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Edge Magic Total Labeling & Edge Trimagic Total Labeling of Some Graphs

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ABSTRACT
An edge magic total labeling of a graph $G = (V, E)$ with p vertices and q edges is a bijection $f: V(G) \cup E(G) \to \{1,2,3, \dots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is a magic constant K. An edge trimagic total labeling of a graph $G = (V, E)$ with p vertices and q edges is a bijection $f: V(G) \cup E(G) \to \{1,2,3, \dots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is any of the distinct constant K_1, K_2, K_3.
In this paper we prove that H −graph of a path P_n ,Alternate triangular belt graph ,Braid graph , Semi Jahangir graph , m −Join of H_n ,F −tree,H −super subdivision of a path P_n are edge magic total graphs and H −graph of a path P_n ,Alternate triangular belt graph ,Braid graph , Semi Jahangir graph , m −Join of H_n , H −super subdivision of a path P_n , F −tree,$H \odot K_1$ graph of a path P_n are edge trimagic total graphs.
KEYWORDS
Edge magic total graph , Edge trimagic total graph ,H −graph of a path P_n ,Alternate triangular belt graph ,Braid graph.

INTRODUCTION

All the graphs in this paper are finite and undirected. The symbols $V(G)$& $E(G)$ denotes the vertex set and edge set of a graph G. An excellence reference on this subject is the survey by J. Gallian [3].Magic labeling was introduced by Kotzing and Rosa[1] defined by a graph G with bijection $f: V(G) \cup E(G) \to \{1,2,3, \dots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is a magic constant K. In 1996, Ringel and Llado called this labeling as edge magic. In 2001, Wallis introduced this as edge magic total labeling.
.In 2013, Jayasekaran et al.[2] introduced an edge trimagic total labeling of graphs. An edge trimagic total labeling of a graph $G = (V, E)$ with p vertices and q edges is a bijection $f: V(G) \cup E(G) \to \{1,2,3, \dots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is any of the distinct constant K_1, K_2, K_3. A graph which admits an edge trimagic total labeling is called an edge trimagic total graph. [2]
Definition: The H graph of path P_n is the graph obtained from two copies of P_n with vertices $u_1, u_2, \dots, u_n$&$v_1, v_2, \dots, v_n$ by joining the vertices $u_{\frac{n+1}{2}}$&$v_{\frac{n+1}{2}}$ by an edge if n is odd and the vertices $u_{\frac{n}{2}+1}$ &$v_{\frac{n}{2}}$ if n is even. [9]

Definition: Let $L_n = P_n \times P_2$ be the ladder graph with vertex set u_k&v_k $k = 1,2, \dots, n$.The triangular belt is obtained from a ladder by adding an edge $u_k v_{k+1}$,$\forall k = 1,2, \dots, n-1$.This graph is denoted by $ATB(n)$.[5]

Definition: The Braid graph is obtained from a pair of paths $P_n{}'$ and $P_n{}''$. Let $u_1, u_2, \dots, u_n$ be vertices of $P_n{}'$ and $v_1, v_2, \dots, v_n$ be vertices of $P_n{}''$. To obtaine braid graph join i^{th} vertex of path $P_n{}'$ with $(i+1)^{th}$ vertex of path $P_n{}''$ and i^{th} vertex of path $P_n{}''$ with $(i+2)^{th}$ vertex of path $P_n{}'$ with the new edges for all $1 \le i \le n-2$. [5]
Definition: A semi Jahangir graph is $S(J_n)$ a connected graph with a vertex set $V(SJ_n) = \{u, u_k: 1 \le k \le n\} \cup \{s_k: 1 \le k \le n-1\}$

And edge set $E(G) = \{u_k s_k: 1 \le k \le n-1\} \cup \{s_k u_{k+1}: 1 \le k \le n-1\} \cup \{u_k u: 1 \le k \le n\}$.[6]

Definition: m −Joins of H graph is a graph where each of H graph denoted by H_{n_1} by an edge e_1 with H graph denoted by H_{n_2} , H graph denoted by H_{n_2} by an edge e_2 with H graph denoted by H_{n_3} and so on with H graph denoted by $H_{n_{m-1}}$ by an edge e_{m-1} with H graph denoted by H_{n_m} such that $n_1 = n_2 = n_m$.[8]

Definition: Let G be a graph. A graph obtained from G by replacing each edge e_i by a H graph in such a way that the ends of e_i are merged with a pendant vertex in P_2 and pendant vertex in P'_2 is called H super subdivision of G is denoted by $HSS(G)$, where the H graph is a tree on 6 vertices in which exactly two vertices of degree 3.[7]

MAIN RESULTS

Theorem 2.1.1 H −graph of a path P_n is an edge magic total graph.

Proof: Let $G = H$ graph of a path P_n.P_n be the path $u_1, u_2, \dots, u_n$.we can obtain H −graph by considering two copies

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ofP_n. $V(G) = \{u_k, v_k : 1 \le k \le n\}$and

$$E(G) = \{(u_k u_{k+1}): 1 \le k \le n-1\} \cup \left\{\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right): n \text{ is } odd\ or \left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right): n\ is\ even\right\} \cup \{v_k v_{k+1}: 1 \le k \le n-1\}.$$

$$So, |V(G)| = 2n \,\&\, |E(G)| = 2n-1.$$

Define $f: V(G) \cup E(G) \to \{1,2,3,\dots,4n-1\}$,

Case :1n is even.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u_{2k}) = n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v_{2k-1}) = \frac{n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v_{2k}) = \frac{3n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

Case:2n is odd.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u_{2k}) = n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v_{2k-1}) = \frac{3n-1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(v_{2k}) = \frac{n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

For both cases we define following edge function as,

$$f(u_k u_{k+1}) = 4n - k \qquad 1 \le k \le n-1$$

$$f(v_k v_{k+1}) = 3n - k \qquad 1 \le k \le n-1$$

$f\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 3n$, n is even

$f\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 3n$, n is odd

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K = 5n + 1$.So, H −graph of a path P_n is an edge magic total graph.

Illustration: An edge magic total labeling of H graph of a pathP_6 and H graph of a path$P_7$is shown in Figure-1(a) and Figure-1(b) respectively.

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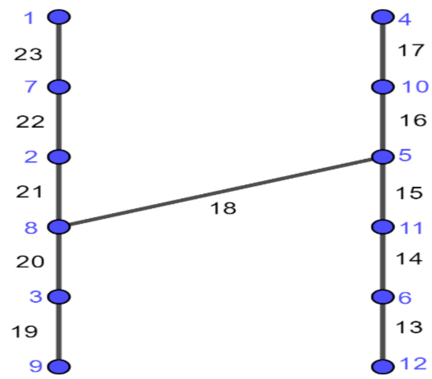

Figure-1(a)H graph of a pathP_6 with $K = 31$

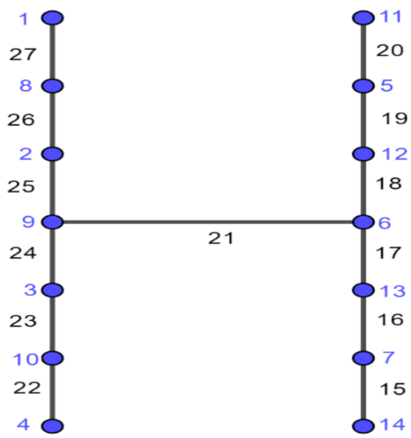

Figure-1(b) ***H* graph of a pathP_7 with $K = 36$**

Theorem 2.1.2 H −graph of a path P_n is an edge trimagic total graph.

Proof: Let $G = H$ graph of a path P_n.P_n be the path $u_1, u_2, \dots, u_n$.we can obtain H −graph by considering two copies ofP_n.

$$V(G) = \{u_k, v_k : 1 \le k \le n\}$$

$$E(G) = \{(u_k u_{k+1}): 1 \le k \le n-1\} \cup \left\{\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right): n \text{ is odd or } \left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right): n \text{is even}\right\} \cup \{v_k v_{k+1}: 1 \le k \le n-1\}.$$

$$So, |V(G)| = 2n \& |E(G)| = 2n-1.$$

Define $f: V(G) \cup E(G) \to \{1,2,3, \dots, 4n-1\}$.

Case:1n is even.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u_{2k}) = n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v_{2k-1}) = \frac{n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v_{2k}) = \frac{3n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

Case:$2n$ is odd.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u_{2k}) = n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v_{2k-1}) = \frac{3n-1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(v_{2k}) = \frac{n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

For this we define following edge function,

$$f(u_k u_{k+1}) = 4n - 1 - k \qquad 1 \le k \le n-1$$

$$f(v_k v_{k+1}) = 3n - k \qquad 1 \le k \le n-1$$

$$f\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 4n - 1 \qquad , \qquad n \text{ even}$$

$$f\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 4n - 1 \qquad , \qquad n \text{ odd}$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K_1 = 5n$, $K_2 = 5n + 1$ &$K_3 = 6n$.

So, H −graph of a path P_n is an edge trimagic total graph.

