Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1442 - 1454
Received 05 March 2021; Accepted 01 April 2021.

Solutions to Fuzzy Inventory Model with Fuzzy Demand Rate Using Heptagonal Fuzzy Numbers

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Abstract

The objective of this article focuses a solution to fuzzy inventory model with fuzzy demand rate. Mathematical model has been described for finding the cycle length and total inventory cost in both crisp and fuzzy environments. Robust ranking is used to defuzzify the total cost function and this model is solved by using GP technique and Kuhn Tucker method. Next, comparative analysis between GP technique and Kuhn Tucker method. Additionally, a numerical examples and sensitivity analysis are given to the proposed model.

Keywords:Total cost, GP technique, Kuhn-tucker condition, Robust's Ranking defuzzification.

1. Introduction

In general, the analytic complexity of inventory models depends on whether the demand for an item is deterministic or probabilistic. In real life, demand is usually probabilistic, but in some cases the simpler deterministic approximation may be acceptable. Within either category, the demand may or may not vary with time .[3] Burwell, T. H., Dave, D. S., Fitzpatrick, K. E., & Roy, M. R. (1997) investigated an Economic lot size model for price-dependent demand under quantity and freight discounts. The Kuhn tucker conditions provide the most unifying theory for all non linear programming problems. The main contribution of this model is the development of the general Kuhn tucker necessary conditions for determining the stationary points. [5] MangasarianO.L, [6] McCormick G.P solved nonlinear Programming. [9] Yang J H, Cao B Y (2005) presented Geometric programming with fuzzy relation equation constraints. [4] Cao B Y (1993) describes an extended fuzzy geometric programming. The basic experiment towards concepts of fuzziness was made by zadeh [10]. Several researchers have carried out examine on several fuzzy numbers. [13] Zimmermann H.J, proposed results obtained by Fuzzy set theory and its applications.

This paper is standardized as follows. Section 2 contains the notations and arithmetic operations used for this model. The G.P technique development is formulated both crisp and fuzzy set in Section 3. A Section 4 describes the solution procedure for Kuhn tucker condition both crisp and fuzzy set. Numerical examples, sensitivity analysis and graphs are provided in Section 5 related to our models. Finally, conclusion is included in section 6.

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2. Model's Criterion and Enlargement

2.1 Notation:

d- Demand

O- Ordering Cost

H- Holding Cost

C- Purchase Cost

S- Shortage Cost

L- Cycle Length

θ- Time 0≤θ≤L

λ- Shortage Period

TIC-Total Inventory Cost.

2.2 Robust's Ranking

Robust's ranking which satisfy compensation, linearity, and additively properties and provides results which are consistent with human intuition. If $\tilde{A}_H$ is a fuzzy number then the ranking is defined by

$$R(\tilde{A}_H) = \int_0^1 0.5(a_{h\alpha}^L, a_{h\alpha}^U)d\alpha$$

Where $(a_{h\alpha}^L, a_{h\alpha}^U)$ is the α level cut of fuzzy number $\tilde{A}_H$.

Proposed Ranking

If $(h_1, h_2, h_3, h_4, h_5, h_6, h_7)$ are heptagonal fuzzy numbers, then

$$(a_{h\alpha}^L, a_{h\alpha}^U) = \{(h_2 - h_1)\alpha + h_1, h_4 - (h_4 - h_3)\alpha, (h_6 - h_5)\alpha + h_5, h_7 - (h_7 - h_5)\alpha\}$$

Hence, the fuzzy version of heptagonal ranking is

$$R(\tilde{A}_H) = \int_0^1 0.5\{(h_2 - h_1)\alpha + h_1, h_4 - (h_4 - h_3)\alpha, (h_6 - h_5)\alpha + h_5, h_7 - (h_7 - h_5)\alpha\} d\alpha .$$

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2.3 Arithmetic operations on heptagonal fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$, $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$ be two heptagonal fuzzy numbers. Then

