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On the Partition Dimension of Honey Comb, Hexagonal Cage Networks and Quartz Network

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ABSTRACT

Let *v* and *P* be a vertex and subset *V(G)* of a simple connected graph *G* and the distance between *v* and *P* is defined by $d(v,P) = \min\{d(v,x), x \in P\}$. Let $\pi = \{P_1, P_2,,,, P_k\}$ be a *K* resolving vertex partition of *G* then a distinct vertex representation of a vertex *v* with respect to π is $\{d(v,P_1), d(v,P_2), .. d(v,P_K)\}$. The partition dimension of G is the minimum cardinality of the resolving partition set $\pi$and it is denoted by $pd(G)$. In this paper we have determined partition dimension of Honeycomb Cage Network of dimension n with two-layer, Hexagonal Cage Network and Quartz Network.

KEYWORDS

Partition Dimension, Hexagonal cage network, Honeycomb Cage network, Quartz Network.

Introduction

Harary and Milter [1] explained independently the concepts of resolvability and location in graphs. The same structure was defined in a graph by Slater[2]. Several authors developed the theoretical concepts of this topic work after some years [1,3,4,5,6,7,]. Theapplication of this concepts is useful in the following, field of Chemistry, Pattern recognition problems, image processing and robot Navigation in Networks. [1,6].

Let $w_i$be a vertex of a connected simple graph G and (v_1, v_2) be a pair of vertices in g. let $d(v_1, v_2)$ be the distance betweenv_1 and v_2. A vertex w_i is said to resolve v_1 and v_2 if $d(w_i, v_1) \neq d(w_i, v_2)$. A set of vertices W of G is called a resolving set of G if every pair of vertices (v_i, v_j) resolved by atlest one vertices $w_j \in W$. A resolving set of G with least cardinalityis called basis of G. The cardinality of metric basis is called metric dimension of G. The more results based on metric dimension of graphs studied in [3,4,5,6,7]

Let *v* and *P* be a vertex and subset if *V(G)* of a simple connected graph *G* and the distance between *v* and *P* is defined by $d(v,P) = \min\{d(v,x), x \in P\}$. Let $\pi = \{P_1, P_2,,,, P_k\}$ be a K resolving vertex partition of G then a vertex representation of a vertex v with respect to π is $\{d(v,P_1), d(v,P_2), .. d(v,P_K)\}$. The partition dimension of G is the least cardinality of the resolving partition set $\pi$and it is denoted by $pd(G)$.The more results are related to partition dimension of graphs is studied in [5,8,9,10].

For a non-trivial connected graph G,$pd(G) - 1 \leq \dim(G)$. The partition dimension and metric dimension are related in [8]. The upper bound of partition dimension is depend on metric dimension but the lower bound does not depend on metric dimension. Chartrand et al [8] proved that the partition dimension of a graph is 2 and n iff$G = P_n$and $G = K_n$ with order n.

The partition dimension of n-cycle, Petersen graph, 3-cube studied in [8,11]. Bharati Rajan et all studied Partition dimension of Honeycomb Networks Hexagonal Network and circulant Networks [12]. Chris Monica et all [13] studied the partition dimension of some classes of graphs like Hive Network, Honeycomb Rhombic mesh, Honeycomb Rectangular mesh.

Honeycomb Cage Networks of Dimension n with 2 Layer

Consider two copy of honeycomb Network namely $HC_1(n)$ and $HC_2(n)$. Connect all boundary vertices of degree two from one copy of $HC_1(n)$ to $HC_2(n)$. The resulting graph of Honeycomb cage Network of dimension n with 2 layers. The representation of vertices in Honeycomb cage network is $u = (l_1, m_1, n_{1,}0)$ in $HC_1(n)$ and $u' = (l_2, m_2, n_2, 1)$ respectively. The number of vertices, edges and faces in a honeycomb cage network is $12n^2$ $18n^2$ and

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$6n^2 + 2$.[14]. We solved partition dimension of Honeycomb Cage network is 4. A two-dimensionalHoneycob cage Network with resolving partition set is depicted Figure 1.

