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General Service Time Distribution of Fuzzy Queue Using Parametric Non - Linear Programming

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ABSTRACT

In this Paper we propose a mathematical programming technique to find the membership functions of the performance measures of a fuzzy single server queue in steady state. The closed form of the steady state solution of M/G/1 queue is used in coordination with the parametric non - linear programming approach and the formulae for FM/FG/1 fuzzy queue are derived. An effective algorithm is found to determine the optimum solution for various possibilities of α. Numerical examples have been shown for understanding the solution procedure. Sensitivity for standard deviation equal to zero is done as a special case.

KEYWORDS

FM/FM/1 Queue, Time Independent Solution, Parametric Non - Linear Programming, Fuzzy Sets.

Introduction

Queuing theory have many applications in our real life. Its first application started in the area of telecommunications later extended to almost all areas where waiting time plays a role. In crisp queues the arrival and service are already random in nature and can be states as “arrival rate is approximately 5 per hour” etc. This vagueness can be precisely removed by the usage of fuzzy membership functions. The basic approach for investigation of such Fuzzified stochastic models was developed by Zadeh. The fuzzy arrival and fuzzy service rates can be best described by the linguistic terms like “Very low, Low, Moderate, High and Very High”.

Transient analysis of a queue model is significantly important in studying the conduct of the system that wants a lengthier warm up period to attain stability. The time-dependent solution in a simple and clear form for M/M/1 queue system is obtained in a direct way by Parthasarathy and Sharafali. In all the literature investigated many authors converted only the steady state queue models to fuzzy queues. So we present the Fuzzified transient solution to a single server model treating only the basic parameters are fuzzy in nature.

Literature Survey

Visalakshi and Suvith [1] have represented the arrival rate and service rate in queue models as a pentagon fuzzy number. The probabilities for no customer in the system and the probability that the server is busy are given as range of fuzzy numbers for various α-cut.

R.-J. LI and E. S. LEE [2] considered M/F/1 where the arrival is Poisson but the service is treated as a fuzzy random variable and FM/FM/1 fuzzy queue where both arrival and service are treated as a fuzzy random variable. The authors in [3] carefully taken two kinds of fuzzy arithmetic, one depending on Zadeh’s extension principle and the next constructed on α-cut strategy. The minimum and maximum values of the system measures are established using the parametric non-linear program (PNLP). The analytical time independent solutions for average number of clients in the system and waiting time in the organization are derived with parametric non-linear program procedure. Finally a numerical problem is taken to describe the procedure of the formulae.

Dong – Yuh Yang et al [4] considered a batch arrival Markovian queue which is formulated as a parametric non - linear programming for maximum and minimum values separately. Ramesh R and Hari Ganesh A [5] have demonstrated the finite capacity single server queue model under fuzzy environment using Wingspans fuzzy ranking

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technique. They considered the retention of discouraged customers. They proposed the ranking function for triangular and trapezoidal fuzzy numbers. They have taken a real time example of a temple in Trichy district to demonstrate their model. The accurate values and effectiveness of the model is shown by the solved example and corresponding graphs are drawn.

Many authors have considered the fuzziness in queue model as the service and arrival rate measures as fuzzy variable. Negi and Lee [6] proposed a strategy to incorporate both randomness and fuzziness in deriving the model. Botzoris et al [7] have used the same strategy to develop the multi - server model. That is they employed fuzzy statistics to create estimators for the model parameters. NAFE (non – asymptotic fuzzy estimator is developed by the authors to instrument fuzzy statistics into queue models. V. Ashok Kumar [8] took a queue model with unreliable server and converted it into the fuzzy environment by taking the service, arrival, breakdown and repair rates as fuzzy numbers. An effective algorithm is established using parametric non – linear programming to find the performance measures of the model for various levels of α. Matlab is used to solve the model and using an example it is demonstrated. Geetha et al [9] considered the k – phase Erlang model. They derived the inverse membership function of the model taking service and arrival rates as fuzzy in nature.

