

## The Limited Problem of Less Parameters and the Configuration of the Depression Curve at Unreliable Water Filtration in Soils

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**Abstract:** The article discusses the issues of compiling hydraulic models for water systems that are part of the general model of water exchange. The subject of research in this article is the determination of the depression curve for unsteady filtration flow through the furrow when irrigating cotton. The filtration calculation scheme based on the assumption of the unsteady character of filtration in a porous medium by solving the Boussinesq equation.

**Key words:** stochastic processes, groundwater, filtration ratio, depression curve, Boussinesq equation.

### Introduction

When considering the problem of coupled mass transfer by interacting flows, it is necessary to study a number of issues on the hydraulic substantiation of the models and their numerical implementation. This is the creation of the principles of conjugation of mathematical models corresponding to various branches of the hydrological cycle; analysis of mathematical problems of correctness and study of the qualitative properties of solutions of the corresponding initial-boundary value problems. A special place occupied by the development of numerical methods and effective algorithms for calculating the boundary value problems arising here for systems of equations of a non-standard form, complicated by various features (nonlinearity, degeneration, etc.). Models of water systems (reservoirs, streams, furrows, ground basins, etc.) included in the general model of water exchange are not the same in complexity and have different dimensions. It is especially important that the characteristic time scales of transient, stochastic processes for surface and ground waters differ by orders of magnitude. The last circumstance is the great importance in creating a pattern of interrelated processes of water runoff and infiltration. In this regard, there is a need for fundamental research to study the above scientific problems.

### Methods

Methods were used in the process of research that were adopted for natural and field conditions, the theory of unsteady water filtration in soils and grounds, modeling the dynamics and direction of hydrological, hydrogeological and soil reclamation processes using modern technical means of observation and mathematical methods. Experimental studies of water consumption of plants against a natural background using a lysimeter. Management and monitoring of the components of the water-

salt balance on lysimeters. Phenological observations on cotton. Graphic and statistical processing of results.

### Results and Discussion

Free-flow filtration flows with a free surface, on which the fluid pressure is constant and equal to the external atmospheric pressure, are typical for the filtration of irrigation water through soils, caused by furrow irrigation of cotton.

The theory of filtration for such water management problems was developed [17, 18, 19, 20, 21]. In [7, 8, 9, 10] it is noted that in the exact setting, the study of free-flow motion presents significant mathematical difficulties. Of particular importance are the works of P.Y. Polubarinova - Kochina, who obtained some exact solutions to the problem of unpressurized filtration flow through a heterogeneous medium. Thus, the problems of gravity filtration remain of theoretical interest, which was pointed out [1, 2, 3, 4, 5, 6, 11, 12, 13], and their solution is the practical importance for predicting the characteristics of the movement of filtration flows in applied land reclamation tasks [6, 14, 26, 27, 28].

The application of the above given in the field leads to the conservation of irrigation water depending on water distribution in irrigation and drainage systems as well as the usage of groundwater, etc. [14,15,16,17,22,23,24,25].

In the case of free-flow filtration of a liquid through a furrow into the soil-ground, caused by the difference in the hydrostatic pressure of the liquid in a heterogeneous medium, the area of fluid movement is limited from above by a free surface, called the depression surface, at each point of which constant pressure acts. The section of the depression surface along the movement of the fluid through the porous medium is a depression curve. The resulting gap between the outlet of the depression curve and groundwater is a wedging surface.

In contrast, the problems of free-flow filtration of unsteady one-dimensional flows of an incompressible fluid, including the unsteady flow regime under linear and nonlinear filtration laws, have not been sufficiently studied. This is due to the complexity of their mathematical description and obtaining the shape of the depression surface.

The subject of research in this article is the determination of the depression curve for unsteady filtration flow through the furrow when irrigating cotton.

The filtration calculation scheme is based on the assumption of a non-stationary the nature of filtration in a porous medium. Stationary filtration modes are interpreted as limiting (at "large" times) filtration flow states. The starting identity is the continuity condition for unsteady motion:  $\partial h / \partial t + \partial q / \partial x = 0$ , moreover, by virtue of the Dupuis condition,  $q = -kh \frac{\partial h}{\partial x}$ ,  $k$  - filtration coefficient. Then for the distribution  $h(t, x)$  we obtain the Boussinesq equation (1):

$$m \frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \left( kh \frac{\partial h}{\partial x} \right), (1)$$

Where:  $t$  - time,  $x$  - coordinate,  $h = h(t, x)$  - depth of the filtration flow,  $h_e \leq h_0 < h \leq H$ ,  $m$  - coefficient of porosity,  $0 < m < 1$ .