Illustration: An edge trimagic total labeling of H graph of a pathP_8 and H graph of a pathP_9 is shown in Figure-2(a) and Figure- 2(b) respectively.

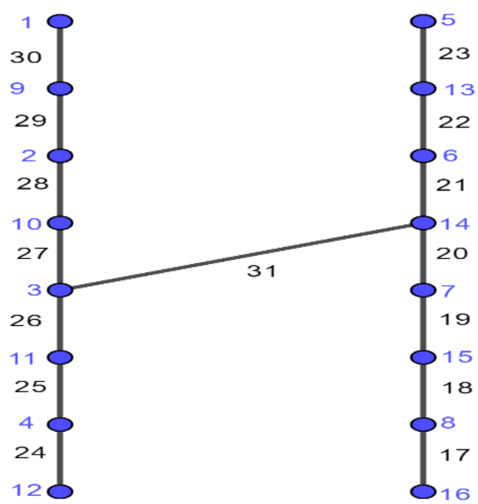

Figure-2(a)H graph of a pathP_8 with $K_1 = 40$, $K_2 = 41$, $K_3 = 48$

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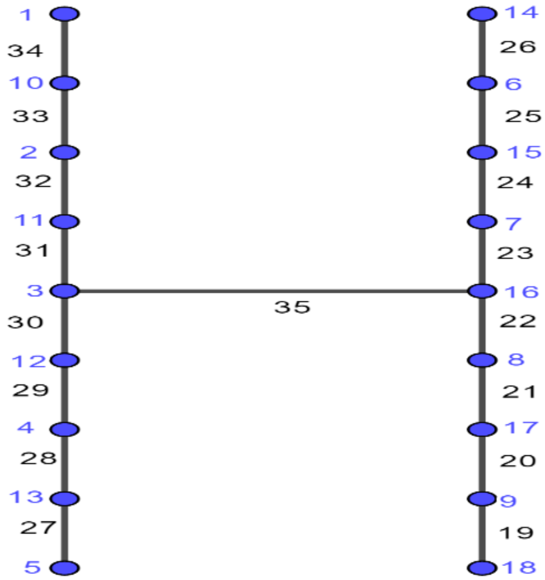

Figure-2(b) ***H* graph of a pathP_9 with $K_1 = 45$, $K_2 = 46$, $K_3 = 54$**

Theorem 2.1.3 An alternate triangular belt$ATB(n)$ is an edge magic total graph.

Proof: Let $G = ATB(n)$ with $V(G) = \{u_k : 1 \le k \le n\} \cup \{v_k : 1 \le k \le n\}$

$E(G) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{v_k v_{k+1} : 1 \le k \le n-1\} \cup \{u_k v_k : 1 \le k \le n\} \cup \{u_{2k-1} v_{2k} : 1 \le k \le \frac{n}{2}\} \cup \{u_{2k+1} v_{2k} : 1 \le k \le \frac{n-2}{2}\}$, n is even.

$E(G) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{v_k v_{k+1} : 1 \le k \le n-1\} \cup \{u_k v_k : 1 \le k \le n\} \cup \{u_{2k-1} v_{2k} : 1 \le k \le \frac{n-1}{2}\} \cup \{u_{2k+1} v_{2k} : 1 \le k \le \frac{n-1}{2}\}$, n is odd.

$$So, |V(G)| = 2n \ , \qquad |E(G)| = 4n - 3.$$

Define $f : V(G) \cup E(G) \to \{1,2,3, \dots, 6n-3\}$ as follows.

$$f(u_k) = 2k - 1 \qquad 1 \le k \le n$$

$$f(v_k) = 2k \qquad 1 \le k \le n$$

For this we define following edge function as,

$$f(u_k u_{k+1}) = 6n - 4k \qquad 1 \le k \le n-1$$

$$f(v_k v_{k+1}) = 6n - 2 - 4k \qquad 1 \le k \le n-1$$

$$f(u_{2k-1} v_{2k}) = 6n + 3 - 8k \quad 1 \le k \le \frac{n}{2}$$

$$f(u_{2k+1} v_{2k}) = 6n - 1 - 8k \quad 1 \le k \le \frac{n-2}{2}$$

$$f(u_k v_k) = 6n + 1 - 4k \qquad 1 \le k \le n$$

Case:2n is odd.

$$f(u_k) = 2k - 1 \qquad 1 \le k \le n$$

$$f(v_k) = 2k \qquad 1 \le k \le n$$

For this we define following edge function as,

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$$f(u_k u_{k+1}) = 6n - 4k \qquad 1 \le k \le n-1$$

$$f(v_k v_{k+1}) = 6n - 2 - 4k \qquad 1 \le k \le n-1$$

$$f(u_{2k-1} v_{2k}) = 6n + 3 - 8k \quad 1 \le k \le \frac{n-1}{2}$$

$$f(u_{2k+1} v_{2k}) = 6n - 1 - 8k \quad 1 \le k \le \frac{n-1}{2}$$

$$f(u_k v_k) = 6n + 1 - 4k \qquad 1 \le k \le n$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K = 6n$. So, an alternate triangular belt $ATB(n)$ is an edge magic total graph.

Illustration: An edge magic total labeling of $ATB(4)$and $ATB(5)$ is shown in Figure-3(a) and Figure-3(b) respectively .

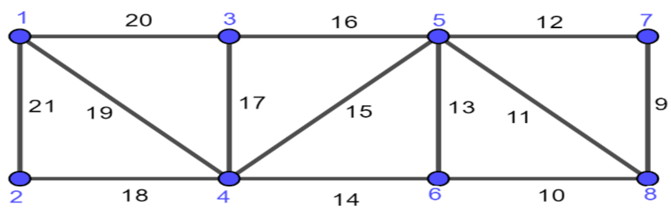

Figure-3(a) $\boldsymbol{ATB(4)}$ **with** $\boldsymbol{K = 24}$

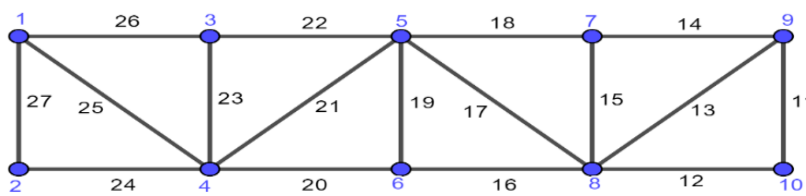

Figure-3(b)$\boldsymbol{ATB(5)}$ **with** $\boldsymbol{K = 30}$

Theorem 2.1.4 An alternate triangular belt$ATB(n)$ is an edge trimagic total graph.

Proof: Let $G = ATB(n)$ with $V(G) = \{u_k: 1 \le k \le n\} \cup \{v_k: 1 \le k \le n\}$

$E(G) = \{u_k u_{k+1}: 1 \le k \le n-1\} \cup \{v_k v_{k+1}: 1 \le k \le n-1\} \cup \{u_k v_k: 1 \le k \le n\} \cup \{u_{2k-1} v_{2k}: 1 \le k \le \frac{n}{2}\} \cup \{u_{2k+1} v_{2k}: 1 \le k \le \frac{n-2}{2}\}$ if n is even.

$E(G) = \{u_k u_{k+1}: 1 \le k \le n-1\} \cup \{v_k v_{k+1}: 1 \le k \le n-1\} \cup \{u_k v_k: 1 \le k \le n\} \cup \{u_{2k-1} v_{2k}: 1 \le k \le \frac{n-1}{2}\} \cup \{u_{2k+1} v_{2k}: 1 \le k \le \frac{n-1}{2}\}$ if n is odd.

$$So, |V(G)| = 2n \& |E(G)| = 4n - 3.$$

Define $f: V(G) \cup E(G) \rightarrow \{1,2,3, \dots, 6n-3\}$.

Case:1n is even.

$$f(u_k) = 2k - 1 \qquad 1 \le k \le n$$

$$f(v_k) = 2k \qquad 1 \le k \le n$$

For this we define following edge function,

$$f(u_k u_{k+1}) = 6n - 2 - 4k \qquad 1 \le k \le n-1$$

$$f(v_k v_{k+1}) = 6n - 4 - 4k \qquad 1 \le k \le n-2$$

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$$f(v_{n-1}v_n) = 6n - 4$$

$$f(u_{2k-1}v_{2k}) = 6n + 1 - 8k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u_{2k+1}v_{2k}) = 6n - 3 - 8k \qquad 1 \le k \le \frac{n-2}{2}$$

$$f(u_k v_k) = 6n - 1 - 4k \qquad 1 \le k \le n - 1$$

$$f(u_n v_n) = 6n - 3$$

Case:2n is odd.

$$f(u_k) = 2k - 1 \qquad 1 \le k \le n$$

$$f(v_k) = 2k \qquad 1 \le k \le n$$

For this we define following edge function,

$$f(u_k u_{k+1}) = 6n - 2 - 4k \qquad 1 \le k \le n - 1$$

$$f(v_k v_{k+1}) = 6n - 4 - 4k \qquad 1 \le k \le n - 2$$

$$f(v_{n-1}v_n) = 6n - 4$$

$$f(u_{2k-1}v_{2k}) = 6n + 1 - 8k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u_{2k+1}v_{2k}) = 6n - 3 - 8k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u_k v_k) = 6n - 1 - 4k \qquad 1 \le k \le n - 1$$

$$f(u_n v_n) = 6n - 3$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K_1 = 6n - 2$, $K_2 = 10n - 6$ &$K_3 = 10n - 4$.