1) The addition of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \oplus \tilde{B} = (a1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7)$.
2) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6, a_7b_7)$.
3) $\frac{1}{\tilde{B}} = (\frac{1}{b7}, \frac{1}{b6}, \frac{1}{b5}, \frac{1}{b4}, \frac{1}{b3}, \frac{1}{b2}, \frac{1}{b1})$, then the division of $\tilde{A}$ and $\tilde{B}$ is $\frac{\tilde{A}}{\tilde{B}} = (\frac{a1}{b7}, \frac{a2}{b6}, \frac{a3}{b5}, \frac{a4}{b4}, \frac{a5}{b3}, \frac{a6}{b2}, \frac{a7}{b1})$.

3. Geometric Programming Technique

3.1 Crisp Model Using GP Technique

$$TIC(L) = Cd + \frac{O}{L} + \frac{hd\theta^2}{2L} + Sd\frac{(L-\theta)^2}{2L} \tag{1}$$

Here (C- θS) d is constant.

Hence the total cost becomes,

$$TIC(L) = \frac{1}{L}\left(O + \frac{(h+S)\theta^2 d}{2}\right) + \frac{SdL}{2} \tag{2}$$

Using the GP techniques to eqn (1), we get

$$t(\ell) = \left(\frac{O + \frac{(h+S)\theta^2 d}{2}}{\ell_1}\right)^{\ell_1} \left(\frac{Sd}{2\ell_2}\right)^{\ell_2} \tag{3}$$

S.t

$$\begin{aligned} \ell_1 + \ell_2 &= 1 \\ -\ell_1 + \ell_2 &= 0 \end{aligned} \tag{4}$$

Solving we get,

$$\ell_1 = \ell_2 = \frac{1}{2}$$

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i.e., $$t(\ell)=\left(\frac{O+\frac{(h+S)\theta^2 d}{2}}{1/2}\right)^{1/2}\left(\frac{Sd}{2\,1/2}\right)^{1/2} \tag{5}$$

From primal-dual relation

$$\frac{1}{L}\left(O+\frac{(h+S)\theta^2 d}{2}\right)=\ell_1 t(\ell) \tag{6}$$

$$\frac{SdL}{2}=\ell_2 t(\ell) \tag{7}$$

Solving the above eqns, we get the cycle length and total cost is given as follows

$$L=\sqrt{\frac{2}{Sd}\left(O+\frac{\theta^2 d}{2}(h+S)\right)}$$

$$TIC(L)=\sqrt{2Sd\left(O+\frac{\theta^2 d}{2}(h+S)\right)}+(C-S\theta)d$$

3.2 Fuzzy Model Using GP Technique

In this model we take the criterions ordering cost and demand are heptagonal fuzzy numbers.

Then from eqn (1) we have

$$\tilde{TIC}(L)=\frac{1}{L}\left(\tilde{O}+\frac{(h+S)\theta^2\tilde{d}}{2}\right)+\frac{S\tilde{d}L}{2} \tag{8}$$

Where $d=(d_1,d_2,d_3,d_4,d_5,d_6,d_7)$, $O=(O_1,O_2,O_3,O_4,O_5,O_6,O_7)$.

Using robust ranking defuzzification for heptagonal fuzzy numbers

Corresponding dual form of eqn (8) is given by

$$t(\ell)=\left(\frac{2(O_1+O_2+O_3+O_4+2O_5+O_6+O_7)+(h+S)\theta^2(d_1+d_2+d_3+d_4+2d_5+d_6+d_7)}{8\ell_1}\right)^{\ell_1}\left(\frac{S(d_1+d_2+d_3+d_4+2d_5+d_6+d_7)}{8\ell_2}\right)^{\ell_2}$$

Subject to,

$$\ell_1+\ell_2=1$$
$$-\ell_1+\ell_2=0$$

Solving we get,

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$$\ell_1 = \ell_2 = \frac{1}{2}$$

Putting the values in the above dual form then we get the optimal solutions are obtained from GP technique as follows.

