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Markovian Two Server Single Vacation Queue with Heterogeneous Arrival and Service Rates

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ABSTRACT

In this paper, we consider a two heterogeneous server queue. In which, the servers takes a single vacation, if there are no customers in the system at a service completion point. The inter-arrival times and the service times follows negative exponential distributions and the arrival rates depends on the server's state. Each arriving customer requires exactly one server for its service. On arrival, if the customer finds one server free he enters in to that server for service or if the customer finds both the server free he selects the server using random selection or if the customer finds both servers busy he waits for the a server become free. The queue discipline is FCFS. The vacation period follows negative exponential distribution. For this model, the steady state results have been obtained. Some performance measures have been calculated and numerical models are given.

Keywords
Heterogeneous, Single vacation, Two server queue, Steady state

Introduction

In the literature, many of work on more than one server queueing system the servers are homogeneous. This is only valid on when the service procedure is mechanically controlled. In [8], Neuts and Takahashi have pointed out that two heterogeneous servers in the queueing system, analytical results are intractable. Even though, some investigators focused their studies on two heterogeneous servers in queue. Karlin and McGregor [6]. In $M/M/S$ queue we obtained the busy period distribution. Krishnamoorthi [7] a Poisson queue with two heterogeneous servers are considered and with contravention of the First-come-First-serve principle also. In Singh [9] consider a Markovian queuing system with balking and two heterogeneous servers. The capacity of the slower server and obtains the optimal service rates are obtained by the author. Singh [10] consider a Markovian queue with number of servers depending upon the length of the queue. Desmit [4], [5] presented a tactic to identify distribution of waiting times and lengths of the queue for the queue $GI/H_2/S$.

Barcelo [3] consider an approximation of the average waiting time of $M/H_{2b}/S$ queue. Shin and Moon [11] have been enacted almost accurate analysis of $M/G/C$ queue. Arkat and Farahanim [2],The method of partial-fraction decomposition have been used to $M/H_2/2$ queue. Zhernovyi [12] analyzed waiting line with swapping the service modes and threshold blocking of input flow. Zhernovyi and Kopytko [13] examined the swapping the service mode with Markovian waiting line.

In a vacation queue, the server stopovers serving to the customers fully during the entire duration. After the completion of vacation, the server stays idle and waits for the arrivals if no customer is waiting in the queue in a single vacation policy (refer to Tian et al. [14]) The server begins the service,if there is at least a customer waiting in the line at the vacation instantly completed. Altman and Yechiali [1] obtained a inclusive analysis of the impatient behavior of only one server lines for both the vacation cases and obtained various closed form results. Using

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matrix geometric method, the steady-state system size and virtual time distributions of the $M/M/1$ queueing model with single working vacations were obtained by Tian et al. [14].

In this paper, we consider an heterogeneous two server single vacation queue. The arrival rate dependents on the server states. The two servers serve the customers using two different exponential distributions. Using Probability generating function method, this model has been examined. Remaining of this paper is systematized as follows: model description and analysis are given in section 2. In section 3, some system performance measures have been calculated. In section 4, we carried out a numerical study.

Model Description and Analysis

We consider two server heterogeneous queueing system with a single waiting line of unlimited capacity. Customers arrive at the system conferring to a Poisson process. The service times of customers follows exponential distribution. If no customer in the system at a service completion epoch the servers take vacation of random period, follows negative exponential distribution with rate θ. After completion of a single vacation period the servers return to the system independent of the number of customers in the queue. The arrival and service rates are defined as follows:

- During service the arrival rate is λ_1
- During vacation the arrival rate is λ_2
- Service rate of first server is μ_1
- Service rate of second server is μ_2, $\mu_1 > \mu_2$

For the analysis the following probabilities have been defined in steady state:

$p_{0,1}$ = Probability that the servers are idle and there are no customers in the system;

$p_{n,0}$= Probability that the severs are in vacation and there are n customers in the system; $n \geq 0$

$p_{n,1}$= Probability that the severs are busy and there are n customers in the system; $n \geq 1$

Using birth-death arguments we have derived the difference equations.

$$\lambda_1 p_{0,1} = \theta p_{0,0} \quad (1)$$
$$(\lambda_1 + \mu_1 + \mu_2) p_{n,1} = \lambda_1 p_{n-1,1} + (\mu_1 + \mu_2) p_{n+1,1} + \theta p_{n,0};\ n \geq 1 \quad (2)$$
$$(\lambda_2 + \theta) p_{0,0} = (\mu_1 + \mu_2) p_{1,1} \quad (3)$$
$$(\lambda_2 + \theta) p_{n,0} = \lambda_2 p_{n-1,0};\ n \geq 1 \quad (4)$$

and the normalizing condition

$$\sum_{n=0}^{\infty} p_{n,0} + \sum_{n=0}^{\infty} p_{n,1} = 1.$$

The following probability generating functions (P.G.F.'s) have been defined for the analysis

$$P_0(z) = \sum_{n=0}^{\infty} p_{n,0} z^n\ , P_1(z) = \sum_{n=0}^{\infty} p_{n,1} z^n$$