So, an alternate triangular belt $ATB(n)$ is an edge trimagic total graph.

Illustration: An edge trimagic total labeling of $ATB(5)$ and$ATB(6)$is shown in Figure-4(a) and Figure-4(b) respectively.

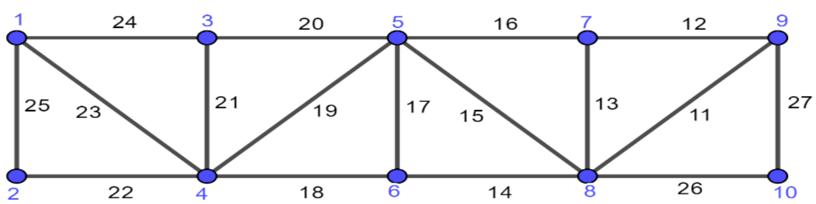

Figure-4(a) $ATB(5)$with $K_1 = 28$, $K_2 = 44$, $K_3 = 46$

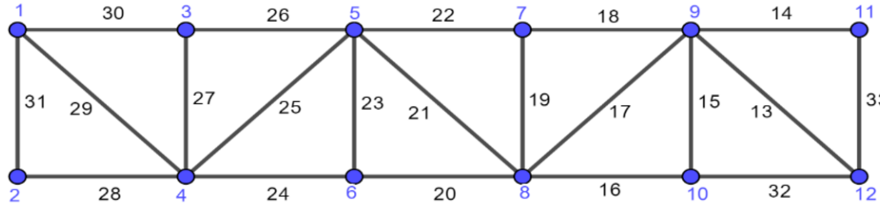

Figure-4(b)$ATB(6)$with $K_1 = 34$, $K_2 = 54$, $K_3 = 56$

Theorem 2.1.5 The Braid graph$B(n)$ is an edge magic total graph.

Proof: Let $G = B(n)$ with $V(G) = \{u_k : 1 \le k \le n\} \cup \{v_k : 1 \le k \le n\}$

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$E(G) = \{u_k u_{k+1}: 1 \leq k \leq n-1\} \cup \{v_k v_{k+1}: 1 \leq k \leq n-1\} \cup \{u_k v_{k+1}: 1 \leq k \leq n-1\} \cup \{u_{k+2} v_k: 1 \leq k \leq n-2\}$.

$$So, |V(G)| = 2n, \qquad |E(G)| = 4n-5.$$

Define $f: V(G) \cup E(G) \rightarrow \{1,2,3,\ldots,6n-5\}$ as follows.

$$f(u_k) = 2k-1 \qquad 1 \leq k \leq n$$

$$f(v_k) = 2k \qquad 1 \leq k \leq n$$

For this we define following edge function as

$$f(u_k u_{k+1}) = 6n-1-4k \qquad 1 \leq k \leq n-1$$

$$f(v_k v_{k+1}) = 6n-3-4k \qquad 1 \leq k \leq n-1$$

$$f(u_k v_{k+1}) = 6n-2-4k \qquad 1 \leq k \leq n-1$$

$$f(u_{k+2} v_k) = 6n-4-4k \qquad 1 \leq k \leq n-2$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K = 6n-1$.So, an alternate triangular belt $AT(B_n)$ is an edge magic total graph.

Illustration: An edge magic total labeling of $B(4)$and$B(5)$is shown in Figure-5(a) and Figure-5(b) respectively.

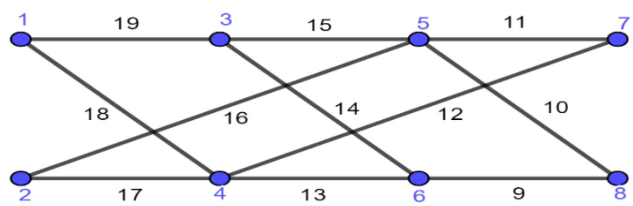

Figure-5(a) $B(4)$ with $K = 23$

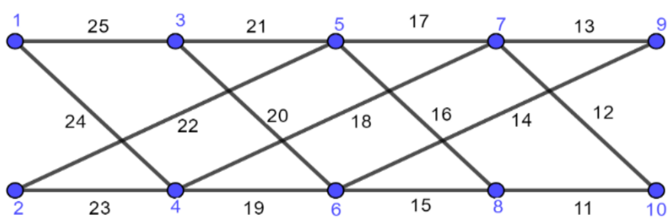

Figure-5(b)$B(5)$ with $K = 29$

Theorem 2.1.6 The Braid graph$B(n)$ is an edge trimagic total graph.

Proof: Let $G = B(n)$ with $V(G) = \{u_k: 1 \leq k \leq n\} \cup \{v_k: 1 \leq k \leq n\}$

$E(G) = \{u_k u_{k+1}: 1 \leq k \leq n-1\} \cup \{v_k v_{k+1}: 1 \leq k \leq n-1\} \cup \{u_k v_{k+1}: 1 \leq k \leq n-1\} \cup \{u_{k+2} v_k: 1 \leq k \leq n-2\}$.

$$So, |V(G)| = 2n \& |E(G)| = 4n-5.$$

Define $f: V(G) \cup E(G) \rightarrow \{1,2,3,\ldots,6n-5\}$.

$$f(u_k) = 2k-1 \qquad 1 \leq k \leq n$$

$$f(v_k) = 2k \qquad 1 \leq k \leq n$$

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For this we define following edge function,

$$f(u_k u_{k+1}) = 6n - 3 - 4k \qquad 1 \le k \le n-1$$

$$f(v_k v_{k+1}) = 6n - 5 - 4k \qquad 1 \le k \le n-2$$

$$f(u_{n-1} v_n) = 6n - 6$$

$$f(v_{n-1} v_n) = 6n - 5$$

$$f(u_k v_{k+1}) = 6n - 4 - 4k \qquad 1 \le k \le n-2$$

$$f(u_{k+2} v_k) = 6n - 6 - 4k \qquad 1 \le k \le n-2$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K_1 = 6n - 3$, $K_2 = 10n - 9$, $K_3 = 10n - 7$.

So, Braid graph $B(n)$ is an edge trimagic total graph.

Illustration: An edge trimagic total labeling of $B(3)$and $B(4)$is shown in Figure-6(a) and Figure-6(b) respectively.

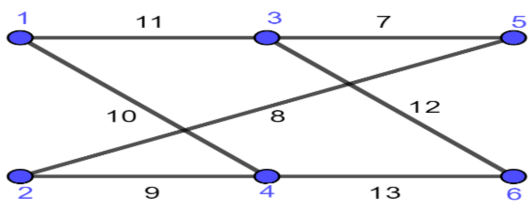

Figure-6(a) $\boldsymbol{B(3)}$**with** $\boldsymbol{K_1 = 15}$, $\boldsymbol{K_2 = 21}$, $\boldsymbol{K_3 = 23}$

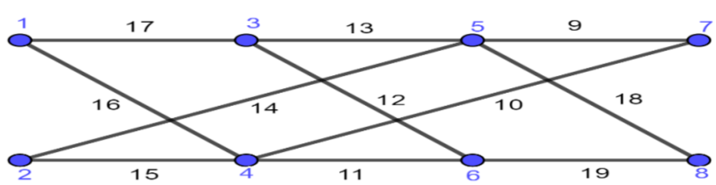

Figure-6(b)$\boldsymbol{B(4)}$**with** $\boldsymbol{K_1 = 21}$, $\boldsymbol{K_2 = 31}$, $\boldsymbol{K_3 = 33}$

Theorem 2.1.7 The semi Jahangir graphSJ_n is an edge magic total graph.

Proof: Let $G = SJ_n$ with $V(G) = \{u, u_k : 1 \le k \le n\} \cup \{s_k : 1 \le k \le n-1\}$

$$E(G) = \{u_k s_k : 1 \le k \le n-1\} \cup \{s_k u_{k+1} : 1 \le k \le n-1\} \cup \{u_k u : 1 \le k \le n\}$$

$$So, |V(G)| = 2n, \qquad |E(G)| = 3n - 2.$$

Define $f: V(G) \cup E(G) \to \{1,2,3, \dots, 5n-2\}$ as follows.

$$f(u_k) = k \qquad 1 \le k \le n$$

$$f(s_k) = n + k \qquad 1 \le k \le n-1$$

$$f(u) = 3n$$

For this we define following edge function as,

$$f(u_k s_k) = 5n - 2k \qquad 1 \le k \le n-1$$

$$f(s_k u_{k+1}) = 5n - 1 - 2k \qquad 1 \le k \le n-1$$

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$$f(u_k u) = 3n - k \qquad 1 \le k \le n$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K = 6n$. So, Semi Jahangir graph SJ_n is an edge magic total graph.