$$L = \sqrt{\frac{\left(2\left(O_1 + O_2 + O_3 + O_4 + 2O_5 + O_6 + O_7\right) + \theta^2\left(d_1 + d_2 + d_3 + d_4 + 2d_5 + d_6 + d_7\right)\left(h + S\right)\right)}{S\left(d_1 + d_2 + d_3 + d_4 + 2d_5 + d_6 + d_7\right)}}$$

$$TIC(L) = \sqrt{\frac{S\left(d_1 + d_2 + d_3 + d_4 + 2d_5 + d_6 + d_7\right)}{16}\left(2\left(O_1 + O_2 + O_3 + O_4 + 2O_5 + O_6 + O_7\right) + \theta^2\left(d_1 + d_2 + d_3 + d_4 + 2d_5 + d_6 + d_7\right)\left(h + S\right)\right)} + \frac{(C - S\theta)\left(d_1 + d_2 + d_3 + d_4 + 2d_5 + d_6 + d_7\right)}{4}$$

4. Kuhn-Tucker Condition Approach

4.1 Crisp Model Using Kuhn-Tucker Condition

The crisp total inventory cost is given by,

$$TIC(L) = Cd + \frac{O}{L} + \frac{hd\theta^2}{2L} + Sd\,\frac{(L-\theta)^2}{2L}$$

Differentiating partially w.r.t L and equating it to zero then implies the cycle length is,

$$L = \sqrt{\frac{2}{Sd}\left(O + \frac{\theta^2 d}{2}(h+S)\right)}$$

4.2 Fuzzy Model Using Kuhn-Tucker Condition

The fuzzy total inventory cost is given by,

$$\widetilde{TIC}(L) = C\tilde{d} + \frac{\tilde{O}}{L} + \frac{h\tilde{d}\theta^2}{2L} + S\tilde{d}\,\frac{(L-\theta)^2}{2L}$$

Suppose $d = (d_1, d_2, d_3, d_4, d_5, d_6, d_7)$,are heptagonal fuzzy numbers. Using robust ranking defuzzification for heptagonal fuzzy numbers and then we solve the cycle length is given as follows.

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$$\tilde{TIC}(L)=\frac{1}{4}\begin{bmatrix}\frac{1}{L}\left(O_1+\frac{(h+S)\theta^2 d_1}{2}\right)+\frac{Sd_1L}{2}+\left(C-\ \theta S\right)d_1+\\ \frac{1}{L}\left(O_2+\frac{(h+S)\theta^2 d_2}{2}\right)+\frac{Sd_2L}{2}+\left(C-\ \theta S\right)d_2+\\ \frac{1}{L}\left(O_3+\frac{(h+S)\theta^2 d_3}{2}\right)+\frac{Sd_3L}{2}+\left(C-\ \theta S\right)d_3+\\ \frac{1}{L}\left(O_4+\frac{(h+S)\theta^2 d_4}{2}\right)+\frac{Sd_4L}{2}+\left(C-\ \theta S\right)d_4+\\ \frac{1}{L}\left(2O_5+\frac{2(h+S)\theta^2 d_5}{2}\right)+\frac{2Sd_5L}{2}+2\left(C-\ \theta S\right)d_5+\\ \frac{1}{L}\left(O_6+\frac{(h+S)\theta^2 d_6}{2}\right)+\frac{Sd_6L}{2}+\left(C-\ \theta S\right)d_6+\\ \frac{1}{L}\left(O_7+\frac{(h+S)\theta^2 d_7}{2}\right)+\frac{Sd_7L}{2}+\left(C-\ \theta S\right)d_7\end{bmatrix}$$

Differentiating partially w.r.t L and equating it to zero.