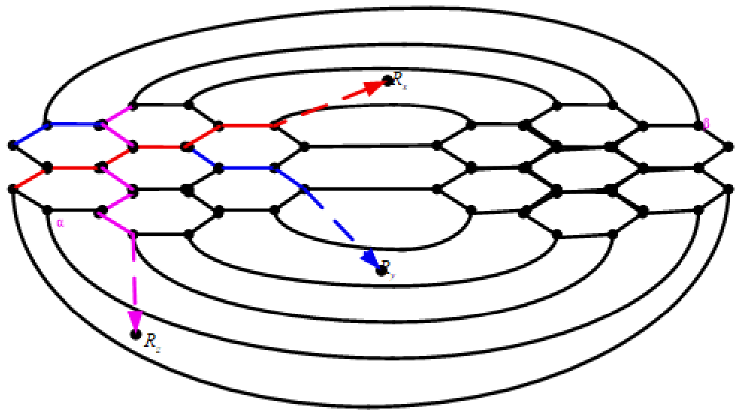

Fig. 1. Honeycomb cage network of dimension n with two layer

Lemma 1.1

For anyr_1 and $r_2 N_{r_1}(\alpha) \cap N_{r_2}(R_{X,n}) = 0$ or 1 or 2.

Lemma 2.2

For anyr_1 and $r_2 N_{r_1}(\beta) \cap N_{r_2}(R_{X,n}) = 0$ or 1 or 2 or 3.

Theorem 1.3

Let G be a Honeycomb Cage Network of dimension n with two layersthen$pd(G) = 4$.

Proof

Let $P_1 = \{\alpha\}, P_{2=}\{\beta\}, P_3 = \{V(R_{X,n})\}$ and $P_4 = V(G) - \{P_1 \cup P_2 \cup P_3\}$ Where$V(R_{X,n})$ denote the path induced by the vertex at$X = n$.Let $u = (l_1, m_1, n_{1,} t_1)$and $v = (l_2, m_2, n_2, t_2)$be two vertices of *G*.

Enough to prove $\pi = \{P_1, P_2, P_3, P_4\}$ is a resolving partition of *G*.

To prove π is a resolving partition we consider three cases namely when u and v are in the first and second copy of the Honeycomb Cage network and u and v are in different copy of Honeycomb cage network with two layers.

Case 1 $[t_1 = t_2]$

If u and v are in the first and second copy of honeycomb cage network then we have the following sub cases.

Subcase 1.1

For $l_1 = l_2$if $u, v \in R_X$then $d(u, \alpha) \neq d(v, \alpha)$that is $d(u, P_1) \neq d(v, P_1)$ and $u, v \in R'_X$ then$d(u, \beta) \neq d(v, \beta)$that is $d(u, P_2) \neq d(v, P_2)$

Subcase 1.2

For $m_1 = m_2$ if$u, v \in R_Y$ then$d(u, P_3) \neq d(v, P_3)$ and $u, v \in R'_Y$ then$d(u, \alpha) \neq d(v, \alpha)$that is $d(u, P_1) \neq d(v, P_1)$.

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Subcase 1.3

For $n_1 = n_2$ if$u, v \in R_Z$ then$d(u, \beta) \neq d(v, \beta)$that is $d(u, P_2) \neq d(v, P_2)$ and $u, v \in R'_z$ then$d(u, P_3) \neq d(v, P_3)$.

Subcase 1.4

For $l_1 \neq l_2, m_1 \neq m_2, n_1 \neq n_2$suppose $d(u, P_1) = d(v, P_1)$or $d(u, P_2) \neq d(v, P_2)$ then $d(u, P_3) \neq d(v, P_3)$.

Case 2 $\{t_1 \neq t_2\}$

suppose $u \in HC_1(n)$ and $v \in HC_2(n)$ then we have the following cases.

Subcase 2.1

For $l_1 = l_2$ either $d(u, P_1) = d(v, P_1)$ or $d(u, P_1) \neq d(v, P_1)$ suppose $d(u, P_1) = d(v, P_1)$ then $u \in HC_1(n)$ and $v \in HC_2(n)$are partition resolved by $d(u, P_2) \neq d(v, P_2)$.

Subcase 2.2

For $m_1 = m_2$, If $u \in HC_1(n)$ and $v \in HC_2(n)$ then $d(u, P_1) \neq d(v, P_1)$ or $d(u, P_2) \neq d(v, P_2)$.