Illustration: An edge magic total labeling of $SJ_5$is shown in Figure-7.

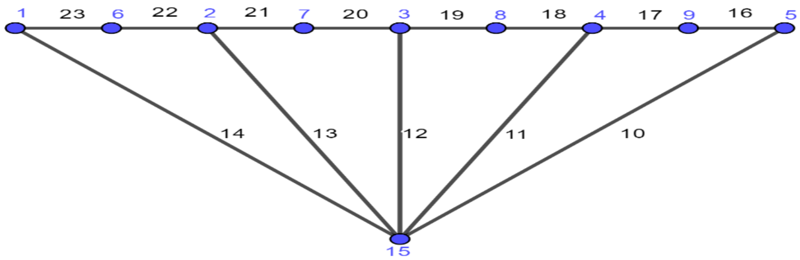

Figure-7 $SJ_5$with $K = 30$

Theorem 2.1.8 The semi Jahangir graphSJ_n is an edge trimagic total graph.

Proof: Let $G = SJ_n$

$$V(G) = \{u, u_k : 1 \le k \le n\} \cup \{s_k : 1 \le k \le n-1\}$$

$$E(G) = \{u_k s_k : 1 \le k \le n-1\} \cup \{s_k u_{k+1} : 1 \le k \le n-1\} \cup \{u_k u : 1 \le k \le n\}$$

$$So, |V(G)| = 2n \& |E(G)| = 3n - 2.$$

Define $f: V(G) \cup E(G) \to \{1,2,3, \dots, 5n-2\}$.

$$f(u_k) = k \qquad 1 \le k \le n$$

$$f(s_k) = n + k \qquad 1 \le k \le n-1$$

$$f(u) = 3n$$

For this we define following edge function,

$$f(u_k s_k) = 5n - 2k \qquad 1 \le k \le n-1$$

$$f(s_k u_{k+1}) = 5n - 1 - 2k \qquad 1 \le k \le n-1$$

$$f(u_1 u) = 2n$$

$$f(u_{k+1} u) = 3n - 1 - k \qquad 1 \le k \le n-2$$

$$f(u_n u) = 3n - 1$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K_1 = 6n$, $K_2 = 5n + 1$, $K_3 = 7n - 1$.

So, Semi Jahangir graph SJ_n is an edge trimagic total graph.

Illustration: An edge trimagic total labeling of $SJ_5$is shown in Figure-8.

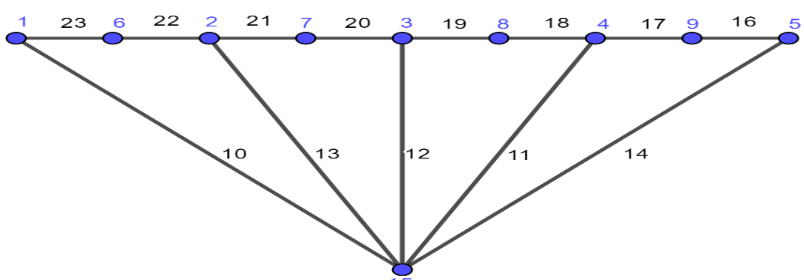

Figure-8 $SJ_5$with $K_1 = 30$, $K_2 = 26$, $K_3 = 34$

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Theorem 2.1.9 m-Join of H_n is an edge magic total graph.

Proof: Let $G = m$-Join of H_n

$$V(G) = \{u_k, v_k : 1 \le k \le n\} \cup \{u'_k, \quad v'_k : 1 \le k \le n\} \bigcup \ldots \cup \{u^m{}_k, \quad v^m{}_k : 1 \le k \le n\}$$

$E(G) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{u'_k u'_{k+1} : 1 \le k \le n-1\} \cup \ldots \{u^m{}_k u^m{}_{k+1} : 1 \le k \le n-1\} \cup \{v_k v_{k+1} : 1 \le k \le$
n−1Uv′kv′k+1:1≤k≤n−1...vmkvmk+1:1≤k≤n−1U vnu′1Uv′nu′′1... Uvm−1num1Uun2+1vn2Uu′n2+1v′n2...
$\cup \left\{u^m{}_{\frac{n}{2}+1} v^m{}_{\frac{n}{2}}\right\}$ if n is even.

$E(G) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{u'_k u'_{k+1} : 1 \le k \le n-1\} \cup \ldots \{u^m{}_k u^m{}_{k+1} : 1 \le k \le n-1\} \cup \{v_k v_{k+1} : 1 \le k \le$
n−1Uv′kv′k+1:1≤k≤n−1...vmkvmk+1:1≤k≤n−1U vnu′1Uv′nu′′1...
$\cup \{v^{m-1}{}_n u^m{}_1\} \cup \left\{u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right\} \cup \left\{u'_{\frac{n+1}{2}} v'_{\frac{n+1}{2}}\right\} \cup \ldots \cup \left\{u^m{}_{\frac{n+1}{2}} v^m{}_{\frac{n+1}{2}}\right\}$ if n is odd.

$$So, |V(G)| = 2mn + 2n, |E(G)| = 2mn + 2n - 1.$$

Define $f : V(G) \cup E(G) \to \{1,2,3, \ldots, 4mn + 4n - 1\}$,

Case:1 n is even.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u_{2k}) = mn + n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v_{2k-1}) = \frac{n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v_{2k}) = \frac{2mn + 3n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u'_{2k-1}) = n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u'_{2k}) = mn + 2n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v'_{2k-1}) = \frac{3n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v'_{2k}) = \frac{2mn + 5n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u''_{2k-1}) = 2n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u''_{2k}) = mn + 3n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v''_{2k-1}) = \frac{5n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v''_{2k}) = \frac{2mn + 7n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

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$$f(u^m{}_{2k-1}) = mn + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u^m{}_{2k}) = 2mn + n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v^m{}_{2k-1}) = \frac{2mn + n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v^m{}_{2k}) = \frac{4mn + 3n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

Case:2n is odd.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u_{2k}) = mn + n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v_{2k-1}) = \frac{2mn + 3n - 1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(v_{2k}) = \frac{n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u'{}_{2k-1}) = n + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u'{}_{2k}) = mn + 2n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v'{}_{2k-1}) = \frac{2mn + 5n - 1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(v'{}_{2k}) = \frac{3n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u''{}_{2k-1}) = 2n + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u''{}_{2k}) = mn + 3n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v''{}_{2k-1}) = \frac{2mn + 7n - 1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(v''{}_{2k}) = \frac{5n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

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$$f(u^m{}_{2k-1}) = mn + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u^m{}_{2k}) = 2mn + n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v^m{}_{2k-1}) = \frac{4mn + 3n - 1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

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$$f(v^m{}_{2k}) = \frac{2mn+n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

For both cases we define following edge function as,

$$f(u_k u_{k+1}) = 4mn + 4n - k \qquad 1 \le k \le n-1$$

$f\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 4mn + 3n$, n is even

$f\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 4mn + 3n$, n is odd

$$f(v_k v_{k+1}) = 4mn + 3n - k \qquad 1 \le k \le n-1$$

$$f(v_n u'_1) = 4mn + 2n$$

$$f(u'_k u'_{k+1}) = 4mn + 2n - k \qquad 1 \le k \le n-1$$

$f\left(u'_{\frac{n}{2}+1} v'_{\frac{n}{2}}\right) = 4mn + n$, n is even

$f\left(u'_{\frac{n+1}{2}} v'_{\frac{n+1}{2}}\right) = 4mn + n$, n is odd

$$f(v'_k v'_{k+1}) = 4mn + n - k \qquad 1 \le k \le n-1$$

$$f(v'_n u''_1) = 4mn$$

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$$f(v^{m-1}{}_n u^m{}_1) = 2mn + 4n$$

$$f(u^m{}_k u^m{}_{k+1}) = 2mn + 4n - k \qquad 1 \le k \le n-1$$

$f\left(u^m{}_{\frac{n}{2}+1} v^m{}_{\frac{n}{2}}\right) = 2mn + 3n$, n is even

$f\left(u^m{}_{\frac{n+1}{2}} v^m{}_{\frac{n+1}{2}}\right) = 2mn + 3n$, n is odd

$$f(v^m{}_k v^m{}_{k+1}) = 2mn + 3n - k \qquad 1 \le k \le n-1$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K = 5mn + 5n + 1$.So, m-Join of H_n graph is an edge magic total graph.

Illustration: An edge magic total labeling of 1-Join of $H_3$and 2-Join of H_4 graph is shown in Figure-9(a) and Figure-9(b) respectively.