Then, multiplying equation (2) by z^n, and then adding equation (1) and summing all possible values of n, we get

$$P_1(z) = \frac{[\theta(\mu_1+\mu_2)(1-z)+\lambda_1 z(\lambda_2+\theta)]p_{0,0} - \lambda_1 \theta z P_0(z)}{\lambda_1[\lambda_1 z^2 - (\lambda_1+\mu_1+\mu_2)z + \mu_1 + \mu_2]} \quad (5)$$

Similarly, we get from (3) and (4)

$$P_0(z) = \frac{(\lambda_2+\theta)p_{0,0}}{\lambda_2(1-z)+\theta} \quad (6)$$

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Substituting (6) in (5), we get

$$P_1(z) = \frac{[\theta(\mu_1+\mu_2)(1-z)[\lambda_2(1-z)+\theta]+\lambda_1\lambda_2(\lambda_2+\theta)z(1-z)]p_{0,0}}{\lambda_1[\lambda_1 z^2-(\lambda_1+\mu_1+\mu_2)z+\mu_1+\mu_2][\lambda_2(1-z)+\theta]} \quad (7)$$

where $P_1(z)$ and $P_0(z)$ is the P.G.F. of number of customers in the system when the servers are in busy and vacation and respectively.

To find $\boldsymbol{p_{0,0}}$

By applying L'Hospital rule to (7) and $z = 1$, we get

$$P_1(1) = \frac{[\theta^2(\mu_1+\mu_2)+\lambda_1\lambda_2(\lambda_2+\theta)]p_{0,0}}{\lambda\theta(\mu_1+\mu_2-\lambda_1)} \quad (8)$$

Similarly $z = 1$ in (6), we get

$$P_0(1) = \frac{(\lambda_2+\theta)p_{0,0}}{\theta} \quad (9)$$

Using normalizing condition, we get from (8) and (9)

$$p_{0,0} = \frac{\lambda_1\theta(\mu_1+\mu_2-\lambda_1)}{\theta^2(\mu_1+\mu_2)+\lambda_1(\lambda_2+\theta)(\mu_1+\mu_2+\lambda_2-\lambda_1)}$$

System Performance Measures

The following system performances are obtained using straight forward calculations.

(i). Mean number of customers in the system when the servers are in vacation

$$L_{sv} = \left(\frac{d}{dz}P_0(z)\right)_{z\to 1} = \frac{\lambda_1\lambda_2(\mu_1+\mu_2-\lambda_1)(\lambda_2+\theta)}{\theta[\theta^2(\mu_1+\mu_2)+\lambda_1(\lambda_2+\theta)(\mu_1+\mu_2+\lambda_2-\lambda_1)]}$$

(ii). Mean number of customers in the system when the servers are busy

$$L_{sb} = \frac{\lambda_1\theta^3(\mu_1+\mu_2)+\lambda_1\lambda_2(\lambda_2+\theta)[(\theta+\lambda_2)(\mu_1+\mu_2)-\lambda_1\lambda_2]}{\theta(\lambda_1-(\mu_1+\mu_2))[\theta^2(\mu_1+\mu_2)+\lambda_1(\lambda_2+\theta)(\mu_1+\mu_2+\lambda_2-\lambda_1)]}$$

(iii) Probability that the server is on vacation

$$P_v = \frac{\lambda_1(\lambda_2+\theta)(\mu_1+\mu_2-\lambda_1)}{\theta^2(\mu_1+\mu_2)+\lambda_1(\lambda_2+\theta)(\mu_1+\mu_2+\lambda_2-\lambda_1)}$$

(iv) Probability that the server is busy

$$P_b = \frac{\theta^2(\mu_1+\mu_2)+\lambda_1\lambda_2(\lambda_2+\theta)}{\theta^2(\mu_1+\mu_2)+\lambda_1(\lambda_2+\theta)(\mu_1+\mu_2+\lambda_2-\lambda_1)} \ .$$

A Numerical Study

In this section, we calculate numerically, the system performance using the expression obtained in the above section. The numerical results are presented in figures. Figure 1, explains the effect of $\boldsymbol{\lambda_1}$ on mean number of customers in the system when the server is busy. The graphs are drawn for both homogeneous and heterogeneous models. The graphs shown an upward trend initially and then steeply increases for both models. Also for higher service rates the queue length comparatively decreases, as expected. The Figure 2, analyzes the effect of $\boldsymbol{\lambda_1}$ on mean number of

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customers in the system when the server is on vacation. The graphs are drawn for both homogeneous and heterogeneous models. The graphs shown an upward trend initially and then reverse the direction. Figure 3 and 4, presents the Probability that the server is busy and, is on vacation, for larger values of $\boldsymbol{\lambda_1}$ the probability is low for homogeneous model compare to heterogeneous model. In the case of vacation probability we experience the converse trend.

