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Minimize the Transportation cost on Fuzzy Environment

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Abstract: Recently,many author has stated various results on Fuzzy transportation problem. In this paper, construct the FTP model into LPP model and proposed an algorithm to obtain the optimal solution for the fuzzy environmental. A numerical example is included.

Key Words: LPP, Feasible solution, FTP, TFN

1.INTRODUCTION

Today world market is very highly competitive for particularly industrial people are facing, how to deliver the product in better ways for the demand customers because those customers become stronger. Transportation gives us the best solution for the above problems. Hitchcock [8] introduced the transportation model and represented as LPP models which can be solved by the simplex method. It's very special mathematical structure. Charnes and Cooper [10] proposed new method, it represented as Stepping Stone method. Dantzig and Thapa [3] applied the simplex method to obtain the optimal solution for the transportation problem. Later many authors stated various results and algorithms on Transportation problem. But, Due to some natural calamities the transportation parameters not certain, In this case, we could not find the optimal solution in the crisp transportation problem. Such a case Fuzzy set theory can help us to give optimal solution.Fuzzy set theory [12,2] "R.E.Bellman and L.A.Zaeh", which was developed by Zadeh in 1960's and fuzzy optimization techniques [5] "S.Fang and C.F.Hu provide a useful and efficient tool for optimization such systems. Optimization under a fuzzy environment is called fuzzy Optimization. The study on the theory and methodology of the fuzzy optimization has been active since the concept of fuzzy decision and the decision model under fuzzy environments, Various models are approaches to fuzzy linear programming [6,7,4,10], fuzzy dynamic programming with the applications proposed by [5] Kacprzyk, and A.O.Esogbue, fuzzy linear programming with fuzzy numbers proposed by [11]. In this paper, we proposed an algorithm to obtain the optimal solution to the fuzzy transportation problem.

2. PRELIMINARIES

2.1 Definition: [8]It is a special type of linear programming problem which arises in many practical applications in the beginning it was formulated for determining the optimal shipping pattern, so it is called transportation problem. The conventional and very well known transportation problem consists in transporting a certain product from each *m* origins i= 1, 2…m to any one of *d*estinations j=1, 2…n. The origins are production facilities with respective capacities a_1, a_2….a_m and the destinations are warehouses with required levels of demand b_1.b_2, b_3…b_n. For the transport of a unit of the given product from the i^{th} source to j^{th}destination a cost c_{ij} is given for which, without loss of generality, we can assume $c_{ij} \geq 0 \;, \forall i, j$. Hence, one must determine the amounts x_{ij} to be transported from all the origins i= 1, 2, 3….m to all the destinations j =1, 2, 3...n in such a way that the total cost is minimized. This problem can be suitably modeled as a linear programming problem. Thus the conventational linear programming problem can be mathematically expressed as;

$$Minimize\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$Subject\ to\ \sum_{j=1}^{n} x_{ij} \leq a_i$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j \qquad x_{ij} \geq 0\ \forall\ i, j$$

$$\sum_{i=1}^{m} a_i = \sum_{j=i}^{n} b_j \quad (balanced conditions)$$

2.2 Definition: [1] A fuzzy number is a convex fuzzy set $\overline{A}$ in which possess the following properties.

(i) $\overline{A}(x) = 1 \quad for\ some\ x \in R$

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(ii) For each $\alpha \in (0,1]$, ${}^{\alpha}\overline{A} = \{x \in R : A(x) \geq \alpha\}$ must be a closed interval.

(iii) The support of A = $\{\alpha \in (0,1] \, ; \, \overline{A}(x) = \alpha, \; x \in R\}$ must be bounded subset of R.

2.3 Definition: [1] Let $\overline{A}$ be a fuzzy set and α be a real number in the interval [0, 1]. The crisp set A_α defined by $A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\}$ is called α - cut of $\overline{A}$. The crisp set $A_\alpha = \{x \in X : \mu_A(x) > \alpha\}$ is called strong α - cut of $\overline{A}$.