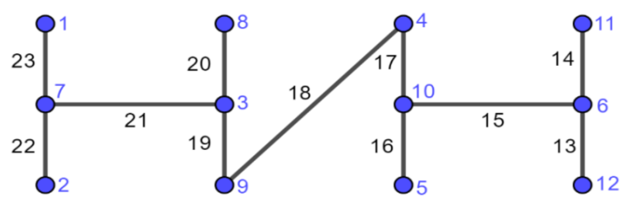

Figure-9(a) 1-Join of H_3 with $K = 31$

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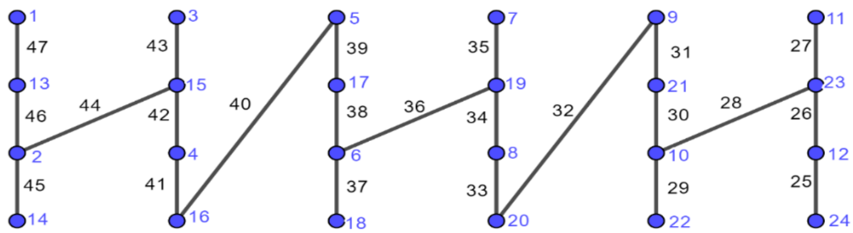

Figure-9(b) 2-Join of H_4 with $K = 61$

Theorem 2.2.0 m-Join of H_n is an edge trimagic total graph.

Proof: Let $G = m$ -Join of H_n

$$V(G) = \{u_k\ , v_k\ : 1 \le k \le n\} \cup \{u'_k\ ,\qquad v'_k\ : 1 \le k \le n\} \bigcup \ldots \cup \{u^m{}_k\ ,\qquad v^m{}_k : 1 \le k \le n\}$$

$E(G) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{u'_k u'_{k+1} : 1 \le k \le n-1\} \cup \ldots \{u^m{}_k u^m{}_{k+1} : 1 \le k \le n-1\} \cup \{v_k v_{k+1} : 1 \le k \le$
$n-1 \cup v'kv'k+1:1\le k\le n-1 \ldots vmkvmk+1:1\le k\le n-1 \cup\ \ vnu'1 \cup v'nu''1 \ldots\ \ \cup vm-1num1 \cup un2+1vn2 \cup u'n2+1v'n2 \ldots$
$\cup \left\{u^m{}_{\frac{n}{2}+1} v^m{}_{\frac{n}{2}}\right\}$ if n is even.

$E(G) = \{u_k u_{k+1} : 1 \le k \le n-1\} \cup \{u'_k u'_{k+1} : 1 \le k \le n-1\} \cup \ldots \{u^m{}_k u^m{}_{k+1} : 1 \le k \le n-1\} \cup \{v_k v_{k+1} : 1 \le k \le$
$n-1 \cup v'kv'k+1:1\le k\le n-1 \ldots vmkvmk+1:1\le k\le n-1 \cup$ $vnu'1 \cup v'nu''1 \ldots$
$\cup \{v^{m-1}{}_n u^m{}_1\} \cup \left\{u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right\} \cup \left\{u'_{\frac{n+1}{2}} v'_{\frac{n+1}{2}}\right\} \cup \ldots \cup \left\{u^m{}_{\frac{n+1}{2}} v^m{}_{\frac{n+1}{2}}\right\}$ if n is odd.

$$So, |V(G)| = 2mn + 2n \& |E(G)| = 2mn + 2n - 1.$$

Define $f : V(G) \cup E(G) \to \{1,2,3, \ldots ,4mn + 4n - 1\}$.

Case:1 n is even.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u_{2k}) = mn + n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v_{2k-1}) = \frac{n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v_{2k}) = \frac{2mn + 3n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u'_{2k-1}) = n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u'_{2k}) = mn + 2n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v'_{2k-1}) = \frac{3n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v'_{2k}) = \frac{2mn + 5n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u''_{2k-1}) = 2n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u''_{2k}) = mn + 3n + k \qquad 1 \le k \le \frac{n}{2}$$

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$$f(v''_{2k-1}) = \frac{5n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v''_{2k}) = \frac{2mn + 7n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

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$$f(u^m_{2k-1}) = mn + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u^m_{2k}) = 2mn + n + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v^m_{2k-1}) = \frac{2mn + n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

$$f(v^m_{2k}) = \frac{4mn + 3n}{2} + k \qquad 1 \le k \le \frac{n}{2}$$

Case:2n is odd.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u_{2k}) = mn + n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v_{2k-1}) = \frac{2mn + 3n - 1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(v_{2k}) = \frac{n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u'_{2k-1}) = n + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u'_{2k}) = mn + 2n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v'_{2k-1}) = \frac{2mn + 5n - 1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(v'_{2k}) = \frac{3n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u''_{2k-1}) = 2n + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u''_{2k}) = mn + 3n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v''_{2k-1}) = \frac{2mn + 7n - 1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(v''_{2k}) = \frac{5n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

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$$f(u^m{}_{2k-1}) = mn + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(u^m{}_{2k}) = 2mn + n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(v^m{}_{2k-1}) = \frac{4mn + 3n - 1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(v^m{}_{2k}) = \frac{2mn + n + 1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

For this we define following edge function,

$$f(u_k u_{k+1}) = 4mn + 4n - k \qquad 1 \le k \le n-1$$

$f\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 4mn + 3n$ $\qquad$ n even

$f\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 4mn + 3n$ $\qquad$ n odd

$$f(v_k v_{k+1}) = 4mn + 3n - k \qquad 1 \le k \le n-1$$

$$f(v_n u'_1) = 4mn + 2n$$

$$f(u'_k u'_{k+1}) = 4mn + 2n - k \qquad 1 \le k \le n-1$$

$f\left(u'_{\frac{n}{2}+1} v'_{\frac{n}{2}}\right) = 4mn + n$ $\qquad$ n even

$f\left(u'_{\frac{n+1}{2}} v'_{\frac{n+1}{2}}\right) = 4mn + n$ $\qquad$ n odd

$$f(v'_k v'_{k+1}) = 4mn + n - k \qquad 1 \le k \le n-1$$

$$f(v'_n u''_1) = 4mn$$

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$$f(v^{m-1}{}_n u^m{}_1) = 2mn + 4n$$

$$f(u^m{}_k u^m{}_{k+1}) = 2mn + 4n - k \qquad 1 \le k \le n-1$$

$f\left(u^m{}_{\frac{n}{2}+1} v^m{}_{\frac{n}{2}}\right) = 2mn + 3n$ $\qquad$ n even

$f\left(u^m{}_{\frac{n+1}{2}} v^m{}_{\frac{n+1}{2}}\right) = 2mn + 3n$ $\qquad$ n odd

$$f(v^m{}_k v^m{}_{k+1}) = 2mn + 3n - k \qquad 1 \le k \le n-3$$

$$f(v^m{}_{n-2} v^m{}_{n-1}) = 2mn + 2n + 1$$

$$f(v^m{}_{n-1} v^m{}_n) = 2mn + 2n + 2$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K_1 = 5mn + 5n$, $K_2 = 5mn + 5n + 1$, $K_3 = 5mn + 5n + 2$.So, m −Join of H_n is an edge trimagic total graph.

Illustration: An edge trimagic total labeling of 1-Join of H_3 and 2-Join of H_4 graph is shown in Figure-10(a) and Figure-10(b) respectively.

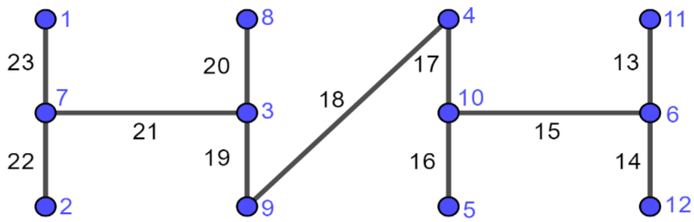

Figure-10(a) 1-Join of H_3 with $K_1 = 30$, $K_2 = 31$, $K_3 = 32$

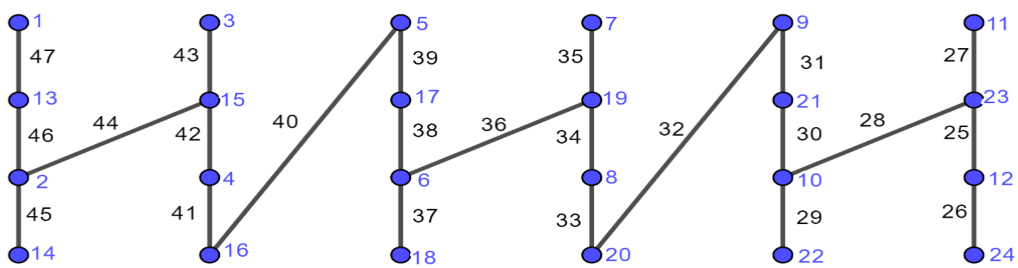

Figure-10(b) 2-Join of H_4 with $K_1 = 60$, $K_2 = 61$, $K_3 = 62$

Theorem 2.2.1. The $H -$ super subdivision of a path $HSS(P_n)$ is an edge magic total graph.