The parametric non – linear programming helps in determining the closed form solution to many fuzzy systems. In case the expressions are very huge then those equations can be solved numerically. The remaining work of the paper is prepared as follows. A brief literature is collected in section 2 and section 3 deals with the introduction to fuzzy concepts and theorems related to it. The parametric non – linear approach are explained in section 4 and extended the concept to the queue model being investigated. The crisp M/G/1 queue is discussed in section 5 and its fuzzy model is described in section 6. The membership functions for several performance measures like probability of zero clients in the model, probability of n clients in the model are developed in this section. Two numerical examples are taken and solved completely to understand the procedure. Robust ranking method is given in section 7. A sensitivity analysis where the standard deviation is zero for service is also discussed. In section 9 results and conclusions are discussed. Finally the references are listed.

Fuzzy Concepts

The transition probability matrix of the embedded fuzzy Markov Chain is represented by

$$\tilde{P} = [\tilde{P}_{ij}] = \begin{bmatrix} \tilde{P}_0 & \tilde{P}_1 & \tilde{P}_2 & \cdots \\ 0 & \tilde{P}_0 & \tilde{P}_1 & \cdots \\ 0 & 0 & \tilde{P}_0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where $\tilde{P}_{ij}$ are defined by

$$\mu_{\tilde{P}_{ij}(x_{ij})} = \sup_{t \in R^+} \left\{ \mu_{\tilde{S}}(t) \middle| x_{ij} = \frac{exp(-\lambda t)(\lambda t)^{j-i+1}}{(j-i+1)!} \right\}$$

Parametric Non – Linear Equation

An equation of the form Optimize $Z = f(x, y, z)$ subject to $x_1 < x < x_2$; $y_1 < y < y_2$; $z_1 < z < z_2$ is called as an parametric non - linear equation.

Formulation of FM/FM/1 fuzzy queue as a parametric programming problem:

Consider a single server fuzzy queue system. The inter arrival time $\hat{A}$ and service time $\tilde{S}$ be approximately known and are denoted by the fuzzy sets below.

$$\tilde{A} = \left\{ \left(a, \mu_{\tilde{A}}(a) \right) / a \in X \right\}$$
$$\tilde{S} = \left\{ \left(s, \mu_{\tilde{S}}(s) \right) / s \in Y \right\}$$

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Here X and Y are universal crisp sets and $\mu_{\tilde{A}}(a)$, $\mu_{\tilde{S}}(s)$ are the their membership functions. The $\propto -$ cut of $\tilde{A}(\alpha)$ and $\tilde{S}(\alpha)$ are

$$\tilde{A}(\alpha) = \{a \in X / \mu_{\tilde{A}}(a) \geq \alpha\}$$
$$\tilde{S}(\alpha) = \{s \in Y / \mu_{\tilde{S}}(a) \geq \alpha\}$$

where$0 < \alpha \leq 1$.

Using α-cut, the arrival rate and service rate can be denoted by various levels of confidence intervals [0,1]. Thus the crisp queue can be got from the fuzzy queue for different α-cuts. The membership function of the performance measure $p(\tilde{A}, \tilde{S})$ is defined as $\mu_{p(a,s)}(z) = \sup \min\{\mu_{\tilde{A}}(a), \mu_{\tilde{S}}(s) / \ z = p(a,s)\}$. The construction of membership function is same as doing the derivation of $\propto \ -$ cuts for $\mu_{p(a,s)}(z)$.

Thus the parametric programming problem takes the following form.

$l_{p(\propto)} = \min p(a,s)$
Such that $l_{\tilde{A}(\alpha)} \leq a \leq u_{\tilde{A}(\alpha)}$ and $l_{S(\alpha)} \leq a \leq u_{S(\alpha)}$.
Similarly, $u_{p(\propto)} = \max\, p(a,s)$
Such that $l_{\tilde{A}(\alpha)} \leq a \leq u_{\tilde{A}(\alpha)}$ and $l_{S(\alpha)} \leq a \leq u_{S(\alpha)}$.
If both $l_{p(\propto)}$ and $u_{p(\propto)}$ can be solved for its inverse then the left shape and right shape functions can be obtained from their membership function $\mu_{p(\tilde{A},\tilde{S})}\,(z)$ can be constructed as follows.

$$\mu_{p(\tilde{A},\tilde{S})}\,(z) = \begin{cases} L(z) & z_1 \leq z \leq z_2 \\ R(z) & z_2 \leq z \leq z_3 \\ 0 & otherwise \end{cases}$$

If both $l_{p(\propto)}$ and $u_{p(\propto)}$ cannot be solved for its inverse then the membership function cannot be derived. But we can construct the graph of $\mu_{p(\tilde{A},\tilde{S})}\,(z)$ from the α-cut concept. This procedure is used to solve FM/FG/1 queue system.