Subcase 2.3

For $n_1 = n_2$, If $u \in HC_1(n)$ and $v \in HC_2(n)$ then $d(u, P_3) \neq d(v, P_3)$ or $d(u, P_2) \neq d(v, P_2)$.

Subcase 2.4

For $l_1 \neq l_2, m_1 \neq m_2, n_1 \neq n_2$ If$u \in HC_1(n)$ and $v \in HC_2(n)$ then
$d(u, P_1) \neq d(v, P_1)$ or $d(u, P_3) \neq d(v, P_3)$.
Hence $pd(G) = 4$.

Hexagonal Cage Network

The Hexagonal cage network is constructed from connecting all the boundary vertices from one copy of Hexagonal network $HX_1(n)$to second copy of Hexagonal network $HX_2(n)$. Then the resulting graph is called Hexagonal cage network of dimension n. The Number of vertices, edges and faces in the Hexagonal cage network is $6n^2 - 6n + 2$, $18n^2 - 24n + 6$, and $12n^2 - 18n + 6$ respectively [15].The representation of vertices in Hexagonal cage network is $u = (l_1, m_1, n_{1,} 0)$ in $HX_1(n)$ and $u' = (l_2, m_2, n_2, 1)$ in $HX_2(n)$ respectively.An X channel, Y channel and Z channel in Hexagonal cage network is denoted by L_x, L_y and $L_z$in $HX_1(n)$ and $L_x{}' L_y{}'$ and $L_z{}'$in $HX_2(n)$respectively. Here $L_{X,n}$ denoted by the path induced by the vertices in the first copy of Hexagonal cage network and $L_{Z,n}{}'$ denoted by path inducted by the vertices in the second copy of Hexagonal cage network. A four-dimension Hexagonal cage network shows in figure 2.

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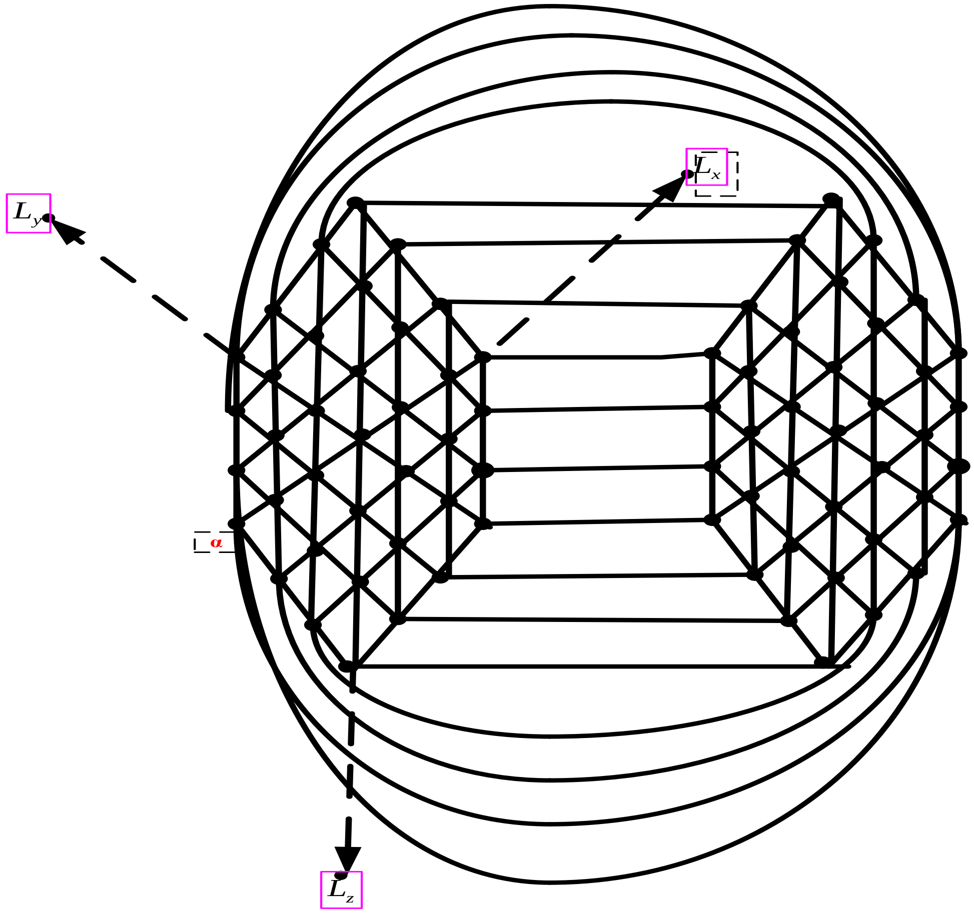