Proof: Let $G = HSS(P_n)$,

$$V(G) = \{u_k , \quad u^{(1)}_{k(k+1)}, u^{(1)}_{(k+1)k}, u^{(2)}_{k(k+1)}, u^{(2)}_{(k+1)k} \ : 1 \le k \le n-1\} \cup \{u_n\}$$

$E(G) = \{ u_k u^{(1)}_{k(k+1)} , \ u^{(1)}_{k(k+1)} u^{(2)}_{k(k+1)} , u^{(1)}_{k(k+1)} u^{(1)}_{(k+1)k} , u^{(1)}_{(k+1)k} u^{(2)}_{(k+1)k} , u_{k+1} u^{(1)}_{(k+1)k} : 1 \le k \le n-1\}$. $So, |V(G)| = 5n-4$, $|E(G)| = 5n-5$.

Define $f: V(G) \cup E(G) \to \{1,2,3, \dots ,10n-9\}$ as follows.

Case :1 n is even.

$$f(u_{2k-1}) = 5n + 1 - 5k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u_{n-2k}) = 5k + 1 \qquad 1 \le k \le \frac{n-2}{2}$$

$$f\left(u^{(1)}_{(n+1-2k)(n+2-2k)}\right) = 5k - 2 \qquad 1 \le k \le \frac{n}{2}$$

$$f\left(u^{(1)}_{2k(2k-1)}\right) = 5n - 1 - 5k \qquad 1 \le k \le \frac{n}{2}$$

$$f\left(u^{(1)}_{2k(2k+1)}\right) = 5n - 2 - 5k \qquad 1 \le k \le \frac{n-2}{2}$$

$$f\left(u^{(1)}_{(n+1-2k)(n-2k)}\right) = 5k - 1 \qquad 1 \le k \le \frac{n-2}{2}$$

$$f\left(u^{(2)}_{(2k-1)2k}\right) = 5n - 5k \qquad 1 \le k \le \frac{n}{2}$$

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$$f\left(u^{(2)}_{(n-2k)(n-2k+1)}\right) = 5k \qquad 1 \le k \le \frac{n-2}{2}$$

$$f\left(u^{(2)}_{(2k+1)2k}\right) = 5n-3-5k \qquad 1 \le k \le \frac{n-2}{2}$$

$$f\left(u^{(2)}_{(n-2k)(n-2k-1)}\right) = 5k+2 \qquad 1 \le k \le \frac{n-2}{2}$$

$f\left(u^{(2)}_{n(n-1)}\right) = 1$, $f(u_n) = 2$

Case:2n is odd.

$$f(u_{2k}) = 5n-1-5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u_{n-2k}) = 5k+1 \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(1)}_{(n+1-2k)(n-2k)}\right) = 5k-1 \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(1)}_{(2k-1)2k}\right) = 5n+1-5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(1)}_{(2k+1)2k}\right) = 5n-3-5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(1)}_{(n+1-2k)(n+2-2k)}\right) = 5k-2 \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(2)}_{2k(2k-1)}\right) = 5n-5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(2)}_{(n-2k)(n-2k+1)}\right) = 5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(2)}_{2k(2k+1)}\right) = 5n-2-5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(2)}_{(n-2k)(n-2k-1)}\right) = 5k+2 \qquad 1 \le k \le \frac{n-3}{2}$$

$f\left(u^{(2)}_{n(n-1)}\right) = 1$, $f(u_n) = 2$

For both cases we define following edge function as,

$$f\left(u_k u^{(1)}_{k(k+1)}\right) = 5n-8+5k \qquad 1 \le k \le n-1$$

$$f\left(u_{k+1} u^{(1)}_{(k+1)k}\right) = 5n-4+5k \qquad 1 \le k \le n-2$$

$$f\left(u^{(1)}_{k(k+1)} u^{(1)}_{(k+1)k}\right) = 5n-6+5k \qquad 1 \le k \le n-1$$

$$f\left(u^{(1)}_{k(k+1)} u^{(2)}_{k(k+1)}\right) = 5n-7+5k \qquad 1 \le k \le n-1$$

$$f\left(u^{(1)}_{(k+1)k} u^{(2)}_{(k+1)k}\right) = 5n-5+5k \qquad 1 \le k \le n-2$$

$$f\left(u_n u^{(1)}_{n(n-1)}\right) = 10n-10$$

$$f\left(u_{n(n-1)} u^{(1)}_{n(n-1)}\right) = 10n-9$$

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Hence for each edge $f(u) + f(v) + f(uv)$ will form $K = \frac{25n-17}{2}$, n is odd and $K = \frac{25n-18}{2}$, n is even. So, $HSS(P_n)$ graph is an edge magic total graph.

Illustration: An edge magic total labeling of $HSS(P_3)$ and $HSS(P_4)$ is shown in Figure-11(a) and Figure-11(b) respectively.

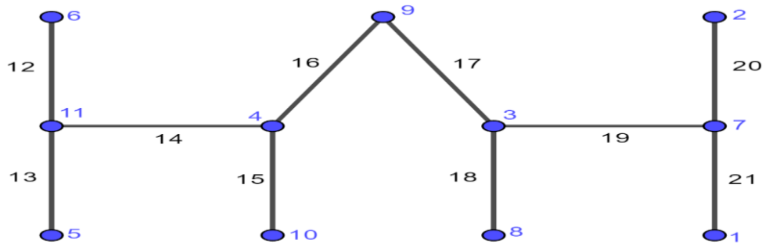

Figure-11(a) $\boldsymbol{HSS(P_3)}$ **with** $\boldsymbol{K = 29}$

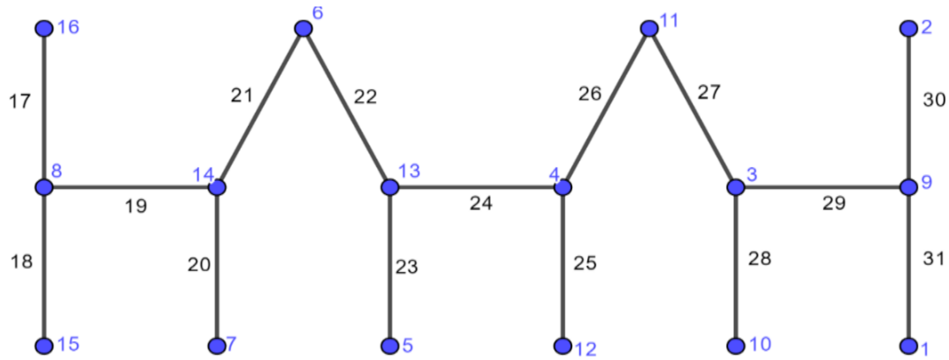

Figure-11(b) $\boldsymbol{HSS(P_4)}$ **with** $\boldsymbol{K = 41}$

Theorem 2.2.2 The $H -$ super subdivision of a path $HSS(P_n)$ is an edge trimagic total graph.

Proof: Let $G = HSS(P_n)$

$$V(G) = \{u_k, u^{(1)}_{k(k+1)}, u^{(1)}_{(k+1)k}, u^{(2)}_{k(k+1)}, u^{(2)}_{(k+1)k} : 1 \leq k \leq n-1\} \cup \{u_n\}$$

$$E(G) = \{ u_k u^{(1)}_{k(k+1)}, u^{(1)}_{k(k+1)} u^{(2)}_{k(k+1)}, u^{(1)}_{k(k+1)} u^{(1)}_{(k+1)k}, u^{(1)}_{(k+1)k} u^{(2)}_{(k+1)k}, u_{k+1} u^{(1)}_{(k+1)k} : 1 \leq k \leq n-1\}$$

$$So, |V(G)| = 5n - 4 \,\&\, |E(G)| = 5n - 5.$$

Define $f : V(G) \cup E(G) \to \{1,2,3, \ldots, 10n - 9\}$.

