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Indian Second Wave Common COVID-19 Equation Analysis with SEIR Model and Effect of Time Delay

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ABSTRACT

This articleis the second wave on Indian pandemic for SEIR (Susceptible-Exposed-Infective-Recovery) model with effect of time delay. We derived COVID-19 equations analysis part only (COVID-19 equation Analysis 1- COVID-19 equation Analysis 9). The mathematical analysis is very helpful for new researchers in Mathematical biology and COVID-19 part.

Keywords:COVID-19, SEIR model, second wave on Indian pandemic

1. Introduction

The same story is continued on first wave for COVID-19. Already we know that the history of the COVID-19 spread. Recently it has been published lot of papers from COVID-19 cases([1]-[15]).But this paper deals the new common COVID-19 equation with mathematical part only.

2. Indian second wave common COVID-19 equation

We consider the common COVID-19 equation on SEIR model [1]:

$$\begin{cases} \dfrac{dS(t)}{dt} = A - dS(t) - \beta S(t)I(t) - \beta_1 S(t)I(t), \\ \dfrac{dE(t)}{dt} = \beta S(t)I(t) + \beta_1 S(t)I(t) - \mu E(t) - dE(t), \\ \dfrac{dI(t)}{dt} = \mu E(t) - \left(\eta + d\right) I(t), \\ \dfrac{dR(t)}{dt} = \eta I(t-\tau) - dR(t-\tau). \end{cases} \tag{1}$$

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The key function of the complete paper is equation (1) and Table 1 gives the parameter description.

Table 1 Parametervalues of host model.

Parameter	**Description**	**Range**
A	Number of new born host	
d	Death rate of host	1.99–2.15
β	Transmission rate from infected sourceto suspected host	3-12.5
β_1	Transmission rate from infected host to suspected host	10-14 days
μ	Incubation period of host	6.5-37.4
η	Infectious period of host	18-130

3. COVID-19 equation Analysis 1

Consider the system of equations' initial values:

$$\begin{cases} S(a) \geq 0, & E(a) \geq 0, \quad I(a) \geq 0, \quad R(a) \geq 0, \\ S(0) > 0, & E(0) > 0, \quad I(0) > 0, \quad R(0) > 0, \end{cases} \quad a \in [-\tau, 0]. \tag{2.1}$$

4. COVID-19 equation Analysis 2

Where $(S(t), E(t), I(t), R(t))$ of (1) satisfying conditions (2.1), we have $\lim_{t\to+\infty} \sup S(t) \leq \frac{A}{d}$.

Then, $t_1 > 0$ such that $S(t_1) > \frac{A}{d}$ and $\dot{S}(t_1) > 0$, we have

$\dot{S}(t_1) = A - dS(t_1) - (\beta + \beta_1) S(t_1) I(t_1) \leq -(\beta + \beta_1) S(t) I(t) \leq 0.$ $S(t_1) > \frac{A}{d}$ whichcontradiction is to $\dot{S}(t_1) > 0.$

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5. COVID-19 equation Analysis 3

From (1):

$$S(t) = S(0)e^{-\int_0^t (d+(\beta+\beta_1)I(\theta))d\theta} + \int_0^t Ae^{-\int_\varepsilon^t (d+(\beta+\beta_1)I(\theta))d\theta} d\varepsilon,$$

$$E(t) = E(0)e^{-(\mu+d)t} + \int_0^t (\beta+\beta_1) S(\varepsilon) I(\varepsilon) e^{-(\mu+d)(t-\varepsilon)} d\varepsilon,$$

$$I(t) = I(0)e^{-(\eta+d)t} + \int_0^t \mu E(\varepsilon) e^{-(\eta+d)(t-\varepsilon)} d\varepsilon,$$

$$R(t) = R(0)e^{-dt} + \int_0^t \eta I(\varepsilon-\tau) e^{-dt} d\varepsilon.$$

Let $N(t) = \left[S(t) + E(t) + \frac{(\mu+d)}{2\mu} I(t) + \frac{d}{\eta A} R(t+\tau) \right]$,

Let $c = \min\left\{ d, \frac{\mu+d}{2}, \eta+d, d \right\}$.