$$\frac{\partial \tilde{TIC}(L)}{\partial L}=0$$

$$\frac{\partial \tilde{TIC}(L)}{\partial L}=\frac{-1}{L^2}\left(\left(O_1+O_2+O_3+O_4+2O_5+O_6+O_7\right)+\frac{(h+S)\theta^2\left(d_1+d_2+d_3+d_4+2d_5+d_6+d_7\right)}{2}\right)$$

$$+\frac{S\left(d_1+d_2+d_3+d_4+2d_5+d_6+d_7\right)}{2}=0$$

Therefore,

$$\tilde{L}=\sqrt{\frac{2\left(O_1+O_2+O_3+O_4+2O_5+O_6+O_7\right)+(h+S)\theta^2\left(d_1+d_2+d_3+d_4+2d_5+d_6+d_7\right)}{S\left(d_1+d_2+d_3+d_4+2d_5+d_6+d_7\right)}}$$

Fuzzy optimal cycle length using Kuhn –tucker condition:

Suppose L be a heptagonal fuzzy number $\tilde{L}=\left(L_1,L_2,L_3,L_4,L_5,L_6,L_7\right)$ with $0<L_1\le L_2\le L_3\le L_4\le L_5\le L_6\le L_7$.

The fuzzy total inventory cost is,

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$$\tilde{TIC}(L)=\begin{bmatrix} \frac{1}{L_7}\left(O_1+\frac{(h+S)\theta^2 d_1}{2}\right)+\frac{Sd_1L_1}{2}+(C-\theta S)d_1, \\ \frac{1}{L_6}\left(O_2+\frac{(h+S)\theta^2 d_2}{2}\right)+\frac{Sd_2L_2}{2}+(C-\theta S)d_2, \\ \frac{1}{L_5}\left(O_3+\frac{(h+S)\theta^2 d_3}{2}\right)+\frac{Sd_3L_3}{2}+(C-\theta S)d_3, \\ \frac{1}{L_4}\left(O_4+\frac{(h+S)\theta^2 d_4}{2}\right)+\frac{Sd_4L_4}{2}+(C-\theta S)d_4, \\ \frac{1}{L_3}\left(O_5+\frac{(h+S)\theta^2 d_5}{2}\right)+\frac{Sd_5L_5}{2}+(C-\theta S)d_5, \\ \frac{1}{L_2}\left(O_6+\frac{(h+S)\theta^2 d_6}{2}\right)+\frac{Sd_6L_6}{2}+(C-\theta S)d_6, \\ \frac{1}{L_1}\left(O_7+\frac{(h+S)\theta^2 d_7}{2}\right)+\frac{Sd_7L_7}{2}+(C-\theta S)d_7 \end{bmatrix}$$

We defuzzify the total inventory cost using robust's ranking method.

$$\tilde{TIC}(L)=\frac{1}{4}\begin{bmatrix} \frac{1}{L_7}\left(O_1+\frac{(h+S)\theta^2 d_1}{2}\right)+\frac{Sd_1L_1}{2}+(C-\theta S)d_1+ \\ \frac{1}{L_6}\left(O_2+\frac{(h+S)\theta^2 d_2}{2}\right)+\frac{Sd_2L_2}{2}+(C-\theta S)d_2+ \\ \frac{1}{L_5}\left(O_3+\frac{(h+S)\theta^2 d_3}{2}\right)+\frac{Sd_3L_3}{2}+(C-\theta S)d_3+ \\ \frac{1}{L_4}\left(O_4+\frac{(h+S)\theta^2 d_4}{2}\right)+\frac{Sd_4L_4}{2}+(C-\theta S)d_4+ \\ \frac{2}{L_3}\left(O_5+\frac{(h+S)\theta^2 d_5}{2}\right)+\frac{2Sd_5L_5}{2}+2(C-\theta S)d_5+ \\ \frac{1}{L_2}\left(O_6+\frac{(h+S)\theta^2 d_6}{2}\right)+\frac{Sd_6L_6}{2}+(C-\theta S)d_6+ \\ \frac{1}{L_1}\left(O_7+\frac{(h+S)\theta^2 d_7}{2}\right)+\frac{Sd_7L_7}{2}+(C-\theta S)d_7 \end{bmatrix}$$

Now differentiating partially w.r.t $L_1, L_2, L_3, L_4, L_5, L_6, L_7$ respectively and equate it to zero.