Replacement of variables

$$t := \frac{kt}{mH} > 0, x = \frac{x}{H}, 0 < x < \lambda := l/H, \lambda \leq \Lambda := L/H, u := h/H, u_0 < u < 1, u_0 = h_0/H,$$

reduces the Boussinesq equation to dimensionless form (2):

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left( u \frac{\partial u}{\partial x} \right). (2)$$

In what follows, the strokes on the arguments are omitted. Thus, equation (2) takes the form (3):

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial}{\partial x} \left( u \frac{\partial u}{\partial x} \right), t > 0, x > 0; (3) \\ u(t, 0) - u_0 &= u(0, x) - 1 = 0. \end{aligned}$$

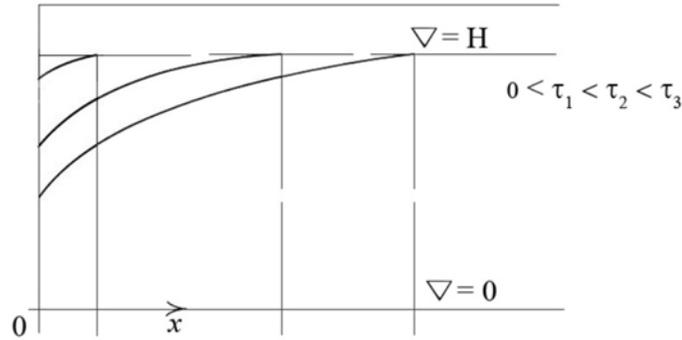
The limiting conditions for equation (3) have the form (4):

$$u(0, x) - H = u(t, 0) - h_0 = 0. (4)$$

Here  $h_0 = h(t, 0)$  is the depth of the filtration flow at the point where the depression curve exits the downstream slope,  $h_0 > h_e \geq 0$ . Then the set of values of the depth of the filtration flow  $\text{Im}(h) = (0 \leq h_0 < h < H)$ .

Let the depression curve represent the straight line  $h = H$ ,  $|x| < \infty$  until the time  $t = 0$ . At the time  $t = 0$ , there is an instantaneous "fast enough" decrease in the ordinate of the half-line  $x < \infty$ , from the value  $h = H$  to the value equal to  $h = h_e$ . The depression curve in the porous medium descends more slowly than the water level in the downstream. The left end of the depression curve lags behind the water edge in the downstream, which leads to a break in the depression curve at the point of pinching out. The separation of the depression curve from the water's edge at the point of descending is due, therefore, to different scales of filtration rates and water movement in the porous medium. Thus, the rate of drop of the pinch-out point in a heterogeneous medium is the order of  $n$ , while the rate of lowering of the groundwater level is  $v_0 = -dh_e/dt \gg n$ . If the lowering of the groundwater level occurred at a rate of the order of  $n$ , then no wedging gap would form and the depth of the filtration flow in the section  $x = 0$  would coincide with the groundwater level. Time equal to  $\tau > 0$ , the depression curve separates from the straight line  $h = H$  and deforms as shown in Figure 1. The pinch-out point approaches the water table, and the right end of the depression curve, continuously touching the ordinate  $h = H$ , slides to the right. A decrease in the depression curve leads to the "piston effect", that is to squeezing water out of the porous medium downward.

The Boussinesq equation and the limiting problem (3) describe the change in soil moisture content. These changes interpreted as translations of perturbations in the depression curve. For example, let the pinch-out point, Figure 1, be shifted downward from the ordinate  $h = H$ . Then a decrease in the pinch-out point leads to deformation of the depression curve. It is easy to check that the equation of the limiting problem (3) admits, among other things, the following solution:  $u = -1/6 \frac{s^2}{\tau}$ . Then the speed  $D$  of displacement of the section with depth  $u = \text{const}$  is  $D = \frac{c}{\sqrt{\tau}}$ . The distance of movement in time  $\tau$  will be  $2C\tau^{1/2}$  (fig. 1).



**Figure 1. Depression curves at different points in time.**

The Boussinesq equation in the limit problem (3) admits a two-parameter, with parameters  $\alpha$  and  $\beta$ , group of transformations of the form:  $z = \alpha t + \beta x$ ,  $u(\tau, x) = u(z)$ . This transformation transforms the partial differential equation (3) into an ODE:

$$\alpha \frac{du}{dz} = \beta^2 \frac{d}{dz} \left( u \frac{du}{dz} \right).$$

The general solution of this equation, depending on two constants of integration,  $c_{1,2}$ , is:  $z + c_2 = c_1/\gamma^2 \ln \frac{c_1}{c_1 + \gamma u} + u/\gamma$ ,  $\gamma = \alpha/\beta^2$ .