$$\frac{d}{dt}\left[N(t)\right] = A - dS(t) - \frac{\mu+d}{2}E(t) - \frac{(\mu+d)(\eta+d)}{2\mu} I(t) + \eta I(t+\tau) - \frac{d^2}{\eta A} R(t+\tau)$$

$$\leq A - dS(t) - \frac{\mu+d}{2}E(t) - \frac{(\mu+d)(\eta+d)}{2\mu} I(t) - \frac{d^2}{\eta A} R(t+\tau)$$

$$< A - c\left[S(t) + E(t) + \frac{(\mu+d)}{2\mu} I(t) + \frac{d}{\eta A} R(t+\tau) \right] = s - cN.$$

6. COVID-19 equation Analysis 4

The basic reproductive ratio of virus for system (1) is given by $R_0 = \frac{(\beta+\beta_1) A\mu}{d}$.

Infection-free equilibrium: $P_0 = \left(\frac{A}{d}, 0, 0, 0 \right)$,

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Absent infection equilibrium:

$$P_1=\left(\frac{(\mu+d)(\eta+d)}{\mu(\beta+\beta_1)},\frac{A}{\mu+d}-\frac{d(\eta+d)}{\mu(\beta+\beta_1)},\frac{A\mu}{(\mu+d)(\eta+d)}-\frac{d}{\beta+\beta_1},0\right),$$

Present infection equilibrium:

$$\bar{P}=\left(\begin{array}{l}\dfrac{\eta(\eta+d)A-(\beta+\beta_1)\mu d}{\eta d(\eta+d)},\dfrac{d(\eta+d)}{\eta(\eta+d)A-(\beta+\beta_1)\mu d},\dfrac{\mu d^2}{\eta(\eta+d)A-(\beta+\beta_1)\mu d},\\ \left[\dfrac{(\beta+\beta_1)\mu\left(\eta(\eta+d)A-(\beta+\beta_1)\mu d\right)}{\eta d\mu}-\mu\right]\end{array}\right).$$

7. COVID-19 equation Analysis 5

$$\begin{cases}\dfrac{dS(t)}{dt}=-dS(t)-\dfrac{(\beta\ +\beta_1)A}{d}I(t),\\ \dfrac{dE(t)}{dt}=\dfrac{(\beta+\beta_1)A}{d}I(t)-\mu E(t)-dE(t),\\ \dfrac{dI(t)}{dt}=\mu E(t)-(\eta+d)I(t),\\ \dfrac{dR(t)}{dt}=\eta I(t)-dR(t).\end{cases} \tag{3.1}$$

The characteristic equation of (3.1) is

$$(\lambda+d)(\lambda+d)\left[\lambda^2+(\mu+\eta+2d)\lambda+(\mu+d)(\eta+d)-\mu(\beta+\beta_1)\right]=0. \tag{3.2}$$

If $\lambda_{1,2}=d$. are the two of the roots of the characteristic equation (3.2) and the other two roots are found by considering $\lambda^2+(\mu+\eta+2d)\lambda+(\mu+d)(\eta+d)-\mu(\beta+\beta_1)=0.$
(3.3)

If $R_0<1$, then $(\mu+d)(\eta+d)-\mu(\beta+\beta_1)>0$ and
$(\mu+\eta+2d)^2-4\left[(\mu+d)(\eta+d)-\mu(\beta+\beta_1)\right]>0$.

We have

$$\lambda_{3,4}=\frac{-(\mu+\eta+2d)\pm\sqrt{(\mu+\eta+2d)^2-4\left[(\mu+d)(\eta+d)-\mu(\beta+\beta_1)\right]}}{2}.$$

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8. COVID-19 equation Analysis 6

Define a Lyapunov functional

$$F=\frac{1}{2}\left[S(t)-\frac{A}{d}\right]^2+\frac{A}{d}E(t)+xI(t)+\eta R(t)+\int_{t-\tau}^{t}S(\alpha)E(\alpha)R(\alpha)d\alpha,$$

$$F'|_{(1.2)}=\left[S(t)-\frac{A}{d}\right]\left[-d\left(S(t)-\frac{A}{d}\right)-(\beta+\beta_1)S(t)I(t)\right]+\frac{A}{d}\left[(\beta+\beta_1)S(t)I(t)-(\mu+d)E(t)\right]+$$
$$x\left[\mu E(t)-(\eta+d)I(t)\right]+\eta\left[\eta I(t)-dR(t)\right]+S(t)E(t)R(t).$$

