Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

Operation Submaximal (Extremally Disconnected) Spaces

P. Sudha[1]

[1]Assistant Professor, PG and Research, Department Of Mathematics,
CAUVERY COLLEGE FOR WOMEN (AUTONOMOUS), Affiliatedto Bharathidasan University,
Trichy-18, Tamilnadu.

Abstract: In this paper we are going to discuss about the characterizations of sub maximal spaces and extremally disconnected spaces via operation in topological spaces.

1. Introduction

Soft set theory [11] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in Economics, Engineering, Social Science, Medical Science, etc. In recent years the development in the fields of soft set theory and its application has been taking place in a rapid pace. This is because of the general nature of parameterization expressed by a soft set. Later Maji et. al. [12] presented some new definitions on soft sets such as a subset, the complement of a soft set and discussed in detail the application of soft set theory in decision making problems [13]. Chen et. al. [4, 5] and Kong et. al. [9] introduced a new definition of soft set parametrization reduction. Xiao et. al. [18] and Pei and Miao [15] discussed the relationship between soft sets and information systems. Also an attempt was made by Kostek [3] to assess sound quality based on a soft set approach. Mushrif et al. [14] presented a novel method for the classification of natural textures using the notions of soft set theory. The algebraic nature of set theories dealing with uncertainties has been studied by some authors. Chang introduced the notion of fuzzy topology and also studied some of its basic properties. Lashin et. al. [10] generalized rough set theory in the framework of topological spaces. Recently, Shabir and Naz [17] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also studied some of basic concepts of soft topological spaces. Later, Aygunoglu et. al. [1], Zorlutuna et. al. [19] and Hussain et. al. [16] continued to study the properties of soft topological spaces. They got many important results in soft topological spaces. In this paper we have study the characterizations of submaximal spaces and extremally disconnected spaces via operation in topological spaces.

2. Preliminaries

Let U be an initial universe set and E_U be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. We will call E_U the universe set of parameters with respect to U.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

Definition 2.1. [11] A pair (F, A) is called a soft set over U if A ⊂ E_U and F : A → P (U), where P (U) is the set of all subsets of U .

Definition 2.2. [6] Let U be an initial universe set and E_U be a universe set of parameters. Let (F, A) and (G, B) be soft sets over a common universe set U and A, B ⊂ E. Then (F, A) is a subset of (G, B), denoted by (F, A) $\tilde{\subseteq}$ (G, B), if A ⊂ B and for all e ∈ A, F (e) ⊂ G(e).
Also (F, A) equals (G, B), denoted by (F, A) = (G, B), if (F, A) $\tilde{\subseteq}$ (G, B) and (G, B) $\tilde{\subseteq}$ (F, A).

Definition 2.3. [12] A soft set (F, A) over U is called a null soft set, denoted by ∅, if e ∈ A, F (e) = ∅.

Definition 2.4. [12] A soft set (F, A) over U is called an absolute soft set, denoted by $\tilde{A}$, if e ∈ A, F (e) = U .

Definition 2.5. [12] The union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where C = A ∪ B, and for all e ∈ C ,

$$H(e) = \begin{cases} F(e) & , if\ e \in A \setminus B \\ G(e) & , if\ e \in B \setminus A \\ F(e) \cup G(e) & , if\ e \in B \cap A \end{cases}$$

We write (F, A) ∪ (G, B) = (H, C).

Definition 2.6. [6] The intersection of two soft sets of (F, A) and (G, B) over a common universe U is the soft set (H, C), where C = A ∩ B, and for all e ∈ C , H (e) = F (e) ∩ G(e). We write (F, A) ∩ (G, B) = (H, C).

Now we recall some definitions and results defined and discussed in [17]-[16]. Henceforth, let X be an initial universe set and E be the fixed nonempty set of parameter with respect to X unless otherwise specified.

Definition 2.7. For a soft set (F, A) over U , the relative complement of (F, A) is denoted by $(F,A)'$ and is defined by $(F,A)'$ = (F', A), where F': A → P (U) is a mapping given by F' (e) = U \F (e) for all e ∈ A.

Definition 2.8. Let τ be the collection of soft sets over X , then τ is called a soft topology on X if τ satisfies the following axioms.

(1) ∅, $\tilde{X}$ belong to τ .
(2) The union of any number of soft sets in τ belongs to τ .
(3) The intersection of any two soft sets in τ belongs to τ .
The triplet (X, τ, E, γ) is called a soft topological space over X .

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

Definition 2.9. Let (X, τ, E, γ) be a soft topological space over X , then the members of τ are said to be soft open sets in X .

Definition 2.10. Let (X, τ, E, γ) be a soft topological space over X . A soft set (F, E) over X is said to be a soft closed set in X , if its relative complement $(F,E)'$ belongs to τ

Proposition 2.11. Let (X, τ, E, γ) be a soft topological space over X . Then one has the following

(1) ∅ , $\tilde{X}$ are soft closed sets over X .
(2) The intersection of any number of soft closed sets is a soft closed set over X .
(3) The union of any two soft closed sets is a soft closed set over X .