Case:1 n is even.

$$f(u_{2k-1}) = 5n + 1 - 5k \qquad 1 \leq k \leq \frac{n}{2}$$

$$f(u_{n-2k}) = 5k + 1 \qquad 1 \leq k \leq \frac{n-2}{2}$$

$$f\left(u^{(1)}_{(n+1-2k)(n+2-2k)}\right) = 5k - 2 \qquad 1 \leq k \leq \frac{n}{2}$$

$$f\left(u^{(1)}_{2k(2k-1)}\right) = 5n - 1 - 5k \qquad 1 \leq k \leq \frac{n}{2}$$

$$f\left(u^{(1)}_{2k(2k+1)}\right) = 5n - 2 - 5k \qquad 1 \leq k \leq \frac{n-2}{2}$$

$$f\left(u^{(1)}_{(n+1-2k)(n-2k)}\right) = 5k - 1 \qquad 1 \leq k \leq \frac{n-2}{2}$$

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$$f\left(u^{(2)}_{(2k-1)2k}\right) = 5n - 5k \qquad 1 \le k \le \frac{n}{2}$$

$$f\left(u^{(2)}_{(n-2k)(n-2k+1)}\right) = 5k \qquad 1 \le k \le \frac{n-2}{2}$$

$$f\left(u^{(2)}_{(2k+1)2k}\right) = 5n - 3 - 5k \qquad 1 \le k \le \frac{n-2}{2}$$

$$f\left(u^{(2)}_{(n-2k)(n-2k-1)}\right) = 5k + 2 \qquad 1 \le k \le \frac{n-2}{2}$$

$f\left(u^{(2)}_{n(n-1)}\right) = 1$, $f(u_n) = 2$

Case:2 n is odd.

$$f(u_{2k}) = 5n - 1 - 5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u_{n-2k}) = 5k + 1 \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(1)}_{(n+1-2k)(n-2k)}\right) = 5k - 1 \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(1)}_{(2k-1)2k}\right) = 5n + 1 - 5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(1)}_{(2k+1)2k}\right) = 5n - 3 - 5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(1)}_{(n+1-2k)(n+2-2k)}\right) = 5k - 2 \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(2)}_{2k(2k-1)}\right) = 5n - 5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(2)}_{(n-2k)(n-2k+1)}\right) = 5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(2)}_{2k(2k+1)}\right) = 5n - 2 - 5k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f\left(u^{(2)}_{(n-2k)(n-2k-1)}\right) = 5k + 2 \qquad 1 \le k \le \frac{n-3}{2}$$

$f\left(u^{(2)}_{n(n-1)}\right) = 1$, $f(u_n) = 2$

For this we define following edge function,

$$f\left(u_k u^{(1)}_{k(k+1)}\right) = 5n - 8 + 5k \qquad 1 \le k \le n-1$$

$$f\left(u_{k+1} u^{(1)}_{(k+1)k}\right) = 5n - 4 + 5k \qquad 1 \le k \le n-2$$

$$f\left(u^{(1)}_{k(k+1)} u^{(1)}_{(k+1)k}\right) = 5n - 6 + 5k \qquad 1 \le k \le n-1$$

$$f\left(u^{(1)}_{k(k+1)} u^{(2)}_{k(k+1)}\right) = 5n - 7 + 5k \qquad 1 \le k \le n-1$$

$$f\left(u^{(1)}_{(k+1)k} u^{(2)}_{(k+1)k}\right) = 5n - 5 + 5k \qquad 1 \le k \le n-2$$

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$$f\left(u_n u_{n(n-1)}^{(1)}\right) = 10n - 9$$

$$f\left(u_{n(n-1)} u_{n(n-1)}^{(1)}\right) = 10n - 10$$

So, $HSS(P_n)$ graph is an edge trimagic total graph. Hence for each edge $f(u) + f(v) + f(uv)$ will form

$K_1 = \frac{25n-19}{2}$, $K_2 = \frac{25n-17}{2}$, $K_3 = \frac{25n-15}{2}$ n is oddand $K_1 = \frac{25n-20}{2}$, $K_2 = \frac{25n-18}{2}$, $K_3 = \frac{25n-16}{2}$ n is even.

Illustration: An edge trimagic total labeling of $HSS(P_3)$and$HSS(P_4)$is shown in Figure-12(a) and Figure-12(b) respectively.

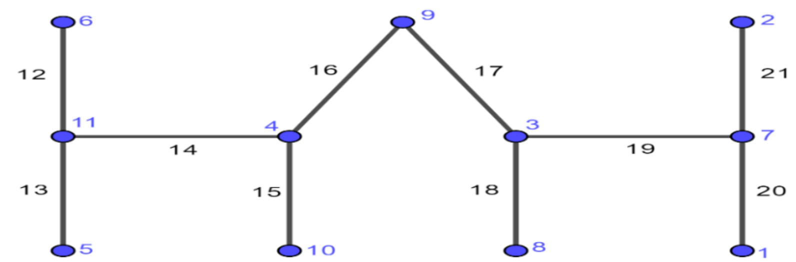

Figure-12(a) $HSS(P_3)$ with $K_1 = 28$, $K_2 = 29$, $K_3 = 30$

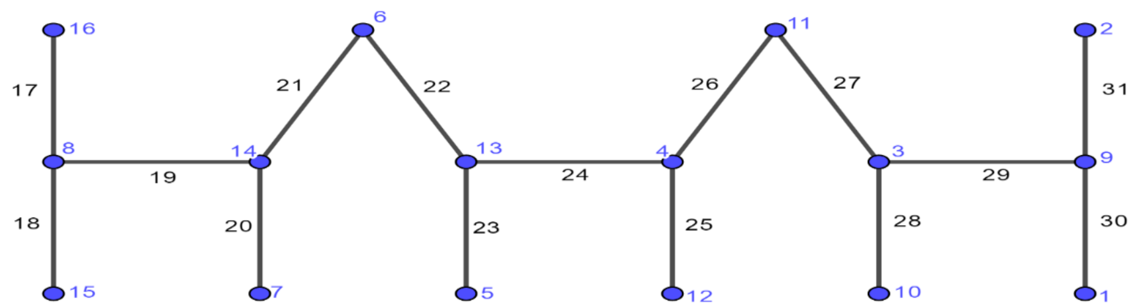

Figure-12(b)$HSS(P_4)$ with $K_1 = 40$, $K_2 = 41$, $K_3 = 42$

Theorem 2.2.3A $F-$ tree $FP_n$is an edge magic total graph .

Proof: Let $G = FP_n$, $V(G) = \{u_k : 1 \leq k \leq n\} \cup \{v, w\}$

$$E(G) = \{(u_k u_{k+1}): 1 \leq k \leq n-1\} \cup \{(u_{n-1} v)\} \cup \{(u_n w)\}$$

So, $|V(G)| = n + 2$&$|E(G)| = n + 1$.

Define $f: V(G) \rightarrow \{1,2,3, \ldots, 2n+3\}$,

Case:1n is even.

$$f(u_{2k-1}) = k \qquad 1 \leq k \leq \frac{n}{2}$$

$$f(u_{2k}) = \frac{n+2}{2} + k \qquad 1 \leq k \leq \frac{n-2}{2}$$

$$f(u_n) = n + 2$$
$$f(v) = n + 1$$
$$f(w) = \frac{n+2}{2}$$

Case:2n is odd.

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$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u_{2k}) = \frac{n+3}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u_n) = \frac{n+3}{2}$$

$$f(v) = \frac{n+1}{2}$$

$$f(w) = n+2$$

For both cases we define following edge function as,

$$f(u_k u_{k+1}) = 2n + 4 - k \qquad 1 \le k \le n-2$$

$$f(u_{n-1}v) = n+5$$

$$f(u_n w) = n+3$$

$$f(u_{n-1}u_n) = n+4$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K = \frac{5n+12}{2}$, n is even and $K = \frac{5n+13}{2}$, n is odd. So, F −tree of path $P_n FP_n$ graph is edge magic total graph.

Illustration: An edge magic total labeling of FP_4 and FP_5 graph is shown in Figure-13(a) and Figure-13(b) respectively

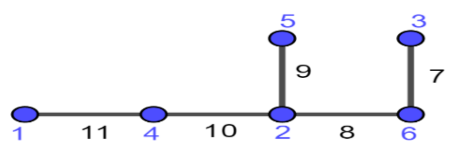

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Figure-13(a)FP_4 with $K = 16$

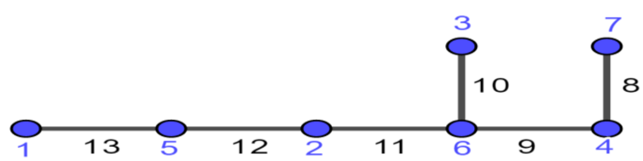

Figure-13(b)FP_5 with $K = 19$

Theorem 2.2.4A $F -$ tree FP_n is an edge trimagic total graph .

Proof: Let $G = FP_n$ with $V(G) = \{u_k : 1 \le k \le n\} \cup \{v, w\}$

$$E(G) = \{(u_k u_{k+1}) : 1 \le k \le n-1\} \cup \{(u_{n-1}v)\} \cup \{(u_n w)\}$$

So, $|V(G)| = n + 2 \& |E(G)| = n + 1$.

Define $f: V(G) \to \{1,2,3, \ldots, 2n+3\}$.

Case:1n is even.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n}{2}$$

$$f(u_{2k}) = \frac{n+2}{2} + k \qquad 1 \le k \le \frac{n-2}{2}$$

$$f(u_n) = n+2$$

$$f(v) = n + 1$$
$$f(w) = \frac{n+2}{2}$$

Case:2n is odd.

$$f(u_{2k-1}) = k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u_{2k}) = \frac{n+3}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(u_n) = \frac{n+3}{2}$$
$$f(v) = \frac{n+1}{2}$$
$$f(w) = n + 2$$

For this we define following edge function,

$$f(u_k u_{k+1}) = 2n + 4 - k \qquad 1 \le k \le n - 2$$

$$f(u_{n-1} v) = n + 5$$

$$f(u_n w) = n + 4$$

$$f(u_{n-1} u_n) = n + 3$$

Hence for each edge $f(u) + f(v) + f(uv)$ will form $K_1 = \frac{5n+10}{2}$, $K_2 = \frac{5n+12}{2}$, $K_3 = \frac{5n+14}{2}$ n is even and $K_1 = \frac{5n+11}{2}$, $K_2 = \frac{5n+13}{2}$, $K_3 = \frac{5n+15}{2}$ n is odd. So, FP_n graph is an edge trimagic total graph.