Since $(\beta+\beta_1)S(t)I(t)=(\beta+\beta_1)\left[S(t)-\frac{A}{d}\right]+\frac{(\beta+\beta_1)A}{d}I(t)$, we have

$$F'|_{(1.2)}=-d\left(S(t)-\frac{A}{d}\right)^2-(\beta+\beta_1)I(t)\left(S(t)-\frac{A}{d}\right)^2-\left[\frac{(\mu+d)A}{d}-x\mu\right]E(t)+$$
$$\left[\frac{(\beta+\beta_1)A^2}{d^2}-x(\eta+d)+\eta^2\right]I(t)+\left[S(t)E(t)-\eta d\right]R(t).$$

Since $R_0<1$ reduces to $\frac{A(\mu+d)}{\mu d}-\frac{(\beta+\beta_1)A^2}{(\eta+d)d^2}>0$, there must be a positive constant $x\left(x\in\left[\frac{(\beta+\beta_1)A^2}{(\eta+d)d^2},\frac{A(\mu+d)}{\mu d}\right]\right)$, such that $\frac{(\mu+d)A}{d}-x\mu>0$ and $\frac{(\beta+\beta_1)A^2}{d^2}-x(\eta+d)+\eta^2>0$.

Suppose $S(t)$, $E(t)$, $I(t)$, $R(t)$ are positive and $S(t)\le\frac{A}{d}$ holds, we have $F'|_{(1.4)}\le 0$ and $F'|_{(1.4)}=0$ iff $(S(t),E(t),I(t),R(t))=\left(\frac{A}{d},0,0,0\right)$.

9. COVID-19 equation Analysis 7

Let $P_1=(\bar{S},\bar{E},\bar{I},0)=\left(\frac{(\mu+d)(\eta+d)}{\mu(\beta+\beta_1)},\frac{A}{\mu+d}-\frac{d(\mu+d)}{\mu(\beta+\beta_1)},\frac{A\mu}{(\mu+d)(\eta+d)}-\frac{d}{\beta+\beta_1},0\right)$, the linearized equations of (1) at P_1 is

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$$\begin{cases} \dfrac{dS(t)}{dt} = -\left(d + (\beta+\beta_1)\bar{I}\right)S(t) - (\beta+\beta_1)\bar{S}I(t), \\ \dfrac{dE(t)}{dt} = (\beta+\beta_1)\bar{I}S(t) - (\mu+d)E(t) + (\beta+\beta_1)\bar{S}I(t), \\ \dfrac{dI(t)}{dt} = \mu E(t) - (\eta+d)I(t), \\ \dfrac{dR(t)}{dt} = \eta I(t-\tau) - dR(t-\tau). \end{cases} \tag{3.4}$$

Therefore, the associated transcendental characteristic equation of (3.4) is,

$$\left(\lambda - \eta e^{-\lambda\tau} + d\right)\left(\lambda^3 + k_1\lambda^2 + k_2\lambda + k_3\right) = 0,$$

Here $k_1 = \mu + \eta + 3d + \dfrac{A}{\mu+d} - \dfrac{d(\mu+d)}{\mu(\beta+\beta_1)}$, $k_2 = (\mu+\eta+2d)\left(d + \dfrac{A}{\mu+d} - \dfrac{d(\mu+d)}{\mu(\beta+\beta_1)}\right)$,
$k_3 = A(\beta+\beta_1)\mu - d(\mu+d)(\eta+d)$.

Let us consider,

$$\lambda^3 + k_1\lambda^2 + k_2\lambda + k_3 = 0.$$

Clearly, if $R_0 > 1$, we have $k_1 = \mu + \eta + 3d + \dfrac{A}{\mu+d} - \dfrac{d(\mu+d)}{\mu(\beta+\beta_1)} > 0$ and
$k_3 = A(\beta+\beta_1)\mu - d(\mu+d)(\eta+d) > 0$ hold; furthermore,

$$k_1k_2 - k_3 = \left(\mu+\eta+3d+\frac{A}{\mu+d} - \frac{d(\mu+d)}{\mu(\beta+\beta_1)}\right)\left((\mu+\eta+2d)\left(d + \frac{A}{\mu+d} - \frac{d(\mu+d)}{\mu(\beta+\beta_1)}\right)\right)$$
$$-A(\beta+\beta_1)\mu - d(\mu+d)(\eta+d)$$