2.4 Definition: [1] A Triangular fuzzy number denoted by $A = [a, a_2, a_3]$, and represented by

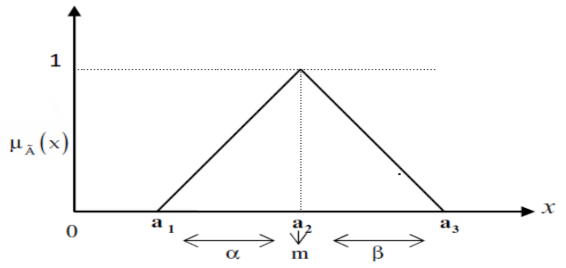

Fig 1. Triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3) = (m, \alpha, \beta)$

$$A(x) = \begin{cases} 0 & x \leq a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x < a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \leq x \leq a_3 \\ 1 & x \geq a_3 \end{cases}$$

2.5 Definition: [3] If $\overline{A} = [\underline{a}, a, \overline{a}]$ is a Triangular Fuzzy Number, then $R_1(\overline{A})$ the Rank of ($\overline{A}$) is defined by $R(\overline{A}) = (2a + \underline{a} + \overline{a})$. It is verified that;

(i) $\overline{A} > \overline{B}$ *iff* $R_1(\overline{A}) > R_1(\overline{B})$

(ii) $\overline{A} < \overline{B}$ *iff* $R_1(\overline{A}) < R_1(\overline{B})$

(iii) $\overline{A} = \overline{B}$ *iff* $R_1(\overline{A}) = R_1(\overline{B})$

2.6 Definition: [9] Roubst ranking technique which satisfies compensation, linearity and additive properties and provides results which are consistent with human intuition, Give a convex fuzzy number and the Roubst Ranking Index is defined by

$R_2(a) = \int_0^1 0.5 \; (a_\alpha^L, a_\alpha^U)\, d\alpha$, where (a_α^L, a_α^U) is α – cut of the fuzzy number.

3. DECISION MAKING UNDER FUZZY ENVIRONMENT AND FUZZY LINEAR PROGRAMMING

A fuzzy decision making model is characterized by a set of goals G_i (i=1, 2…m), along with a set of constraints C_j (j=1, 2,..n), each of which is expressed by a fuzzy set on X. For such a model is decision making, Bellman and Zadeh [4] in their pioneering work, proposed that a fuzzy decision is determined by an appropriate aggregation of fuzzy sets G_i (i=1, 2…m) and C_j (j=1, 2,..n).The main feature in this approach is the symmetry between goals and constraints. Keeping this in mind, they (Bellman and Zadeh) suggested the aggregation operator to be the fuzzy intersection. Thus a fuzzy decision D could be defined as the fuzzy set

$D = (G_1 \cap G_2 \cap ... G_M) \cap (C_1 \cap C_2 \cap .. C_N)$ and $\mu_D : X \to [0,1]$ is given by

$$\mu_D = \min\left(\mu_{G_i}(x), \mu_{C_j}(x)\right)$$

After knowing the fuzzy decision D, the decision $x^* \in X$ is said to be an optimal decision if $\mu_D(x^*) = \max \mu_D(x)$. Another method to solve decision making model is choose an α $s.t.$ $0 < \alpha < 1$, find all the points $x^* \in X$ for which $\mu_D(x^*) \geq \alpha$. These decisions x^* will have at least α degree of membership value.

3.1 Fuzzy Transportation Problem

When a decision maker is uncertain about the precise value of the transportation cost, availability and demand, The Transportation Problem may be formulated into Fuzzy Transportation Problem. The mathematical formulation of Fuzzy Transportation Problem is:

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$$z = \sum_{i=1}^{m}\sum_{j=1}^{n}\overline{C_{ij}}\,\overline{X_{ij}}$$

$$s.to \sum_{i=1}^{m}\overline{X_{ij}} = \overline{b_j}, \sum_{j=1}^{n}\overline{X_{ij}} = \overline{a_j}$$

$$\sum_{i=1}^{m}\overline{a_i} = \sum_{j=1}^{n}\overline{b_j} \text{ , } i=1,2,3,...m,\ j=1,2,3,...n$$

Where- $\overline{X_{ij}}$ Represents the fuzzy variables

3.2 Proposed Algorithm:

Step: 1 Construct the Fuzzy Transportation Table

Step: 2 Convert the Fuzzy Transportation problem into crisp transportation problem

Step: 3 Step (2) problems convert into LPP model

Step: 4 To find the inverse matrix is given by $\bar{b} = B^{-1}b \geq 0$, where $B = \lfloor A_{j1}, A_{j2}....A_{jm} \rfloor$ and $y = c^B B^{-1}$ using simplex multipliers.