FM/FG/1 Queue

Consider the average queue length $L_q = \frac{\lambda^2\, \sigma^2\, + \rho^2}{2(1-\rho)}$.
Substituting $\rho = \frac{\lambda}{\mu}$ we get $L_q = \frac{\lambda^2 [1+\,\sigma^2\, \mu^2]}{2\mu\,(\mu-\lambda)}$

Assuming the parameters $\lambda, \mu,\ \sigma^2$ of the distribution as triangular fuzzy numbers $\lambda = [x_1, x_2, x_3]$, $\mu = [y_1, y_2, y_3]$ and $\sigma^2 = [t_1, t_2, t_3]$, the α-cut of their membership functions are $[x_1 + \alpha, x_3 - \alpha\,]$, $[y_1 + \alpha, y_3 - \alpha\,]$ and $[t_1 + \alpha, t_3 - \alpha\,]$.

$l_{L_q(\alpha)}$ is found using the concept as λ approach lower bound, μ and σ^2 will approach the upper bound. Similarly $u_{L_q(\alpha)}$ is found using the concept as λ approach upper bound, μ and σ^2 will approach the lower bound.

$$l_{L_q} = \frac{(x_1+\alpha)^2[1+(t_3-\alpha)\,(y_3-\alpha)^2]}{2(y_3-\alpha)(y_3-2\alpha-x_1)} \text{ and } u_{L_q} = \frac{(x_3-\alpha)^2[1+(t_1+\alpha)\,(y_1+\alpha)^2]}{2(y_1+\alpha)(y_1+2\alpha-x_3)}$$

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The above equations can be solved as function of α. Using the α-cut the graph of the function can be constructed. But the inverse function is difficult to be found analytically because the above functions are of degree 6.

By taking $\sigma^2 = k$, $l_{L_q} = \frac{(x_1+\alpha)^2[1+K(y_3-\alpha)^2]}{2(y_3-\alpha)(y_3-2\alpha-x_1)}$ and $u_{L_q} = \frac{(x_3-\alpha)^2[1+K(y_1+\alpha)^2]}{2(y_1+\alpha)(y_1+2\alpha-x_3)}$

The above equations can be solved as function of α. Using the α-cut the graph of the function can be constructed. But the inverse function is difficult to be found analytically because the above functions are of degree 4.
Let's consider the particular case when $\sigma^2 = 0$.

$$l_{L_q} = \frac{(x_1+\alpha)^2}{2(y_3-\alpha)(y_3-2\alpha-x_1)} \text{ and } u_{L_q} = \frac{(x_3-\alpha)^2}{2(y_1+\alpha)(y_1+2\alpha-x_3)}$$

The above equations can be solved as function of α. Using the α-cut the graph of the function can be constructed. The inverse function can also be found analytically.

Performance Measure of the FM/FG/1 Queue

Suppose the arrival rate and service rate are represented as fuzzy numbers by $\lambda = [3,4,5]$, $\mu = [7,8,9]$ with standard deviation $\sigma^2 = 0.2$ hours lets calculate the four important measures of the model average queue size, average system size, waiting time in both queue and system. The α-cut of their membership functions are $[3+\alpha, 5-\alpha]$, $[7+\alpha, 9-\alpha]$. The parametric non - linear program for the queue length is, $l_{L_q} = Min\left\{\frac{\lambda^2[1+0.04\,\mu^2]}{2\mu(\mu-\lambda)}\right\}$ such that $3+\alpha < \lambda < 5-\alpha$ and $7+\alpha < \mu < 9-\alpha$ and
$u_{L_q} = Max\left\{\frac{\lambda^2[1+0.04\,\mu^2]}{2\mu(\mu-\lambda)}\right\}$ such that $3+\alpha < \lambda < 5-\alpha$ and $7+\alpha < \mu < 9-\alpha$
where $0 < \alpha < 1$.