From the Routh-Hurwitz criteria,

$$\lambda - \eta e^{-\lambda\tau} + d = 0. \tag{3.6}$$

From (3.6), we have

$$\begin{cases} \upsilon = -(\eta-d)\sin\upsilon t, \\ d = (\eta-d)\cos\upsilon t, \end{cases} \tag{3.7}$$

$$\upsilon^2 = \gamma^2\left[\eta-d\right]^2 - d^2.$$

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If $1 < R_0 < 1 + \frac{(\beta+\beta_1)\mu^2 d}{(\mu+d)(\eta+d)\gamma d}$, then $\upsilon^2 < 0$.

10. COVID-19 equation Analysis 8

Let $S_1(t) = S(t) - \bar{S}, E_1(t) = E(t) - \bar{E}, I_1(t) = I(t) - \bar{I}, R_1(t) = R(t) - \bar{R}$. Therefore, (1) becomes

$$\begin{cases} \frac{dS(t)}{dt} = -\left(d + (\beta+\beta_1)\bar{I}\right)S(t) - (\beta+\beta_1)S(t)I(t) - (\beta+\beta_1)\bar{S}I(t), \\ \frac{dE(t)}{dt} = (\beta+\beta_1)S(t)I(t) + (\beta+\beta_1)\bar{I}S(t) - (\mu+d)E(t) + (\beta+\beta_1)\bar{S}I(t), \\ \frac{dI(t)}{dt} = \mu E(t) - (\eta+d)I(t), \\ \frac{dR(t)}{dt} = \eta I(t-\tau) - dR(t-\tau). \end{cases} \tag{3.8}$$

$$\begin{cases} \frac{dS(t)}{dt} = -\left(d + (\beta+\beta_1)\bar{I}\right)S(t) - (\beta+\beta_1)\bar{S}I(t), \\ \frac{dE(t)}{dt} = (\beta+\beta_1)\bar{I}S(t) - (\mu+d)E(t) + (\beta+\beta_1)\bar{S}I(t), \\ \frac{dI(t)}{dt} = \mu E(t) - (\eta+d)I(t), \\ \frac{dR(t)}{dt} = \eta I(t-\tau) - dR(t-\tau). \end{cases} \tag{3.9}$$

$$C(\lambda) = \lambda^4 + U_1\lambda^3 + U_2\lambda^2 + U_3\lambda + U_4 - \left(V_1\lambda^3 + V_2\lambda^2 + V_3\lambda + V_4\right)e^{-\lambda\tau}, \tag{3.10}$$

Where,

$U_1 = \mu + \eta + 4d + (\beta+\beta_1)\bar{I}$,

$U_2 = d^2 + (\mu+d)(\eta+d) + 2(\mu+\eta+2d)d + (\beta+\beta_1)^2 \mu\overline{SI} + (\beta+\beta_1)\bar{I}d$,

$U_3 = (\mu+\eta+2d)d^2 + 2(\mu+d)(\eta+d)d + (\mu+\eta+2d)(\beta+\beta_1)\bar{I} + (\mu+d)(\eta+d)\bar{I}$
$\quad + (\beta+\beta_1)^2 d\mu\overline{SI}$,

$U_4 = (\mu+d)(\eta+d)d^2 + (\mu+\eta+2d)(\beta+\beta_1)\bar{I}d + (\mu+d)(\eta+d)(\beta+\beta_1)\bar{I}d$,

$V_1 = d$,

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$V_2 = (\beta + \beta_1)\mu\bar{S}$,

$V_3 = 2(\beta + \beta_1)\mu d\bar{S} + (\beta + \beta_1)^2 \mu\overline{SI}$,

$V_4 = (\beta + \beta_1)\mu d\bar{S} + (\beta + \beta_1)^2 \mu d\overline{SI}$.