Step: 5 Consider the coefficient variable for the non-basic cells x_j, obtained the initial basic solution to the given data.($\overline{c_j} = c_j - yA_j = c_j - \sum_{i=1}^{m} y_i a_{ij}$ for j, $\forall \overline{c_j} \leq 0$ the optimal level reached. Otherwise, continue $\exists\ \overline{c_j} > 0$

Step: 6 Choose the introduced basis variable $\overline{c_s} = \sup\{\overline{c_j} / \overline{c_j}\} > 0$

Step: 7 To compute $\overline{A_s} = B^{-1}A_s\ if\ A_s \leq 0$ then stop, The given problem exists in unbounded level $(\overline{a_{ij}} > 0)$ for some I = 1,2,3…m

Step: 8 Choose the leaving vector from the basis cell using the minimum ration $\frac{\overline{b_r}}{a_{rs}} = \min\left(\frac{\overline{b_i}}{\overline{\overline{a_{is}}}}\right), \frac{\overline{b_r}}{a_{rs}} \geq 0$

Step: 9To find the new inverse basis table B^{-1}

Step: 10 Introduce the new multiplier of simplex algorithm by $y = c^B B^{-1}$ by $\frac{b_r}{a_{rs}} \geq 0$

Step: 11 If $y > 0$ then the optimal solution reached stop the procedure otherwise repeat step 3 to 9.

4.NUMERICAL EXAMPLE

Find the Minimum cost of fuzzy transportation problem for the following table

				Supply
	(1,2,3)	(0,3,6)	(5,7,9)	(1,5,6)
	(0,2,4)	(5,6,7)	(0,3,6)	(2,4,7)
Demand	(1,2,4)	(4,5,6)	(0,1,2)	

Using Roubst Ranking function to reconstruct the supply and demand value as follows: $a_1 = 4$, $a_2 = 4$, $b_1 = 2$, $b_2 = 5$, $b_3 = 1$. TO convert the Fuzzy transportation problem into crisp transportation problem and can be written as Linear programming problem as follows

$$Min\, z = 2x_{11} + 3x_{12} + 7x_{13} + 2x_{21} + 6x_{22} + 3x_{23}$$

$$S.to\ \ x_{11} + x_{12} + x_{13} \leq 4,\ x_{21} + x_{22} + x_{23} \leq 4,\ x_{11} + x_{21} \leq 2, x_{12} + x_{22} \leq 5, x_{13} + x_{23} \leq 1$$

Basis Variable	Solution Variable	B_0	B_1	B_2	B_3	B_4	B_5		S_3	S_1	S_5	x_{21}	S_2	S_4
Z	4.81	1	5.84	10.57	10.5	0	0.67		0	0	0	-2	0	0
x_{12}	7.79	0	5.86	9.59	10.5	0	-0.33		0	1	0	0	0	0

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x_{22}	7.79	0	5.86	11.59	10.5	0	-0.33		0	0	0	1	1	0
x_{11}	7.63	0	5.09	12.79	7.92	0	-1		1	0	0	1	0	0
x_{23}	7.71	0	5.14	12.36	9.21	0	0		0	0	0	0	0	1
x_{13}	8.79	0	5.86	17.26	10.5	0	5.43		0	0	1	0	0	0

$$c_k - z_k = \sup\{-(c_j - z_j) > 0,\ i = 1,2,3,4,5,6\}$$
$$= Sup\,\{-(10.5, 5.84, 0.67, 19.07, 10.57, 0)\} < 0$$

The current solution is optimal solution
x_{11}=7.63, x_{12}=7.79,x_{13}=8.79,x_{22}=7.79,x_{23}=7.71 and Min (z) = 170.03

5. CONCLUSION

Here, we proposed an algorithm to obtain the optimal solution on fuzzy transportation problem but the parameters are considered as Triangular Fuzzy numbers. In a complex type of problem optimal solution cannot get easily. Such type of problem our proposed algorithm easily provided the optimal solution for minimum iteration level.

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