Illustration: An edge trimagic total labeling of FP_4 and FP_5 graph is shown in Figure-14(a) and Figure-14(b) respectively.

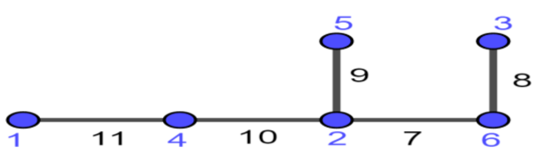

Figure-14(a)FP_4 with $K_1 = 15$, $K_2 = 16$, $K_3 = 17$

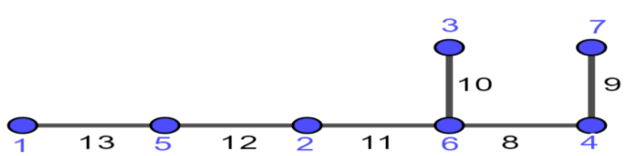

Figure-14(b)FP_4 with $K_1 = 18$, $K_2 = 19$, $K_3 = 20$

Theorem 2.2.5$H \odot K_1$ graph of a path P_n is an edge trimagic total graph.

Proof: Let $G = H \odot K_1$ graph of a path P_n

Consider two copies of path P_n with vertices $u_1, u_2, \dots, u_n$&$v_1, v_2, \dots, v_n$. Join vertices $u_1, u_2, \dots, u_n$ with vertices s_1, $s_2, \dots, s_n$ and $v_1, v_2, \dots, v_n$ with vertices $t_1, t_2, \dots, t_n$ to obtain $H \odot K_1$ graph of a path P_n.

$$V(G) = \{u_k, v_k, s_k, t_k : 1 \le k \le n\} \ \&$$

$E(G) = \{(u_k u_{k+1}) : 1 \le k \le n-1\} \cup \{(v_k v_{k+1}) : 1 \le k \le n-1\} \cup \{(u_k s_k) : 1 \le k \le n\} \cup \{(v_k t_k) : 1 \le k \le$ $nUun+12vn+12$:n is odd . OR

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$$E(G)=\{(u_ku_{k+1}):1\le k\le n-1\}\cup\{(v_kv_{k+1}):1\le k\le n-1\}\cup\{(u_ks_k):1\le k\le n\}\cup\{(v_kt_k):1\le k\le n\}\cup\left\{\left(u_{\frac{n}{2}+1}v_{\frac{n}{2}}\right):n\ is\ even\right\}.$$

So, $|V(G)|=4n$ & $|E(G)|=4n-1$.
Define $f:V(G)\to\{1,2,3,\dots,8n-1\}$.
Case:1 n is even.

$$f(u_{2k-1})=k \qquad 1\le k\le\frac{n}{2}$$

$$f(u_{2k})=n+k \qquad 1\le k\le\frac{n}{2}$$

$$f(v_{2k-1})=\frac{n}{2}+k \qquad 1\le k\le\frac{n}{2}$$

$$f(v_{2k})=\frac{3n}{2}+k \qquad 1\le k\le\frac{n}{2}$$

$$f(s_{2k-1})=3n+k \qquad 1\le k\le\frac{n}{2}$$

$$f(s_{2k})=2n+k \qquad 1\le k\le\frac{n}{2}$$

$$f(t_{2k-1})=\frac{7n}{2}+k \qquad 1\le k\le\frac{n}{2}$$

$$f(t_{2k})=\frac{5n}{2}+k \qquad 1\le k\le\frac{n}{2}$$

For this we define edge function,

$$f(u_ku_{k+1})=6n-k \qquad 1\le k\le n-1$$

$$f(u_{2k-1}s_{2k-1})=8n-2k \qquad 1\le k\le\frac{n}{2}$$

$$f(u_{2k}s_{2k})=8n+1-2k \qquad 1\le k\le\frac{n}{2}$$

$$f(v_kv_{k+1})=5n-k \qquad 1\le k\le n-1$$

$$f(v_{2k-1}t_{2k-1})=7n-2k \qquad 1\le k\le\frac{n}{2}$$

$$f(v_{2k}t_{2k})=7n+1-2k \qquad 1\le k\le\frac{n}{2}$$

$$f\left(u_{\frac{n}{2}+1}v_{\frac{n}{2}}\right)=5n$$

Case:2 n is odd.

$$f(u_{2k-1})=k \qquad 1\le k\le\frac{n+1}{2}$$

$$f(u_{2k})=\frac{n+1}{2}+k \qquad 1\le k\le\frac{n-1}{2}$$

$$f(v_{2k-1})=\frac{7n-1}{2}+k \qquad 1\le k\le\frac{n+1}{2}$$

$$f(v_{2k})=3n+k \qquad 1\le k\le\frac{n-1}{2}$$

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$$f(s_{2k-1}) = \frac{3n-1}{2} + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(s_{2k}) = n + k \qquad 1 \le k \le \frac{n-1}{2}$$

$$f(t_{2k-1}) = 2n + k \qquad 1 \le k \le \frac{n+1}{2}$$

$$f(t_{2k}) = \frac{5n+1}{2} + k \qquad 1 \le k \le \frac{n-1}{2}$$

For this we define edge function

$$f(u_k u_{k+1}) = 8n - k \qquad 1 \le k \le n-1$$

$$f(u_k s_k) = 7n + 1 - k \qquad 1 \le k \le n$$

$$f(v_k t_k) = 6n - k \qquad 1 \le k \le n$$

$$f(v_k v_{k+1}) = 5n - k \qquad 1 \le k \le n-1$$

$$f\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 6n$$

So, $H \odot K_1$ graph of a path P_n is an edge trimagic total graph. Hence for each edge $f(u) + f(v) + f(uv)$ will form $K_1 = \frac{17n+3}{2}$, $K_2 = 10n + 1$, $K_3 = \frac{23n+1}{2}$ n is oddand $K_1 = 7n + 1$, $K_2 = 11n$, $K_3 = 11n + 1$ n is even.

Illustration: An edge trimagic total labeling of $H \odot K_1$ graph of a path P_4 and $H \odot K_1$ graph of a path P_3 is shown in Figure-15(a) and Figure-15(b) respectively.

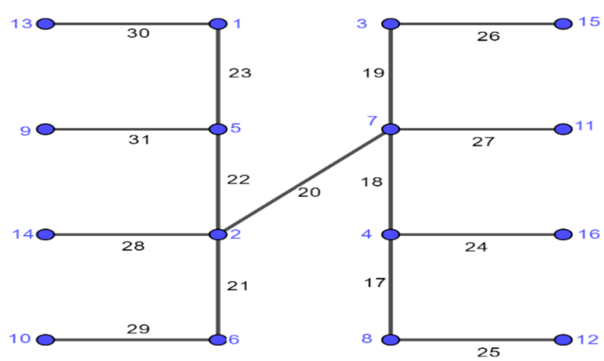

Figure-15(a) $H \odot K_1$ graph of a path P_4 with $K_1 = 29$, $K_2 = 44$, $K_3 = 45$

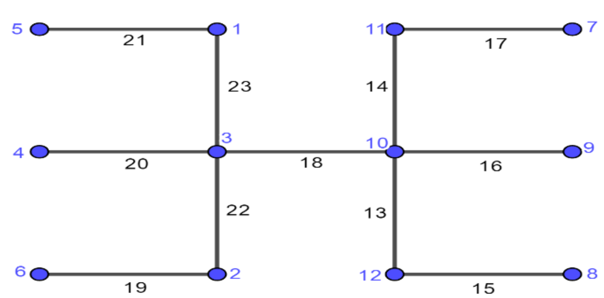

Figure-15(b) $H \odot K_1$ graph of a path P_3 with $K_1 = 27$, $K_2 = 31$, $K_3 = 35$

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CONCLUSION

In this paper we have shown that$H$ −graph of a path P_n ,Alternate triangular belt graph ,Braid graph , Semi Jahangir graph , m −Join of H_n , H −super subdivision of a path P_n are edge magic total graph and H −graph of a path P_n ,Alternate triangular belt graph ,Braid graph , Semi Jahangir graph , m −Join of H_n , H −super subdivision of a path P_n, F −treeare edge magic total graphs and$H$ −graph of a path P_n ,Alternate triangular belt graph ,Braid graph , Semi Jahangir graph , m −Join of H_n , H −super subdivision of a path P_n are edge magic total graph and H −graph of a path P_n ,Alternate triangular belt graph ,Braid graph ,Semi Jahangir graph , m −Join of H_n , H −super subdivision of a path P_n,F −tree,$H \odot K_1$ graph of a path P_n are edge trimagic total graph. We can discuss more similar results for various graphs.

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