11. COVID-19 equation Analysis 9

If $\tau = 0$, Eq. (3.10) become

$$\lambda^4 + (U_1 - V_1)\lambda^3 + (U_2 - V_2)\lambda^2 + (U_3 - V_3)\lambda + U_4 - V_4 = 0 . \tag{3.11}$$

Since $R_0 > 1 + \dfrac{(\beta+\beta_1)\mu^2 d}{(\mu+d)(\eta+d)\gamma d}$, $\bar{S} > 0, \bar{E} > 0, \bar{I} > 0, \bar{R} > 0$. By Routh-Hurwitz criteria, we get

$$W_1 = U_1 - V_1 = \mu + \eta + 3d + (\beta + \beta_1)\bar{I} > 0 ,$$

$$W_2 = (U_1 - V_1)(U_2 - V_2) - (U_3 - V_3)$$

$$= \left(\mu + \eta + 3d + (\beta+\beta_1)\bar{I}\right)\begin{pmatrix} d^2 + (\mu+d)(\eta+d) + 2(\mu+\eta+2d)d + (\beta+\beta_1)^2\mu\overline{SI} + \\ (\beta+\beta_1)\bar{I}d - (\beta+\beta_1)\mu\bar{S} \end{pmatrix}$$
$$-\begin{bmatrix} (\mu+\eta+2d)d^2 + 2(\mu+d)(\eta+d)d + (\mu+\eta+2d)(\beta+\beta_1)\bar{I} + (\mu+d)(\eta+d)\bar{I} - \\ 2(\beta+\beta_1)\mu d\bar{S} - (\beta+\beta_1)^2\mu\overline{SI} \end{bmatrix}$$

$$W_3 = \begin{vmatrix} U_1 - V_1 & U_3 - V_3 & 0 \\ 1 & U_2 - V_2 & U_4 - V_4 \\ 0 & U_1 - V_1 & U_3 - V_3 \end{vmatrix}$$

$$= (U_1 - V_1)\left[(U_2 - V_2)(U_3 - V_3) - (U_1 - V_1)(U_4 - V_4)\right] - (U_3 - V_3)^2 .$$

Let $a = (\mu + d)(\eta + d)$, $b = \beta + \beta_1$ and $c = \mu + \eta + 2d$.

$$W_3 = \left(c + d + b\bar{I}\right)\begin{bmatrix} \begin{pmatrix} d^2 + a + 2cd + b^2\mu\overline{SI} + \\ b\bar{I}d - b\mu\bar{S} \end{pmatrix} \times \begin{pmatrix} cd^2 + 2ad + cb\bar{I} \\ +a\bar{I} - 2b\mu d\bar{S} - b^2\mu\overline{SI} \end{pmatrix} \\ -\left(c + d + b\bar{I}\right)\left(ad^2 + cb\bar{I}d + ab\bar{I}d\right) \end{bmatrix} - \begin{pmatrix} cd^2 + 2ad + cb\bar{I} + a\bar{I} + \\ b^2 d\mu\overline{SI} - 2b\mu d\bar{S} - b^2\mu\overline{SI} \end{pmatrix}^2$$

Clearly, we have $W_3 > 0$.

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$$W_4 = \begin{vmatrix} U_1 - V_1 & U_3 - V_3 & 0 & 0 \\ 1 & U_2 - V_2 & U_4 - V_4 & 0 \\ 0 & U_1 - V_1 & U_3 - V_3 & 0 \\ 0 & 1 & U_2 - V_2 & U_4 - V_4 \end{vmatrix} = k_4 W_3 .$$

$$C(\lambda) = 0, \qquad \mathrm{Re}(\lambda) < 0, \qquad \text{for}\ \tau \in [0, \tau_0), \tag{3.12}$$

$$\upsilon^4 - U_1\upsilon^3 i - U_2\upsilon^2 + U_3\upsilon i + U_4 - \left(-V_1\upsilon^3 i - V_2\upsilon^2 + V_3\upsilon i + V_4\right)\left(\cos\upsilon\tau - i\sin\upsilon\tau\right) = 0 . \tag{3.13}$$