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$$\frac{\partial TIC(L)}{\partial L_1} = 0$$

$$\frac{-1}{L_1^{\,2}}\left(O_7 + \frac{(h+S)\theta^2 d_7}{2} \right) + \frac{Sd_1}{2} = 0$$

$$L_1 = \sqrt{\frac{2O_7 + (h+S)\theta^2 d_7}{Sd_1}}$$

Similarly,

$$\frac{\partial TIC(L)}{\partial L_2} = 0 \Rightarrow L_2 = \sqrt{\frac{2O_6 + (h+S)\theta^2 d_6}{Sd_2}}$$

$$\frac{\partial TIC(L)}{\partial L_3} = 0 \Rightarrow L_3 = \sqrt{\frac{2O_5 + (h+S)\theta^2 d_5}{Sd_3}}$$

$$\frac{\partial TIC(L)}{\partial L_4} = 0 \Rightarrow L_4 = \sqrt{\frac{2O_4 + (h+S)\theta^2 d_4}{Sd_4}}$$

$$\frac{\partial TIC(L)}{\partial L_5} = 0 \Rightarrow L_5 = \sqrt{\frac{2O_3 + (h+S)\theta^2 d_3}{Sd_5}}$$

$$\frac{\partial TIC(L)}{\partial L_6} = 0 \Rightarrow L_6 = \sqrt{\frac{2O_2 + (h+S)\theta^2 d_2}{Sd_6}}$$

$$\frac{\partial TIC(L)}{\partial L_7} = 0 \Rightarrow L_7 = \sqrt{\frac{2O_1 + (h+S)\theta^2 d_1}{Sd_7}}$$

Next, the Kuhn Tucker condition is used to minimize the total cost and to find the solution of

$L_1, L_2, L_3, L_4, L_5, L_6, L_7$.

The Kuhn Tucker conditions are,

$$\nabla f(TIC(L)) - \lambda_i \nabla g(L_i)$$
$$\lambda_i g_i(L_i) = 0$$
$$g_i(L_i) \geq 0$$

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These above simplify to the following, $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7 \le 0$.

$$\nabla f(\tilde{TIC}(L)) - \lambda_i \nabla g(L_i) \Rightarrow$$

$$\tilde{TIC}(L) = \frac{1}{4}\left\{\left[\frac{1}{L_7}\left(O_1 + \frac{(h+S)\theta^2 d_1}{2}\right) + \frac{Sd_1L_1}{2} + (C - \theta S)d_1\right] + \left[\frac{1}{L_6}\left(O_2 + \frac{(h+S)\theta^2 d_2}{2}\right) + \frac{Sd_2L_2}{2} + (C - \theta S)d_2\right]\right.$$
$$+\left[\frac{1}{L_5}\left(O_3 + \frac{(h+S)\theta^2 d_3}{2}\right) + \frac{Sd_3L_3}{2} + (C - \theta S)d_3\right] + \left[\frac{1}{L_4}\left(O_4 + \frac{(h+S)\theta^2 d_4}{2}\right) + \frac{Sd_4L_4}{2} + (C - \theta S)d_4\right] +$$
$$\left[\frac{2}{L_3}\left(O_5 + \frac{(h+S)\theta^2 d_5}{2}\right) + \frac{2Sd_5L_5}{2} + 2(C - \theta S)d_5\right] + \left[\frac{1}{L_2}\left(O_6 + \frac{(h+S)\theta^2 d_6}{2}\right) + \frac{Sd_6L_6}{2} + (C - \theta S)d_6\right] +$$
$$\left.\left[\frac{1}{L_1}\left(O_7 + \frac{(h+S)\theta^2 d_7}{2}\right) + \frac{Sd_7L_7}{2} + (C - \theta S)d_7\right]\right\} - \lambda_1(L_2 - L_1) - \lambda_2(L_3 - L_2) - \lambda_3(L_4 - L_3)$$
$$-\lambda_4(L_5 - L_4) - \lambda_5(L_6 - L_5) - \lambda_6(L_7 - L_6) - \lambda_7(L_1) = 0$$