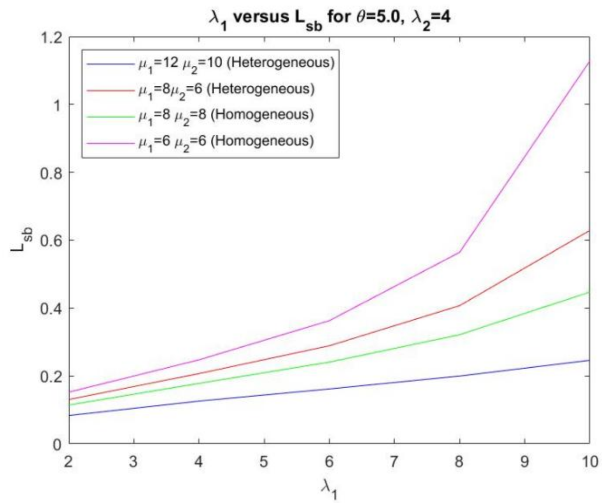

Figure 1. Mean number of customers in the system when the server is busy

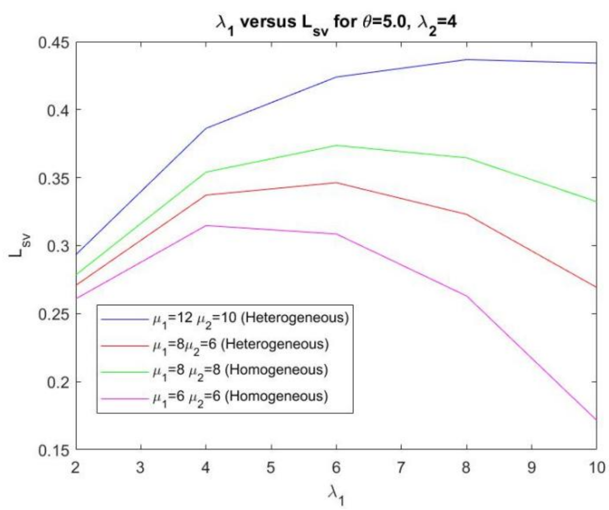

Figure 2. Mean number of customers in the system when the server is on vacation

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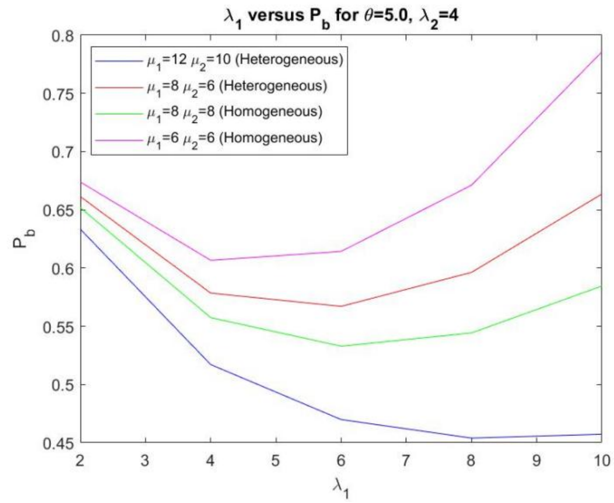

Figure 3. Probability that the server is busy

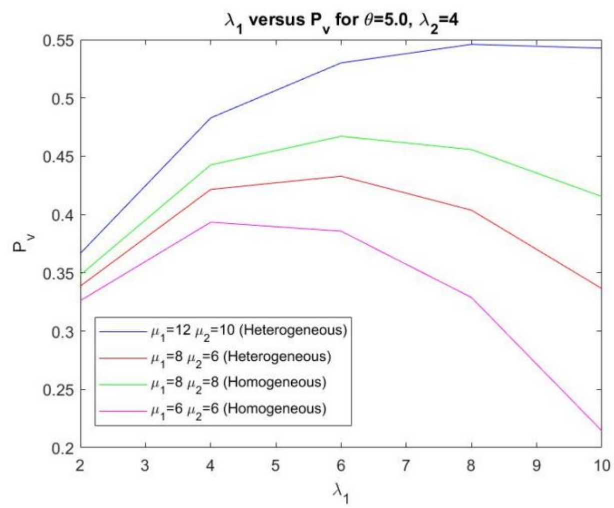

Figure 4. Probability that the server is on vacation

Conclusion

In many real life situations of day-to-day the model proposed can be applied as well as industrial congestion problems like telecommunications, computer communication networks, manufacturing system, etc. For this model, the steady state results have been obtained. Some performance measure and some numerical models are presented. The graphs of L_{sv} and L_s expresses the queue length comparatively decreases as expected, L_{sb} shown an upward trend initially and reverse the direction for both homogeneous and heterogeneous models, and then P_v and P_b represents the probability is low for homogeneous model compare to heterogeneous model.

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