To Find the Mean Queue Length

For l_{L_q}, $\lambda \to 3+\alpha$ and $\mu \to 9-\alpha$.

$$l_{L_q} = \frac{[0.04\alpha^4 - 0.48\alpha^3 + 0.28\alpha^2 + 18.96\alpha + 38.16]}{2(54 - 24\alpha + 2\alpha^2)}$$

For u_{L_q}, $\lambda \to 5-\alpha$ and $\mu \to 7+\alpha$, the u_{L_q} becomes

$$u_{L_q} = \frac{0.04\alpha^4 + 0.16\alpha^3 - 1.6\alpha^2 - 15.6\alpha + 74}{2(14 + 16\alpha + 2\alpha^2)}$$

alpha	lq - l num	lq - l deno	lower lt	lq - l num	lq - l deno	upper lt
0	38.16	108	0.353333	74	28	2.642857
0.1	40.05832	103.24	0.388012	72.42416	31.24	2.318315
0.2	41.95942	98.56	0.425725	70.81734	34.56	2.049113
0.3	43.86056	93.96	0.4668	69.18064	37.96	1.822462
0.4	45.7591	89.44	0.511618	67.51526	41.44	1.629229
0.5	47.6525	85	0.560618	65.8225	45	1.462722
0.6	49.5383	80.64	0.614314	64.10374	48.64	1.317922
0.7	51.41416	76.36	0.673313	62.36048	52.36	1.190995
0.8	53.27782	72.16	0.738329	60.5943	56.16	1.078958
0.9	55.12712	68.04	0.810216	58.80688	60.04	0.979462

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1	56.96	64	0.89	57	64	0.890625

To Find the Mean System Length

For l_{L_s}, $\lambda \to 3+\alpha$ and $\mu \to 9-\alpha$

$$l_{L_s} = \frac{3+\alpha}{9-\alpha} + \frac{[0.04\alpha^4 - 0.48\alpha^3 + 0.28\alpha^2 + 18.96\alpha + 38.16]}{2(54 - 24\alpha + 2\alpha^2)}$$

$$l_{L_s} = \frac{(3+\alpha)2(54 - 24\alpha + 2\alpha^2) + (9-\alpha)[0.04\alpha^4 - 0.48\alpha^3 + 0.28\alpha^2 + 18.96\alpha + 38.16]}{2(9-\alpha)(54 - 24\alpha + 2\alpha^2)}$$

For u_{L_s}, $\lambda \to 5-\alpha$ and $\mu \to 7+\alpha$.

$$u_{L_s} = \frac{3+\alpha}{9-\alpha} + \frac{0.04\alpha^4 + 0.16\alpha^3 - 1.6\alpha^2 - 15.6\alpha + 74}{2(14 + 16\alpha + 2\alpha^2)}$$

$$u_{L_s} = \frac{(3+\alpha)2(14 + 16\alpha + 2\alpha^2) + (9-\alpha)(0.04\alpha^4 + 0.16\alpha^3 - 1.6\alpha^2 - 15.6\alpha + 74)}{2(9-\alpha)(14 + 16\alpha + 2\alpha^2)}$$

alpha	ls - l num	ls - l deno	lower lt	ls - l num	ls - l deno	upper lt
0	38.16	108	0.686667	74	28	2.97619
0.1	40.05832	103.24	0.736326	72.42416	31.24	2.66663
0.2	41.95942	98.56	0.789361	70.81734	34.56	2.412749
0.3	43.86056	93.96	0.846111	69.18064	37.96	2.201772
0.4	45.7591	89.44	0.906967	67.51526	41.44	2.024578
0.5	47.6525	85	0.972382	65.8225	45	1.874487
0.6	49.5383	80.64	1.042886	64.10374	48.64	1.746494
0.7	51.41416	76.36	1.119096	62.36048	52.36	1.636778
0.8	53.27782	72.16	1.201744	60.5943	56.16	1.542373
0.9	55.12712	68.04	1.291698	58.80688	60.04	1.460943
1	56.96	64	1.39	57	64	1.390625

To Find the Mean Queue Waiting Time

For l_{W_q} $\lambda \to 3+\alpha$ and $\mu \to 9-\alpha$.

$$l_{W_q} = \frac{[0.04\alpha^4 - 0.48\alpha^3 + 0.28\alpha^2 + 18.96\alpha + 38.16]}{2(3+\alpha)(54 - 24\alpha + 2\alpha^2)}$$