$$\frac{1}{4}\left[\frac{1}{L_7}\left(O_1 + \frac{(h+S)\theta^2 d_1}{2}\right) + \frac{Sd_1L_1}{2} + (C - \theta S)d_1\right] + \lambda_1 - \lambda_7 = 0$$

$$\frac{1}{4}\left[\frac{1}{L_6}\left(O_2 + \frac{(h+S)\theta^2 d_2}{2}\right) + \frac{Sd_2L_2}{2} + (C - \theta S)d_2\right] - \lambda_1 + \lambda_2 = 0$$

$$\frac{1}{4}\left[\frac{1}{L_5}\left(O_3 + \frac{(h+S)\theta^2 d_3}{2}\right) + \frac{Sd_3L_3}{2} + (C - \theta S)d_3\right] - \lambda_2 + \lambda_3 = 0$$

$$\frac{1}{4}\left[\frac{1}{L_4}\left(O_4 + \frac{(h+S)\theta^2 d_4}{2}\right) + \frac{Sd_4L_4}{2} + (C - \theta S)d_4\right] - \lambda_3 + \lambda_4 = 0$$

$$\frac{1}{4}\left[\frac{2}{L_3}\left(O_5 + \frac{(h+S)\theta^2 d_5}{2}\right) + \frac{2Sd_5L_5}{2} + 2(C - \theta S)d_5\right] - \lambda_4 + \lambda_5 = 0$$

$$\frac{1}{4}\left[\frac{1}{L_2}\left(O_6 + \frac{(h+S)\theta^2 d_6}{2}\right) + \frac{Sd_6L_6}{2} + (C - \theta S)d_6\right] - \lambda_5 + \lambda_6 = 0$$

$$\frac{1}{4}\left[\frac{1}{L_1}\left(O_7 + \frac{(h+S)\theta^2 d_7}{2}\right) + \frac{Sd_7L_7}{2} + (C - \theta S)d_7\right] - \lambda_6 = 0$$

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$$\lambda_1(L_2 - L_1) = 0$$
$$\lambda_2(L_3 - L_2) = 0$$
$$\lambda_3(L_4 - L_3) = 0$$
$$\lambda_4(L_5 - L_4) = 0$$
$$\lambda_5(L_6 - L_5) = 0$$
$$\lambda_6(L_7 - L_6) = 0$$
$$\lambda_7 L_1 = 0$$

If $L_1 > 0 and \lambda_7 L_1 = 0 then \lambda_7 = 0.\ If\ \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$ then $L_7 < L_6 < L_5 < L_4 < L_3 < L_2 < L_1$. It doesn't satisfy the constrains $0 < L_1 \le L_2 \le L_3 \le L_4 \le L_5 \le L_6 \le L_1$.

$$\therefore L_2 = L_1, L_3 = L_2, L_4 = L_3, L_5 = L_4, L_6 = L_5, L_7 = L_6$$
$$i.e., L_1 = L_2 = L_3 = L_4 = L_5 = L_6 = L_7 = \tilde{L}^*$$

$$\tilde{L}^* = \sqrt{\frac{2\left(O_1 + O_2 + O_3 + O_4 + 2O_5 + O_6 + O_7\right) + (h+S)\theta^2\left(d_1 + d_2 + d_3 + d_4 + 2d_5 + d_6 + d_7\right)}{S\left(d_1 + d_2 + d_3 + d_4 + 2d_5 + d_6 + d_7\right)}}$$

