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Unbalanced FTP with Circumcenter of Centroids and Heuristic Method

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Abstract: Many researchers have proposed a variety of methods to solve the Fuzzy Transportation Problems. The intention of this work is to frame a novel procedure for maximizing profit of an unbalanced transportation problem in fuzzy environment. In this work we have applied Circumcenter of Centroids ranking method to convert fuzzy quantities in to crisp quantities. Moreover Heuristic Method for unraveling the Triangular Fuzzy Transportation Problem is proposed. A numerical example is illustrated with the proposed algorithm and the result obtained through this method is compared with the existing methods like North West Corner method, Least cost method, Row Minima Method, Russell's and Vogel's approximation method. Finally the proposed method gives an optimum solution which is explained clearly by comparison chart.

Key words: Heuristic method, Circumcenter of Centroids ranking technique, Unbalanced fuzzy transportation problem, Triangular fuzzy numbers. score function

AMS Subject Classification: 90B06, 90C90, 90C70

1. Introduction

The processes of Transportation ensure the proficient progress and timely availability of raw materials and finished goods. The course of action to meet up the challenge of how to supply the merchandise to the clientele in well-organized way is implemented by a powerful structure called Transportation problems. Hitchcock [6] developed the basic transportation problem. Charnes et al[3] developed the stepping stone method to solve the Transportation problem . Several researchers studied extensively to solve cost minimizing transportation problem in various ways and proposed many methods till date. All the parameters of the transportation problems may not be known precisely due to uncontrollable factors in real world applications. Due to overwhelming aspects in present significance's all the constraints of the transportation problems may not be renowned openly. This kind of unclear information cannot be characterized clearly by picking a random variable from a probability distribution. Zadeh [1] introduced the fuzzy numbers to handle these situations. Zimmermann [16] showed that solutions obtained by fuzzy linear programming are always efficient.Chanas and Kuchta [2] projected the notion of optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers. Saad & Abbas [12] elucidated an algorithm for solving the transportation problems in fuzzy environment. Pandian & Natrajan [9,10] proposed a new algorithm, namely fuzzy zero point method. A new method for solving unbalanced transportation problems are proposed by K. R. Sobha [13] and Krishna Prabha[14]. A new method on ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviations by Chen and Chen [4] .F. Azman ,L.Abdullah [5] ,N. Ravi

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Shankar , P. Phani Bushan Rao[11] have given a review on Ranking Fuzzy Numbers Using the Centroid Point Method.The ranking procedures proposed in the literature use Centroid of trapezoid as reference point, as the Centroid is a balancing point of the trapezoid. In section 2, a new method is proposed here which is based on Circumcenter to rank fuzzy quantities. Allocation table method,Heuristic method for solving dual-hesitant fuzzy transportation problem is introduced by Krishna Prabha et al in 2020 [7].

In this paper, we propose heuristic method with Circumcenter of cerntriod, where the objective is to maximize the profit by converting the maximization problem into a minimization problem for an unbalanced transportation problem. The paper is written as follows, introduction to the concepts were given in section 1.A preface to fuzzy sets and problem formulation is presented in section 2. An algorithm is proposed in section 3. A numerical example is illustrated in section 4, and finally we wrap up the work in section 5.

2. Preliminaries:

In this section some basic definitions and arithmetic operations are reviewed.

Definition1: A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval [0, 1] i.e. A = {(x, μ_A(x); x ∈ X}, Here μ_A: X →[0,1] is a mapping called the degree of membership function of the fuzzy set A and μ_A(x) is called the membership value of x ε X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

Definition2: A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with "ordinary" (single-valued) numbers. In many respects, fuzzy numbers depict the physical world more realistically than single-valued numbers. Fuzzy numbers are used in statistics, computer programming, engineering (especially communications), and experimental science. The concept takes into account the fact that all phenomena in the physical universe have a degree of inherent uncertainty.

Definition3: A triangular fuzzy number A(x) can be represented by A (a, b, c; 1) with the membership function $\mu_A(x)$ given by,

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$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x = b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & otherwise \end{cases}$$

Let $a = [a_1, a_2, a_3,]$ and $b = [b_1, b_2, b_3]$ be two triangular fuzzy numbers then the arithmetic operations on a and b as follows.