For u_{W_q} $\lambda \to 5-\alpha$ and $\mu \to 7+\alpha$.

$$u_{W_q} = \frac{0.04\alpha^4 + 0.16\alpha^3 - 1.6\alpha^2 - 15.6\alpha + 74}{2(3+\alpha)(14 + 16\alpha + 2\alpha^2)}$$

alpha	**Wq - l num**	**wq - l deno**	**lower lt**	**wq - l num**	**wq - l deno**	**upper lt**
0	38.16	108	0.117778	74	28	0.880952
0.1	40.05832	103.24	0.125165	72.42416	31.24	0.747844
0.2	41.95942	98.56	0.133039	70.81734	34.56	0.640348
0.3	43.86056	93.96	0.141455	69.18064	37.96	0.552261
0.4	45.7591	89.44	0.150476	67.51526	41.44	0.479185
0.5	47.6525	85	0.160176	65.8225	45	0.417921
0.6	49.5383	80.64	0.170643	64.10374	48.64	0.36609
0.7	51.41416	76.36	0.181976	62.36048	52.36	0.32189
0.8	53.27782	72.16	0.194297	60.5943	56.16	0.283936

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0.9	55.12712	68.04	0.207748	58.80688	60.04	0.251144
1	56.96	64	0.2225	57	64	0.222656

To Find the Mean System Waiting Time

For l_{W_s} $\lambda \to 3+\alpha$ and $\mu \to 9-\alpha$

$$l_{W_s} = \frac{[0.04\alpha^4 - 0.48\alpha^3 + 0.28\alpha^2 + 18.96\alpha + 38.16]}{2(3+\alpha)(54-24\alpha+2\alpha^2)} + \frac{1}{9-\alpha}$$

For u_{L_s}, $\lambda \to 5-\alpha$ and $\mu \to 7+\alpha$

$$u_{W_s} = \frac{0.04\alpha^4 + 0.16\alpha^3 - 1.6\alpha^2 - 15.6\alpha + 74}{2(3+\alpha)(14+16\alpha+2\alpha^2)} + \frac{1}{7+\alpha}$$

alpha	Ws - l num	ws - l deno	lower lt	Ws - l num	Ws - l deno	upper lt
0	38.16	108	0.228889	74	28	0.992063
0.1	40.05832	103.24	0.237525	72.42416	31.24	0.860203
0.2	41.95942	98.56	0.246675	70.81734	34.56	0.753984
0.3	43.86056	93.96	0.256397	69.18064	37.96	0.667204
0.4	45.7591	89.44	0.266755	67.51526	41.44	0.595464
0.5	47.6525	85	0.277824	65.8225	45	0.535568
0.6	49.5383	80.64	0.28969	64.10374	48.64	0.485137
0.7	51.41416	76.36	0.302458	62.36048	52.36	0.442372
0.8	53.27782	72.16	0.316248	60.5943	56.16	0.405888
0.9	55.12712	68.04	0.331205	58.80688	60.04	0.374601
1	56.96	64	0.3475	57	64	0.347656

Robust Ranking Technique

Ranking techniques are very worth mentioning in the fuzzy numbers method for defuzzification. Many authors have previously proposed different types of procedures to find out the performance of fuzzy queues. It is likely to convert from fuzzy background to crisp background by our suggested ranking method in order to investigate the performance measures of fuzzy queues.

The Robust ranking index $R(\tilde{r})$ is used to find the demonstrative value for the fuzzy number r. It fulfills the linearity property and additive property. Some problems are very complex in nature that alpha cut methods may be too complicated to solve the problem. When the parameters of a problem are specified as a fuzzy number we can defuzzify it using Robust Ranking method. For a convex fuzzy number $\tilde{r}$, the robust ranking index is stated as

$$R(\tilde{r}) = \int_0^1 0.5\,(r_\propto{}^L + r_\propto{}^U)\ d\propto.$$

$$R(\tilde{\lambda}) = \int_0^1 0.5\ (3+5)\ d\propto = 4$$

Similarly, $R(\tilde{\mu}) = \int_0^1 0.5\,(7+9)\ d\propto = 8$. Also given that the standard deviation is $\sigma^2 = 0.02$ we get

$$L_q = \frac{16*0.04+0.25}{2(0.5)} = 0.89$$

$$L_s = \frac{\lambda}{\mu} + L_q = 0.5 + 0.89 = 1.39$$

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$$W_q = \frac{\left(\frac{\lambda^2\sigma^2+\rho^2}{2(1-\rho)}\right)}{\lambda} = \frac{L_q}{\lambda} = \frac{0.89}{4} = 0.225$$
$$W_s = \frac{\lambda^2\,\sigma^2+\rho^2}{2\,\lambda\,(1-\rho)} + \frac{1}{\mu} = W_q + \frac{1}{\mu} = 0.35$$

