 days	After 30-32 days	After 20-22days

Termination:

We can conclude that the model taken into account is highly appropriate to estimate the effect of different drugs into chemotherapy treatment for the spatiotemporal behaviour of the tumor. After identifying the location, size, shape, diffusion and growth rate of specific tumor cells, this model can help to suggest the best type of therapy to maximize the survival time of the patient. Different drugs react differently and vary case to case. So, it is highly recommended to verify for every case.

Advantage of applying the new arrival - Kamal Transformation upon Variational Iteration method is used to reduce the tangible workload for finding the Lagrange Multiplier as well as save the time.

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