Addition:

$a + b = (a_1+b_1, a_2+b_2, {}_3+b_3)$

Subtraction:

$a - b = (a_1-b_1, a_2-b_2, a_3-b_3)$

Multiplication:

a. $b = \frac{a_1}{3}(b_1+b_2+b_3), \frac{a_2}{3}(b_1+b_2+b_3), \frac{a_3}{3}(b_1+b_2+b_3)$ if $R(a) > 0$

$a \,.\, b = \frac{a_3}{3}(b_1+b_2+b_3), \frac{a_2}{3}(b_1+b_2+b_3), \frac{a_1}{3}(b_1+b_2+b_3)$ if $R(a) < 0$

2. a.Ranking Technique:

A new ranking method called circumcentre of centroid is used for the generalized fuzzy number to solve the transportation problems. This approach involves simple computational and is easily understandable.

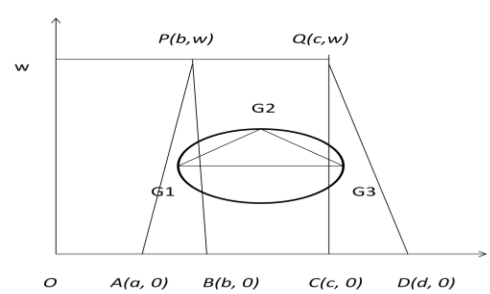

Figure 1: Circumcenterr of Centroids.

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Consider a generalized trapezoidal fuzzy number A= (a, b, c, d: w), (Fig.1) The Centroids of the three triangles are $G_1=\left(\frac{a+2b}{3},\frac{w}{3}\right)$, $G_2=\left(\frac{b+c}{2},\frac{w}{2}\right)$, $G_3=\left(\frac{2c+d}{3},\frac{w}{3}\right)$ respectively. G_1, G_2 and G_3 are non-collinear and they form a triangle.

We define the Circumcenter, S_A (x,y) of the triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number A=(a,b,c,d:w), as

$S_A(x,y)=\left(\frac{a+2b+2c+d}{6},\frac{(2a+b-3c)(2d+c-3b)+5w^2}{12w}\right)$

As a special case, for triangular fuzzy number A= (a, b, c, d:w) i.e., $c = b$ the Circumcenter of centroids is given by

$S_A(x,y) = \left(\frac{a+4b+d}{6},\frac{4(a-b)(d-b)+5w^2}{12w}\right)$

The formula for ranking of generalized trapezoidal fuzzy number is defined as $R(A) = \sqrt{x^2+y^2}$ which is the Euclidean distance from the Circumcenter of the Centroids.

2.b. Problem Formulation

The balanced fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

$$minimize \sum_{i=1}^{p}\sum_{j=1}^{q} c_{ij} * x_{ij}$$

Subject to $\sum_{j=1}^{q} x_{ij} = \tilde{a}_i$, i = 1,2,3,………,p

$\sum_{i=1}^{p} x_{ij} = b_j$, j = 1,2,3,………,q

$\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j$

X_{ij} is a non- negative trapezoidal fuzzy number,

Where p = total number of sources

Q = total number of destinations

a_i = the fuzzy availability of the product at i^{th} source

b_j = the fuzzy demand of the product at j^{th} destination

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c_{ij} = the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination

x_{ij} = the fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost. $\sum_{i=1}^{p} a_i$=1total fuzzy availability of the product,

$\sum_{j=1}^{q} b_j$=1total fuzzy demand of the product

$\sum_{i=1}^{p} \sum_{j=1}^{q} c_{ij} * x_{ij}$=1 total fuzzy transportation cost.

If $\sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j$ then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.

An unbalanced transportation problem is converted into a balanced transportation problem by introducing a dummy origin or dummy destinations which will provide the excess availability or the requirement the cost of transporting a unit from this dummy origin (or dummy destination) to any place is taken to be zero.

After converting the unbalanced problem into a balanced problem, we adopt the usual procedure for solving the transportation problem

3. Algorithm for Heuristic Method

Step-1: Construct a Fuzzy Transportation Table from the transportation problem.

Step-2: Verify if the TP is balanced or not, if not, make it balanced.

Step-3: Find the Penalty for each row and column by computing the difference between the two minimum cost cells for each row and column. These values are called as penalties, P, respectively

Step-4: Find the summations of row and Column cost. These values are called as T

Step-5: Find the product PT by multiplying the penalty 'P' and the total cost 'T

Step-6: Spot out the row/column having minimum 'PT'.

Step-7: Choose the cell having minimum cost in row/column identified in Step-6.