The above values are in accordance with the fuzzy numbers of the performance measures.

Sensitivity Analysis

When the server rate remains the same for all customers (i.e) $\mu = constant$, then the standard deviation $\sigma^2 = 0$. But this constant μ may be a fuzzy number for the system.
where $\rho = \frac{\lambda}{\mu}$.

Illustration

Suppose the arrival rate and same service rate are represented as fuzzy numbers by $\lambda = [3,4,5]$, $\mu = [7,8,9]$ with standard deviation $\sigma^2 = 0$ hours lets calculate the four important measures of the model average queue size, average system size, waiting time in both queue and system. The α-cut of their membership functions are $[3+\alpha, 5-\alpha]$, $[7+\alpha, 9-\alpha]$.

To Find the Mean Queue Length

The parametric non - linear program for the queue length is,

$$l_{L_q} = Min\left\{\frac{\lambda^2}{2\mu(\mu-\lambda)}\right\} \text{ such that } 3+\alpha < \lambda < 5-\alpha \text{ and } 7+\alpha < \mu < 9-\alpha \text{ and}$$
$$u_{L_q} = Max\left\{\frac{\lambda^2}{2\mu(\mu-\lambda)}\right\} \text{ such that } 3+\alpha < \lambda < 5-\alpha \text{ and } 7+\alpha < \mu < 9-\alpha$$

Where 0< α <1.

For l_{L_q}, $\lambda \to 3+\alpha$ and $\mu \to 9-\alpha$. Hence the l_{L_q} becomes, $l_{L_q} = \frac{9+6\alpha+\alpha^2}{2(54-24\alpha+2\alpha^2)}$

For u_{L_q}, $\lambda \to 5-\alpha$ and $\mu \to 7+\alpha$, the u_{L_q} becomes $u_{L_q} = \frac{25-10\alpha+\alpha^2}{2(14+16\alpha+2\alpha^2)}$

The membership values of queue length:

alpha	lq - l num	lq - l deno	lower lt	lq - l num	lq - l deno	upper lt
0	9	108	0.083333	25	28	0.892857
0.1	9.61	103.24	0.093084	24.01	31.24	0.768566
0.2	10.24	98.56	0.103896	23.04	34.56	0.666667
0.3	10.89	93.96	0.1159	22.09	37.96	0.581928
0.4	11.56	89.44	0.129249	21.16	41.44	0.510618
0.5	12.25	85	0.144118	20.25	45	0.45
0.6	12.96	80.64	0.160714	19.36	48.64	0.398026
0.7	13.69	76.36	0.179282	18.49	52.36	0.353132
0.8	14.44	72.16	0.200111	17.64	56.16	0.314103
0.9	15.21	68.04	0.223545	16.81	60.04	0.27998
1	16	64	0.25	16	64	0.25

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To find the membership function $\mu_{L_q}(z) = \begin{cases} L(z) & z_1 \le z \le z_2 \\ R(z) & z_2 \le z \le z_3 \\ 0 & otherwise \end{cases}$ we take the inverse of the above function.

$$\text{Let } x = \frac{9+6z+z^2}{2(54-24z+2z^2)}$$

$$\therefore L(z) = \frac{3(1+8z) \pm 12\sqrt{z^2+2z}}{(4z-1)} \quad 0.083 \le z \le 0.25$$

$$\text{Let } y = \frac{25-10z+z^2}{2(14+16z+2z^2)}$$

$$\therefore R(z) = \frac{-(5+16z) \pm 12\sqrt{z^2+2z}}{(4z-1)} \quad 0.25 \le z \le 0.89$$

$$\text{Thus } \mu_{L_q}(z) = \begin{cases} \frac{3(1+8z) \pm 12\sqrt{z^2+2z}}{(4z-1)} & 0.083 \le z \le 0.25 \\ \frac{-(5+16z) \pm 12\sqrt{z^2+2z}}{(4z-1)} & 0.25 \le z \le 0.89 \\ 0 & otherwise \end{cases}$$