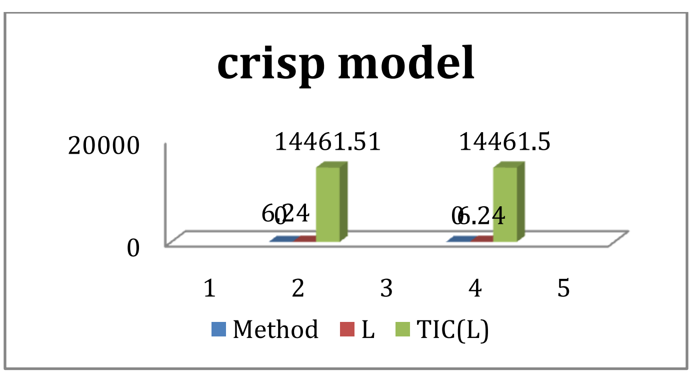
5. Numerical Example

Crisp Model

Let us consider O = \$300 per cycle, h = \$1.5 per unit, C = \$25 per unit, S= \$5 per unit, d = 500 units, θ=5.46.

Method	L	TIC(L)
GP	6.24	14461.51
Kuhn tucker	6.24	14461.51

Graphical representation of the above table is given by,



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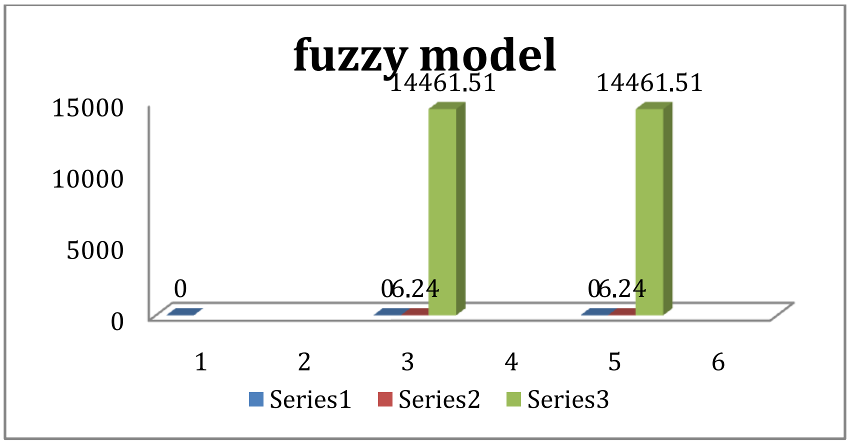
Fuzzy model

Let us consider h = \$1.5 per unit, C = \$25 per unit, S= \$5 per unit, θ=5.46,

O=300= (50, 75,100,150,175,200,275) and D=500=(100,150,200,250,300,325,375).