Step-8: Make maximum feasible allocation to the cell chosen in Step-7, if the cost of this

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cell **is** also minimum in its column/row. Otherwise allocation is avoided and goto Step-9.

Step-9: Identify the row/column having next to lowest 'PT'.

Step-10: Choose the cell having minimum cost in row/column identified in step 9.

Step-11: Make maximum feasible allocation to the cell chosen in Step-10.

Step-12: Cross out the satisfied row/column.

Step-13: Repeat the procedure until all the requirements are satisfied.

Step-14: Now shift this allocation to the original FTT.

Step-15: To conclude, compute the total profit of the FTT. This calculation is the sum of the product of cost and resultant allocated value of the FTT

4. Numerical Example

Consider the given FTTP**,**

Table.1: Triangular Fuzzy Transportation Problem

	$D1$	$D2$	$D3$	$D4$	**Fuzzy Available**
S1	(-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,3)	(0,2,5)
S2	(4,9,16)	(4,8,12)	(2,5,8)	(1,4,7)	(1,6,11)
S3	(2,7,12)	(0,5,10)	(0,5,10)	(4,8,12)	(1,4,8)
Fuzzy Requirement	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	

After converting the triangular fuzzy transportation problem by the Circumcenter of centroids ranking technique discussed in section 2, the crisp table is given by,

Table.2: Crisp Transportation Problem

	$D1$	$D2$	$D3$	$D4$	**Fuzzy Available**
S1	8.45	8.45	8.45	1.35	2.67

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S2	14.61	9.38	5.59	4.71	9.92
S3	10.55	9.35	9.35	9.38	5.48
Fuzzy Requirement	4.71	3.24	4.71	4.87	

Here Total Demand = 17.53 is less than Total Supply=18.07,a dummy demand constraint with o unit cost and with allocation 0.54 is added. The modified Table is,

Table.3: Balanced Transportation Problem

	***D*1**	***D*2**	***D*3**	***D*4**	**D5**	**Fuzzy Available**
S1	8.45	8.45	8.45	1.35	0	2.67
S2	14.61	9.38	5.59	4.71	0	9.92
S3	10.55	9.35	9.35	9.38	0	5.48
Fuzzy Requirement	4.71	3.24	4.71	4.87	0.54	

The given Problem is Maximization, so convert it to minimization by subtracting all the elements from max element (14.61)

Table.4: Converted Transportation Problem

	***D*1**	***D*2**	***D*3**	***D*4**	**D5**	**Fuzzy Available**
S1	6.16	6.16	6.16	13.26	14.61	2.67
S2	0	5.23	9.02	9.9	14.61	9.92
S3	4.06	5.26	5.26	5.23	14.61	5.48
Fuzzy Requirement	4.71	3.24	4.71	4.87	0.54	

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From the above table by implementing the algorithm to get Penalty and Total cost, as bellow.

Table 5: Penalty and Total Cost

1	**D1**	**D2**	**D3**	**D4**	**D5**	**Fuzzy Available**	**ROW PENALTY (P)**	**TOTAL (T)**	**P*T**
S1	6.16	6.16	6.16	13.26	14.61	2.67	0	46.35	0
S2	0	5.23	9.02	9.9	14.61	9.92	5.23	38.76	202.71
S3	4.06	5.26	5.26	5.23	14.61	5.48	1.17	34.42	40.27
Fuzzy Requirement	4.71	3.24	4.71	4.87	0.54				
COLUMN PENALTY	4.06	0.03	0.09	4.67	0				
TOTAL (T)	10.22	16.65	20.44	28.39	43.83				
P*T	41.49	0.5	18.4	132.58	0				

The lowest PT = 0, occurs in row S1.

The minimum cij in this row is c11=6.16.

The maximum allocation in this cell is min(2.67,4.71) = 2.67.

It satisfy supply of S1 and adjust the demand of D1 from 4.71 to 2.04 (4.71 - 2.67=2.04).

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Table 6: Iteration 1 of FTTP

1	*D*1	*D*2	*D*3	*D*4	D5	Fuzzy Available	ROW PENALTY (P)	TOTAL (T)	P*T
S1	6.16(2.67)	6.16	6.16	13.26	14.61	2.67(0)	0	46.35	0
S2	0	5.23	9.02	9.9	14.61	9.92	5.23	38.76	202.71
S3	4.06	5.26	5.26	5.23	14.61	5.48	1.17	34.42	40.27
Fuzzy Requirement	4.71(2.04)	3.24	4.71	4.87	0.54				
COLUMN PENALTY	4.06	0.03	0.09	4.67	0				
TOTAL (T)	10.22	16.65	20.44	28.39	43.83				
P*T	41.49	0.5	18.4	132.58	0				

The lowest PT = 0, occurs in column D5.