Let, not excluding generality,  $\alpha = \beta^2$ ,  $\gamma = 1$ . Then the general solution of this ODE, depending on 2 constants,  $c_1, c_2$ , has the form:

$$z + c_2 = u - c_1 \ln(u + c_1) + c_1 \ln c_1, \\ \alpha t \pm \sqrt{\alpha} x = u - c_1 \ln(u + c_1) + c_3, c_3 := c_1 \ln c_1 - c_2.$$

At  $c_1 = 0$ , a solitary centered flow rate wave  $\alpha t \pm \sqrt{\alpha} x = u + c_3$  is obtained, propagating with a velocity  $D := \left( \frac{\partial x}{\partial t} \right)_u = \frac{\partial(s,u)}{\partial(t,u)} = m\sqrt{\alpha}$  upstream and downstream (respectively upper and lower sign).

For a centered flow rate wave, excluding the parameter  $\alpha$  and passing to the velocity  $D$ , we obtain:  $u + c_3 = \alpha t \pm \sqrt{\alpha} x = D^2 t - Dx = Dt \left( D - \frac{x}{t} \right)$ . A special solution has the form:  $u + c_3 = \frac{x^2}{4t}$ ,  $D = \frac{x}{2t}$ , or, in dimensional form,  $\left( \frac{\partial x}{\partial t} \right)_u = \frac{x}{2t}$ . By integrating this equality, we find that the displacement of the longitudinal coordinate of the depression with a fixed depth is proportional to  $t^{1/2}$ , and the velocity of propagation of the discharge wave will be  $1/t^{1/2}$ . It means that the length of the disturbed region increases as  $\sqrt{t}$  for a lysimeter and as  $\sqrt{t + \pi^2/12}$  for a site near the lysimeter.

Consider the following problem: let there exist a solution  $u = u(z)$ ,  $z = \alpha t + \beta x$ . Then  $u(t, x)$  satisfies the homogeneous linear equation:

$$\frac{1}{\alpha} \frac{\partial u}{\partial t} - \frac{1}{\beta} \frac{\partial u}{\partial x} = 0$$

Admitting a general solution  $z = \text{const}$ ,  $u = u(z)$ . Let the Boussinesq equation hold at the same time. Eliminating the time derivative, we get:  $\frac{1}{\alpha} \frac{\partial}{\partial x} \left( u \frac{\partial u}{\partial x} \right) - \frac{1}{\beta} \frac{\partial}{\partial x} = 0$  This equation admits the first integral:

$u \frac{\partial u}{\partial x} - \frac{\alpha}{\beta} u = \varphi(t)$ , where  $\varphi(t)$  is an arbitrary differentiable function. As you can see, in this equation, the variables separated and the general solution of the resulting equation is:

$$s + \psi(t) = \left(\frac{\beta}{\alpha}\right)^2 \left(\frac{\alpha}{\beta u} - \varphi(t) \ln\left(\varphi(t) + \frac{\alpha}{\beta u}\right)\right)$$

Where  $\psi(t)$  is an arbitrary function.

Formal integration of the Boussinesq equation (3) over the coordinate  $x$  from 0 to  $\infty$  leads to the identity:

$$I_0(t) = \frac{d}{dt} \int_0^{\infty} (1-u) dx = \left(u \frac{\partial u}{\partial x}\right)_{x=0} = \theta_0(t), \quad \theta_0(t) \geq 0.$$

Here  $\theta_0 \geq 0$  is the dimensionless flow rate in the section  $x = 0$ ,  $\theta_0 kH = q(t, 0)$ . The value of the integral  $\lambda(t) := \int_0^{\infty} (1-u) dx$  is interpreted as the length of the active filtration zone and equal to the area of the curved lise, the sides of which are the instantaneous depression curve and the ordinate  $u = 1$ . Thus, the condition is equal to:

$$\frac{d\lambda}{dt} = \theta_0(t) \quad (5)$$

The value of the integral found similarly:

$$I_1 = \frac{d}{dt} \int_0^{\infty} x(1-u) dx = -\left(xu \frac{\partial u}{\partial x}\right)_0 + \frac{1-u_0^2}{2} = \frac{1-u_0^2}{2} \geq 0$$

And, in general,

$$I_m := \frac{d}{dx} \int_0^{\infty} x^m (1-u) dx = \frac{m}{2} \int_0^{\infty} x^{m-1} du^2$$

Integrall<sub>1</sub> = inv, if  $u_0 = \text{const}$ . In this case, there is an invariant  $A = \frac{1-u_0^2}{2}$ ,  $dA/dt = 0$ ,  $t \geq T$ . Here  $T$  is the time to establish a stationary mode, i.e. the time during which the right end of the depression curve reaches the water table.

### Conclusions

1. The Boussinesq equation describes the filtration flow at unsteady flow. The result of solving the Boussinesq equation is the dependence of the depression curve on time and, as a consequence, the size of the drainage interval. In the particular case of stationary flows, its solution determines the Dupuis parabola.

2. The solution of the Boussinesq equation for unsteady flows is associated with significant difficulties and requires a search for approaches to reduce it to a low-order ordinary differential equation (ODE).

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