After the real and imaginary parts have been separated, we have

$$\begin{cases} \left(V_4 - V_2\upsilon^2\right)\cos\upsilon\tau + \left(V_3\upsilon - V_1\upsilon^3\right)\sin\upsilon\tau = \upsilon^4 - U_2\upsilon^2 + U_4, \\ \left(V_1\upsilon^3 - V_3\upsilon\right)\cos\upsilon\tau + \left(V_4 - V_2\upsilon^2\right)\sin\upsilon\tau = U_1\upsilon^3 - U_3\upsilon. \end{cases} \tag{3.14}$$

$$\cos\upsilon\tau = \frac{1}{\Delta}\begin{vmatrix} \upsilon^4 - U_2\upsilon^2 + U_4 & V_3\upsilon - V_1\upsilon^3 \\ U_1\upsilon^3 - U_3\upsilon & V_4 - V_2\upsilon^2 \end{vmatrix}$$

$$= \frac{1}{\Delta}\left[\left(U_1V_1 - V_2\right)\upsilon^6 + \left(V_4 + U_2V_2 - U_1V_3 - U_3V_1\right)\upsilon^4 + \left(U_3V_3 - U_2V_4 - U_4V_2\right)\upsilon^2 + U_4V_4\right]$$

$$= \frac{1}{\Delta}\left(m_1\upsilon^6 + m_2\upsilon^4 + m_3\upsilon^2 + m_4\right).$$

$$\sin\upsilon\tau = \frac{1}{\Delta}\begin{vmatrix} V_4 - V_2\upsilon^2 & \upsilon^4 - U_2\upsilon^2 + U_4 \\ V_1\upsilon^3 - V_3\upsilon & U_1\upsilon^3 - U_3\upsilon \end{vmatrix}$$

$$= -\frac{\upsilon}{\Delta}\left[V_1\upsilon^6 + \left(U_1V_2 - V_3 - U_2V_1\right)\upsilon^4 + \left(U_2V_3 + U_4V_1 - U_3V_2 - U_1V_4\right)\upsilon^2 + U_3V_4 - U_4V_3\right]$$

$$= -\frac{\upsilon}{\Delta}\left(n_1\upsilon^6 + n_2\upsilon^4 + n_3\upsilon^2 + n_4\right).$$

Where,

$$\Delta = \begin{vmatrix} V_4 - V_2\upsilon^2 & V_3\upsilon - V_1\upsilon^3 \\ V_1\upsilon^3 - V_3\upsilon & V_4 - V_2\upsilon^2 \end{vmatrix}$$

$$= \left(V_4 - V_2\upsilon^2\right)^2 + \left(V_3\upsilon - V_1\upsilon^3\right)^2 = V_1\upsilon^6 + \left(V_2 - 2V_1V_3\right)\upsilon^4 + \left(V_3^2 - 2V_2V_4\right)\upsilon^2 + V_4^2$$

$$= \left(g_1\upsilon^6 + g_2\upsilon^4 + g_3\upsilon^2 + g_4\right) > 0 .$$

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Noting $\sin^2 \upsilon\tau + \cos^2 \upsilon\tau = 1$, it follows that

$$\upsilon^{14} + h_1\upsilon^{12} + h_2\upsilon^{10} + h_3\upsilon^8 + h_4\upsilon^6 + h_5\upsilon^4 + h_6\upsilon^2 + h_7 = 0, \tag{3.15}$$

Where,

$$h_1 = \frac{1}{n_1^2}\left(m_1^2 + 2d_1d_2 - g_1^2\right),$$

$$h_2 = \frac{1}{n_1^2}\left(2m_1m_2 + n_2^2 + 2n_1n_3 - 2g_1g_3\right),$$

$$h_3 = \frac{1}{n_1^2}\left(m_2^2 + 2m_1m_3 + 2n_1n_4 + 2n_2n_4 - g_2^2 - 2g_1g_3\right),$$

$$h_4 = \frac{1}{n_1^2}\left(2m_1m_4 + 2m_2m_3 + n_3^2 + 2n_2n_4 - 2g_1g_4 - 2g_2g_3\right),$$

$$h_5 = \frac{1}{n_1^2}\left(m_3^2 + 2m_2m_4 + 2n_3n_4 - g_3^2 - 2g_2g_4\right),$$

$$h_6 = \frac{1}{n_1^2}\left(2m_3m_4 + n_4^2 - 2g_3g_4\right),$$

$$h_7 = \frac{1}{n_1^2}\left(m_4^2 - g_4^2\right).$$

Denoting: $x = \upsilon^2$, (3.15) becomes

$$x^7 + h_1x^6 + h_2x^5 + h_3x^4 + h_4x^3 + h_5x^2 + h_6x + h_7 = 0. \tag{3.16}$$

Assume

(C_1) Eq. (3.16) has only one positive real root;

(C_2)
$\Gamma \square \left[4\upsilon^6 + 3\left(U_1^2 - 2U_2 - V_1^2\right)\upsilon^4 + 2\left(U_2^2 - V_2^2 + 2U_4 + 2V_1V_3 - 2U_1U_3\right)\upsilon^2 + U_3^2 - V_3^2 + 2V_2V_4 - 2U_2U_4\right] > 0$
for any $\upsilon > 0$.