The minimum cij in this column is c25=14.61.

The maximum allocation in this cell is min(9.92,0.54) = 0.54.

It satisfy demand of D5 and adjust the supply of S2 from 9.92 to 9.38 (9.92 - 0.54=9.38).

Proceeding by the given algorithm, the final allocation to the original problem is given below,

Table 7: Optimum Allocation

	$D1$	$D2$	$D3$	$D4$	D5	Fuzzy Available
S1	8.45 **(2.67)**	8.45	8.45	1.35	0	2.67
S2	14.61	9.38 **(3.24)**	5.59 **(4.71)**	4.71 **(1.43)**	0 **(0.54)**	9.92
S3	10.55 **(2.04)**	9.35	9.35	9.38 **(3.44)**	0	5.48
Fuzzy Requirement	4.71	3.24	4.71	4.87	0.54	

The maximum profit

$8.45\times2.67+9.38\times3.24+5.59\times4.71+4.71\times1.43+0\times0.54+10.55\times2.04+9.38\times3.44=139.81$

Here, the number of allocated cells = 7 is equal to m + n - 1 = 3 + 5 - 1 = 7

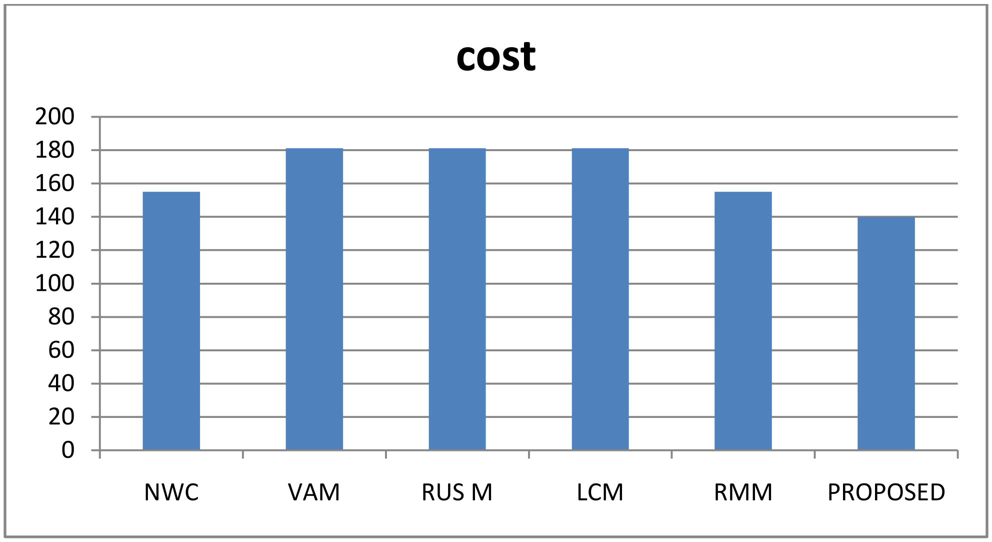
∴ This solution is non-degenerate

The above problem gives the following solutions for various methods tested bellow,

Table 8: Comparison of different methods

North-West Corner method	Vogel's Approximation method	Russell's method	Least cost method	Row Minima Method	Proposed method
155.03	181.14	181.14	181.14	155.03	139.81

Comparison Chart



Heuristic method is comparatively better than other methods for the same problem. Obviously this method gives a better optimal solution when compared with other methods.

5. CONCLUSION

In order to minimize the transportation cost and to maximize the profit we have applied heuristic method to solve a Dual-Hesitant Fuzzy Transportation problem. By comparing our results to the exiting results we have verified that our approach yields a better optimum solution. The proposed algorithm will be very effective for real-life problem. This technique can be used to solve all types of Dual-Hesitant Fuzzy Transportation problems. Consequently this scheme can be utilized to solve the problems in supply chain management, assignment, cloud computing, travelling salesman etc.Further this can be extended to various types of Single-value, cubic, Bipolar, Interval bipolar fuzzy sets etc.

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