Method	$\tilde{L}$	$TIC(L)$
GP	6.24	14461.51
Kuhn tucker	6.24	14461.51

Graphical representation of the above table is given by,

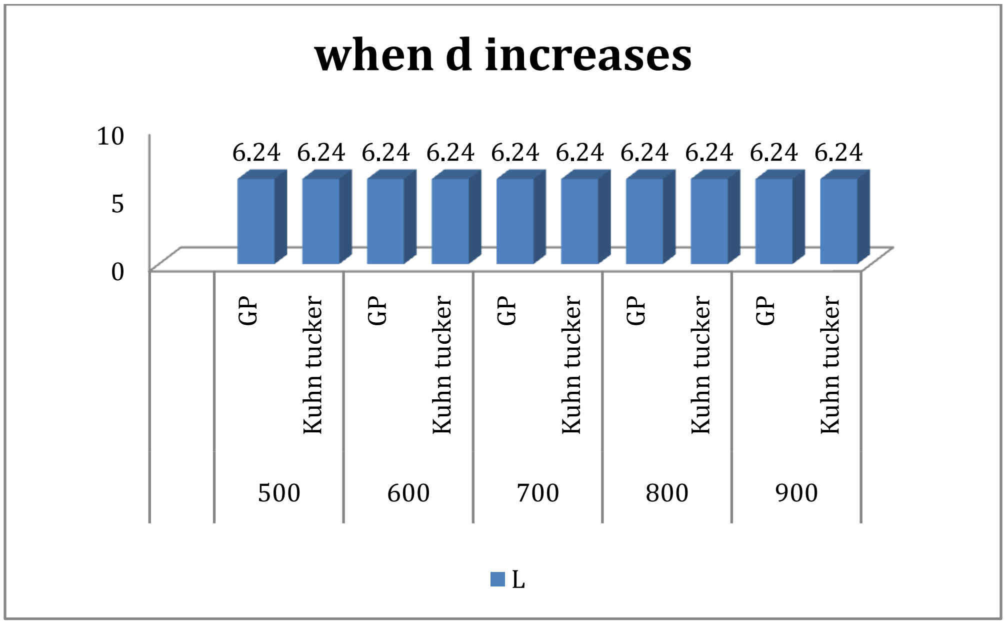


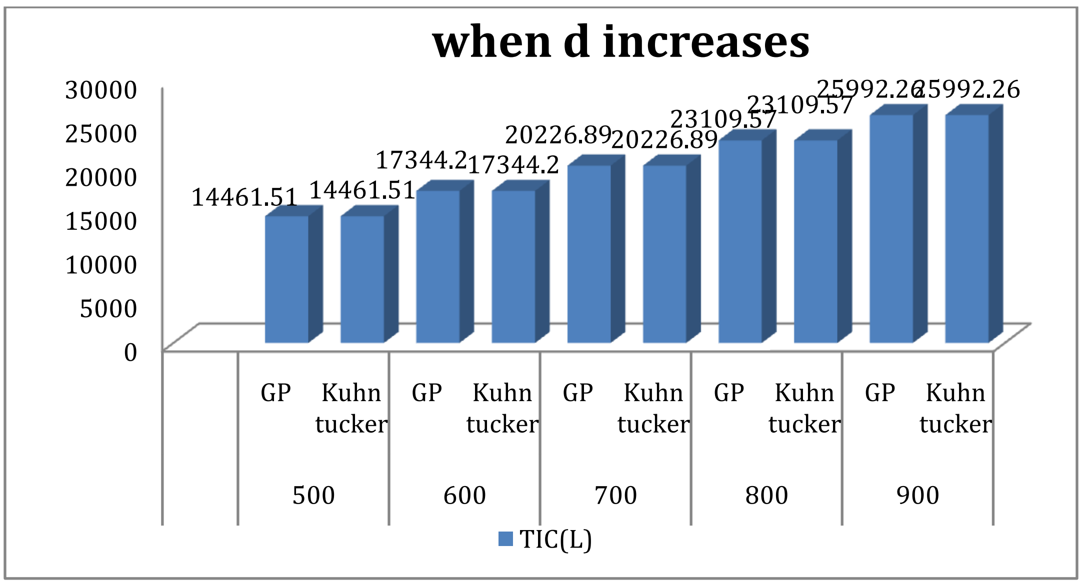
When d increases then its optimal solution TIC (L) are also increases as follows.

D	Method	L	TIC(L)
500	GP	6.24	14461.51
	Kuhn tucker	6.24	14461.51
600	GP	6.24	17344.20
	Kuhn tucker	6.24	17344.20
700	GP	6.24	20226.89
	Kuhn tucker	6.24	20226.89
800	GP	6.24	23109.57
	Kuhn tucker	6.24	23109.57
900	GP	6.24	25992.26
	Kuhn tucker	6.24	25992.26

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1442 - 1454
Received 05 March 2021; Accepted 01 April 2021.

Graphical representation of the above table is given by,





6. Conclusion

In this paper GP technique and Kuhn-tucker conditions for finding the optimal solutions to minimize the total cost and maximize the total profit of fuzzy inventory model in which the ordering cost and demand of the product are taken as heptagonal fuzzy numbers and solved by robust ranking defuzzification method. The results of these proposed models, crisp sense is very close to fuzzy sense. Finally, numerical example is found to illustrate the proposed models.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1442 - 1454
Received 05 March 2021; Accepted 01 April 2021.

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