Figure 2. Hexagonal cage network of dimension 4

Lemma 2.1

For anyr_1 and $r_2 N_{r_1}(\alpha) \cap N_{r_2}(L_{X,n})$ is either empty or singleton.

Lemma 2.2

For anyr_1 and $r_2 N_{r_1}(\alpha) \cap N_{r_2}(L'_{Z,n})$ is either empty or singleton.

Lemma 2.3

Let G be a Hexagonal Cage Network of dimension n then $pd(G) > 3$.

Proof

Hexagonal cage network is constructed for connecting all the boundary vertices from first copy of $HX_1(n)$ to second copy of $HX_2(n)$ and $pd\big(HX(n)\big) > 3$ [12]. Therefore$pd(G) > 3$.

Theorem 2.3

Let G be a Hexagonal cage network then $pd(G) = 4$.

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Proof

Let $P_1 = \{\alpha\}, P_2 = V(L'_{Z,n}), P_3 = V\left(L_{X,n}\right) P_4 = V(G) - \{P_1 \cup P_2 \cup P_3\}$. Let $u = \left(l_1, m_1, n_{1,} t_1\right)$and $v = (l_2, m_2, n_2, t_2)$be two vertices of *G*.

Claim$\pi = \{P_1, P_2, P_3, P_4\}$ is a resolving partition of *G*.

To prove π is a resolving partition we consider two cases namely when u and v are in the first and second copy of the Hexagonal Cage network and u and v are in different copy of Hexagonal cage network.

Case 1$\{t_1 = t_2\}$

Sub case 1.1 If $l_1 = l_2$then $u, v \in L_X$ and $u, v \in L'_X$ For $u, v \in L_X$, $d(u, P_1) \neq d(v, P_1)$ and $u, v \in L'_X$, $d(u, P_3) \neq d(v, P_3)$.

Subcase 1.2 If $m_1 = m_2$then $u, v \in L_Y$ and $u, v \in L'_Y$then, $d(u, P_2) \neq d(v, P_2)$.

Sub case 1.3 If $n_1 = n_2$then $u, v \in L_Z$ and $u, v \in L'_Z$then, $d(u, P_3) \neq d(v, P_3)$.

Sub Case 1.4 If $l_1 \neq l_2, m_1 \neq m_2, n_1 \neq n_2$ for $u, v \in HX_1(n)$ then $d(u, P_3) \neq d(v, P_3)$ and $u, v \in HX_2(n)$ then $d(u, P_1) \neq d(v, P_1)$.

Case 2$\{t_1 \neq t_2\}$

Then $u \in HX_i(n)$ and $v \in HX_j(n) i \neq j$ then we have the following subcases.

Subcase 2.1

For If $l_1 = l_2$Either $d(u, P_1) \neq d(v, P_1)$ or$d(u, P_1) = d(v, P_1)$.

Suppose $d(u, P_1) = d(v, P_1)$ then u and v are partition resolved by$d(u, P_2) \neq d(v, P_2)$.

Subcase 2.2.

For If $m_1 = m_2$then the pair of vertices partition resolved by $d(u, \alpha) \neq d(v, \alpha)$.That is,$d(u, P_1) \neq d(v, P_1)$.

Subcase 2.3

For If $n_1 = n_2$then the pair of vertices partition resolved by$d(u, P_2) \neq d(v, P_2)$.

Subcase 2.4.

If $l_1 \neq l_2, m_1 \neq m_2, n_1 \neq n_2$ in this case either$d(u, P_3) \neq d(v, P_3)$ or$d(u, P_1) \neq d(v, P_1)$.

Hence $pd(G) = 4$.