To Find the Mean System Length

Average system length is given by $L_s = \rho + \frac{\lambda^2\sigma^2+\rho^2}{2(1-\rho)} = \frac{\lambda}{\mu} + L_q$

For l_{L_s} $\lambda \to 3+\alpha$ and $\mu \to 9-\alpha$ $l_{L_s} = \frac{45+6\alpha-3\alpha^2}{2(54-24\alpha+2\alpha^2)}$

For u_{L_s}, $\lambda \to 5-\alpha$ and $\mu \to 7+\alpha$, the u_{L_s} becomes $u_{L_s} = \frac{45+6\alpha-3\alpha^2}{2(14+16\alpha+2\alpha^2)}$

The membership values of system length:

alpha	lq - l num	lq - l deno	lower lt	lq - l num	lq - l deno	upper lt
0	45	108	0.416667	45	28	1.607143
0.1	45.57	103.24	0.441399	45.57	31.24	1.458707
0.2	46.08	98.56	0.467532	46.08	34.56	1.333333
0.3	46.53	93.96	0.495211	46.53	37.96	1.225764
0.4	46.92	89.44	0.524597	46.92	41.44	1.132239
0.5	47.25	85	0.555882	47.25	45	1.05
0.6	47.52	80.64	0.589286	47.52	48.64	0.976974
0.7	47.73	76.36	0.625065	47.73	52.36	0.911574
0.8	47.88	72.16	0.663525	47.88	56.16	0.852564
0.9	47.97	68.04	0.705026	47.97	60.04	0.798967
1	48	64	0.75	48	64	0.75

To find the membership function $\mu_{L_s}(z) = \begin{cases} L(z) & z_1 \le z \le z_2 \\ R(z) & z_2 \le z \le z_3 \\ 0 & otherwise \end{cases}$ we take the inverse of the above function.

Let $x = \frac{45+6z-3z^2}{2(54-24z+2z^2)}$ then

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$$\therefore L(z) = \frac{3(1+8z) \pm 12\sqrt{z^2+1}}{(4z+3)} \quad 0.42 \le z \le 0.75$$

Let $y = \frac{45+6z-3z^2}{2(14+16z+2z^2)}$

$$\therefore R(z) = \frac{(3-16z) \pm 12\sqrt{z^2+1}}{(4z+3)} \quad 0.75 \le z \le 1.6$$

$$\mu_{L_s}(z) = \begin{cases} \frac{3(1+8z) \pm 12\sqrt{z^2+1}}{(4z+3)} & 0.42 \le z \le 0.75 \\ \frac{(3-16z) \pm 12\sqrt{z^2+1}}{(4z+3)} & 0.75 \le z \le 1.6 \\ 0 & otherwise \end{cases}$$

To Find the Mean Queue Waiting Time

For l_{W_q} $\lambda \to 3+\alpha$ and $\mu \to 9-\alpha$.

$$l_{w_q} = \frac{3+\alpha}{2(54-24\alpha+2\alpha^2)}$$

For u_{W_q} $\lambda \to 5-\alpha$ and $\mu \to 7+\alpha$

$$u_{W_q} = \frac{5-\alpha}{2(14+16\alpha+2\alpha^2)}$$

The membership values of queue waiting time:

alpha	lq - l num	lq - l deno	lower lt	lq - l num	lq - l deno	upper lt
0	3	108	0.027778	5	28	0.178571
0.1	3.1	103.24	0.030027	4.9	31.24	0.15685
0.2	3.2	98.56	0.032468	4.8	34.56	0.138889
0.3	3.3	93.96	0.035121	4.7	37.96	0.123815
0.4	3.4	89.44	0.038014	4.6	41.44	0.111004
0.5	3.5	85	0.041176	4.5	45	0.1
0.6	3.6	80.64	0.044643	4.4	48.64	0.090461
0.7	3.7	76.36	0.048455	4.3	52.36	0.082124
0.8	3.8	72.16	0.052661	4.2	56.16	0.074786
0.9	3.9	68.04	0.057319	4.1	60.04	0.068288
1	4	64	0.0625	4	64	0.0625

To find the membership function $\mu_{W_q}(z) = \begin{cases} L(z) & z_1 \le z \le z_2 \\ R(z) & z_2 \le z \le z_3 \\ 0 & otherwise \end{cases}$ we take the inverse of the above function.