Let x_0 be the positive roots of (3.16), denoting $\upsilon_0 = \sqrt{x_0}$. From the above, we get

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$$\tau_i = \frac{1}{\upsilon_0}\left(\arccos\frac{m_1\upsilon_0^6 + m_2\upsilon_0^4 + m_3\upsilon_0^2 + m_4}{g_1\upsilon_0^6 + g_2\upsilon_0^4 + g_3\upsilon_0^2 + g_4} + 2i\pi\right), \qquad i = 0,1,2,...,$$

And

$$\tau_0 = \frac{1}{\upsilon_0}\arccos\frac{m_1\upsilon_0^6 + m_2\upsilon_0^4 + m_3\upsilon_0^2 + m_4}{g_1\upsilon_0^6 + g_2\upsilon_0^4 + g_3\upsilon_0^2 + g_4}, \qquad i = 0.$$

$$\left[\frac{d\lambda}{d\tau}\right]^{-1} = \frac{-\left(4\lambda^3 + 3U_1\lambda^2 + 2U_2\lambda + U_3\right)e^{\lambda\tau}}{\lambda\left(V_1\lambda^3 + V_2\lambda^2 + V_3\lambda + V_4\right)} + \frac{3V_1\lambda^2 + 2V_2\lambda + V_3}{\lambda\left(V_1\lambda^3 + V_2\lambda^2 + V_3\lambda + V_4\right)} - \frac{\tau}{\lambda}. \tag{3.17}$$

Noting (3.14), we have

$$\operatorname{Re}\left[\frac{d\lambda}{d\tau}\right]^{-1} = \frac{1}{\upsilon\nabla}\left\{\begin{array}{l}\left(3U_1\upsilon^2 - U_3\right)\left[\left(V_1\upsilon^3 - V_3\upsilon\right)\cos\upsilon\tau + \left(V_4 - V_4\upsilon^2\right)\sin\upsilon\tau\right] \\ +\left(4\upsilon^3 - 2U_2\upsilon\right)\left[\left(V_4 - V_2\upsilon^2\right)\cos\upsilon\tau - \left(V_1\upsilon^3 - V_3\upsilon\right)\sin\upsilon\tau\right] \\ +\left(V_3 - 3V_1\upsilon^2\right)\left(V_1\upsilon^3 - V_3\upsilon\right) + 2V_2\upsilon\left(V_4 - V_2\upsilon^2\right)\end{array}\right\}$$

$$= \frac{1}{\nabla}\left[\begin{array}{l}4\upsilon^6 + 3\left(U_1^2 - 2U_2 - V_1^2\right)\upsilon^4 + 2\left(U_2^2 - V_2^2 + 2U_4 + 2V_1V_3 - 2U_1U_3\right)\upsilon^2 \\ +U_3^2 - V_3^2 + 2V_2V_4 - 2U_2U_4\end{array}\right] \tag{3.18}$$

Where $\nabla = \left(V_1\upsilon^3 - V_3\upsilon\right)^2 + \left(V_4 - V2\upsilon^2\right)^2 > 0$. If the hypothesis $\left(C_2\right)$ is satisfied, ten $(3.18) > 0$ will hold for any $\upsilon > 0$. So,

$$\text{Sign}\left\{\operatorname{Re}\left[\frac{d\lambda}{d\tau}\right]\Bigg|_{\tau=\tau_0}\right\} = \text{sign}\left\{\operatorname{Re}\left[\frac{d\lambda}{d\tau}\right]^{-1}\Bigg|_{\tau=\tau_0}\right\} \square\ \text{sign}(.) = 1.$$

4. Conclusion

This paper gives only the mathematical analysis part. It is very helpful for beginners of mathematical biology and COVID-19 research. The same SEIR model compare to the real life data, we will get the good result.

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