The Quartz Network

The chemical compound of silicon dioxide with formula Sio_2 is a quartz network. The construction of Quarts Network can be represented in different ways. The 1-dimension of Quartz Network contain a 12-cycle. An n-dimensional Quartz Network contain n^2 number of 12-cycles and these 12-cycles can be represented in the shape of rhombus. A 2 -dimensional quartz network with coordinate system and partition resolving sets shows in fig3.

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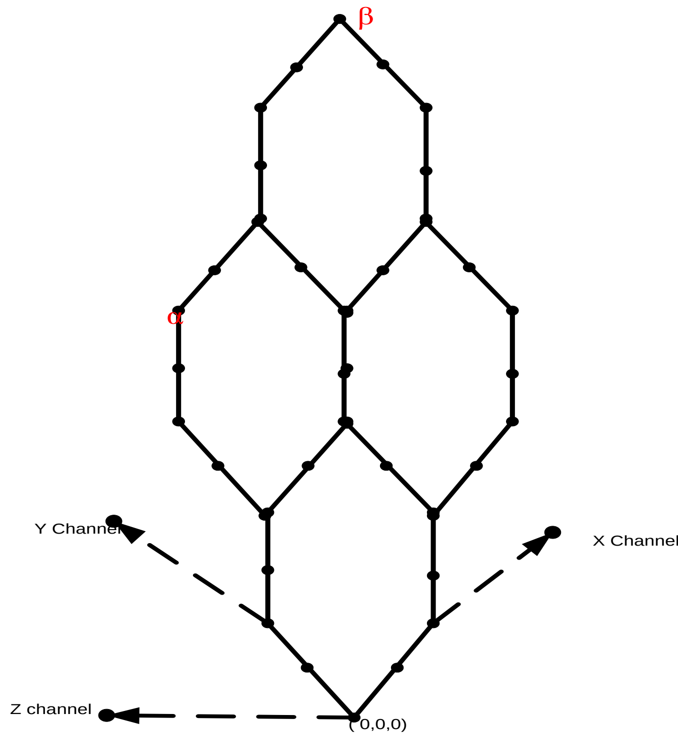

Figure 3. Quartz network of dimension 2 with coordinates system

Theorem 1

Let G be a Quartz Network of dimension *n* then $pd(G) = 3$.

Proof

Let $P_1 = \{\alpha\}, P_2 = \{\beta\}, P_3 = V(G) - \{P_1 \cup P_2\}$ be a resolving partition sets of G. let $u = (l_1, m_1, n_1)$ and $v = (l_2, m_2, n_2)$be two any two vertices of the quartz network.

Enough to Prove $\pi = \{ P_1, P_2, P_3\}$ is a resolving partition set of G.

To prove π is a resolving we have the following discussion.

Case 1.

If $l_1 = l_2$ then $u, v \in the\ X$ channel then $d(u, \alpha) \neq d(v, \alpha)$. That is $d(u, P_1) \neq d(v, P_1)$.

Case 2.

If $m_1 = m_2$for $u, v \in Y$ channel then $d(u, \alpha) \neq d(v, \alpha)$& $d(u, \beta) \neq d(v, \beta)$. That is $d(u, P_1) \neq d(v, P_1)$.That is $d(u, P_2) \neq d(v, P_2)$.

Case 3.

If $n_1 = n_2$ then $u, v \in Z$ channel for $d(u, \beta) = d(v, \beta)$ then u and v resolved by α. That is $d(u, \alpha) \neq d(v, \alpha)$.$d(u, P_1) \neq d(v, P_1)$.

Case 4.

Suppose $l_1 \neq l_2$,$m_1 \neq m_2, n_1 \neq n_2$ for $d(u, \alpha) = d(v, \alpha) d(u, P_1) = d(v, P_1)$. Then u and v resolved by P_2. That is$d(u, P_2) \neq d(v, P_2)$.
Hence $pd(G) = 3$.

Conclusion

In this paper we have studied partition dimension of Honeycomb Cage Network of dimension n with 2-layer, Hexagonal Cage Network and Quartz Network. When compare to other Networks in terms of degree, diameter, total number of edges, costs, bisection width extra with this Honeycomb cage network of dimension n with 2-layer, Hexagonal cage network and quartz network have been considerable.

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