Let $x = \frac{3+z}{2(54-24z+2z^2)}$ then $L(z) = \frac{(1-48z) \pm \sqrt{(1+144z+576z^2)}}{8z}$ $\quad 0.03 \le z \le 0.06$

Let $y = \frac{5-z}{2(14+16z+2z^2)}$ then $R(z) = \frac{-(32z+1) \pm \sqrt{576z^2+144z+1}}{8z}$ $\quad 0.06 \le z \le 0.18$

$$\mu_{W_q}(z) = \begin{cases} \dfrac{(1-48z) \pm \sqrt{(1+144z+576z^2)}}{8z} & 0.03 \le z \le 0.065 \\ \dfrac{-(32z+1) \pm \sqrt{576z^2+144z+1}}{8z} & 0.06 \le z \le 0.18 \\ 0 \quad otherwise & \end{cases}$$

To Find the Mean System Waiting Time

For l_{W_s} $\lambda \to 3+\alpha$ and $\mu \to 9-\alpha$, $l_{W_s} = \frac{3(5-\alpha)}{4(3-\alpha)}$

For u_{W_s}, $\lambda \to 5-\alpha$ and $\mu \to 7+\alpha$, $u_{W_s} = \frac{9+3\alpha}{4(1+\alpha)}$

The membership values of system waiting time:

alpha	lq - l num	lq - l deno	lower lt	lq - l num	lq - l deno	upper lt
0	15	12	1.25	9	4	2.25
0.1	14.7	11.6	1.267241	9.3	4.4	2.113636
0.2	14.4	11.2	1.285714	9.6	4.8	2
0.3	14.1	10.8	1.305556	9.9	5.2	1.903846
0.4	13.8	10.4	1.326923	10.2	5.6	1.821429
0.5	13.5	10	1.35	10.5	6	1.75
0.6	13.2	9.6	1.375	10.8	6.4	1.6875
0.7	12.9	9.2	1.402174	11.1	6.8	1.632353
0.8	12.6	8.8	1.431818	11.4	7.2	1.583333
0.9	12.3	8.4	1.464286	11.7	7.6	1.539474
1	12	8	1.5	12	8	1.5

To find the membership function $\mu_{W_s}(z) = \begin{cases} L(z) & z_1 \le z \le z_2 \\ R(z) & z_2 \le z \le z_3 \\ 0 & otherwise \end{cases}$ we take the inverse of the above function.

Let $x = \frac{15-3z}{12-4z}$ then $\therefore L(z) = \frac{(12z-15)}{(3-4z)}$ $\quad 1.25 \le z \le 1.5$

Let $y = \frac{9+3\alpha}{4(1+\alpha)}$ then $\therefore R(z) = \frac{(4z-3)}{(4z-9)}$ $\quad 1.5 \le z \le 2.25$

$$\mu_{W_s}(z) = \begin{cases} \dfrac{(12z-15)}{(3-4z)} & 1.25 \le z \le 1.5 \\ \dfrac{(4z-3)}{(4z-9)} & 1.5 \le z \le 2.25 \\ 0 \quad otherwise & \end{cases}$$

Conclusion

Parametric non - linear technique has been applied to find the membership function of the performance measures FM/FG/1 model. The average number of customers in the system, in queue and waiting time in the queue, in the system for fuzzy queue has been discussed with numerical example. Further the fuzzy problem has been converted into crisp problem by using Robust Ranking Technique. Thus by applying the R.R.T, we find that the performance measures are well in accordance with the fuzzy values. In real life most of the queuing systems have an unreliable data. In those situations the proposed method can be used to find system measures. This eliminates the shortcomings of point estimation or confidence interval estimation when the factors of the queue system distributions can only got from unreliable sample data. The efficiency and the precise values of our recommended method have been successfully solved by an example and also compared graphically. Sensitivity analysis for standard deviation equal to zero is demonstrated by an example.

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