Definition 2.12. Let (X, τ, E, γ) be a soft topological space and (A, E) be a soft set over X .

(1) The soft interior of (A, E) is the soft set Int(A, E) = ∪{(O, E) : (O, E) is soft open and (O, E) $\tilde{\subseteq}$ (A, E)}.

(2) The soft closure of (A, E) is the soft set Cl(A, E) = ∩{(F, E) : (F, E) is soft closed and (A, E) $\tilde{\subseteq}$ (F, E)}.

Proposition 2.13. Let (X, τ, E, γ) be a soft topological space and let (F, E) and (G, E) be a soft set over X . Then

(1) Cl(Cl(F, E)) = Cl(F, E),
(2) (F, E) $\tilde{\subseteq}$ (G, E) implies Cl(F, E) $\tilde{\subseteq}$ Cl(G, E),
(3) Cl(Cl(F, E)) = Cl(F, E),
(4) (F, E) $\tilde{\subseteq}$ (G, E) implies Cl(F, E) $\tilde{\subseteq}$ Cl(G, E).

Definition 2.14. Let (F, E) be a soft set over X and x ∈ X . We say that x ∈ (F, E) read as x belongs to the soft set (F, E), whenever x ∈ F (α) for all α ∈ E. Note that for x ∈ X , x ∉ (F, E) if x ∉ F (α) for α ∈ E.

Definition 2.15. Let x ∈ X , then (x, E) denotes the soft set over X for which x(α) = {x}, for all α ∈ E.

Definition 2.16. [2] An operation on a soft topology τ over X is called a γ-operation if a mapping from τ to the set $P(X)^E$ and defined by
$\gamma : \tau \to P(X)^E$ such that for each (V, E) ∈ τ , (V, E) $\tilde{\subseteq}$ γ(V, E).

Definition 2.17. [2] A soft set (P, E) is said to be γ-soft open set if for each x $\tilde{\in}$ (P, E), there exists a soft a γ-soft open set (V, E) such that x $\tilde{\in}$ (V, E) $\tilde{\subseteq}$ γ(V, E) $\tilde{\subseteq}$ (P, E). The complement of a γ-soft open set is called a γ-soft closed set. The family of all γ-soft open sets of (X, τ, E, γ) is denoted by τ_γ .

Definition 2.18. [2] Let (X, τ, E, γ) be an operation-soft topological space and (A, E) be a soft set over X . Then

(1) the τ_γ -soft interior of (A, E) is the soft set τ_γ -Int(A, E) =∪{(O, E) : (O, E) is γ-soft open and (O, E) $\tilde{\subseteq}$ (A, E)}.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

(2) the τ_γ -soft closure of (A, E) is the soft set τ_γ -Cl(A, E) = ∩{(F, E) : (F, E) is γ-soft closed and (A, E) $\tilde{\subseteq}$ (F, E)}.

Lemma 2.19. Let (X, τ, E, γ) be an operation-soft topological space. Then we have the following
(1) for every γ-soft open set (G, E) and every subset (A, E) ⊂ X
we have τ_γ -Cl((A, E)) ∩ (G, E) ⊂ τ_γ -Cl((A, E) ∩ (G, E)),
(2) for every γ-closed set F and every subset (A, E) ⊂ X we have τ_γ -Int((A, E) ∩ (F, E) ⊂ τ_γ -Int((A, E)) ∩ (F, E).

Definition 2.20. Let (X, τ, E, γ) be an operation-soft topological space and (A, E) be a soft set over X. Then
(1) e_M is called γ-δ-cluster point of (S, E) if (S, E) ∩ τ_γ -Int(τ_γ -Cl((U, E))) = ∅ , for each τ_γ -open set (U, E) containing e_M .
(2) the family of all γ-δ-cluster point of (S, E) is called the γ-δ- closure of (S, E) and is denoted by τ_γ -Cl_δ ((S, E)).
(3) A subset (S, E) is said to be γ-δ-closed if τ_γ -Cl_δ ((S, E)) = (S, E). The complement of an γ-δ-closed set is said to be an γ-δ-open set.

Definition 2.21. A subset (A, E) of a operation-soft topological space (X, τ, E, γ) is said to be
(1) γ-soft semiopen [7] if (A, E) ⊂ τ_γ -Cl(τ_γ -Int((A, E))).
(2) γ-soft preopen [8] if (A, E) ⊂ τ_γ -Int(τ_γ -Cl((A, E))).
(3) γ-soft α-open if (A, E) ⊂ τ_γ -Int(τ_γ -Cl(τ_γ -Int((A, E)))).
(4) γ-soft β-open if (A, E) ⊂ τ_γ -Cl(τ_γ -Int(τ_γ -Cl((A, E)))).
(5) t_γ -set if τ_γ -Int((A, E)) = τ_γ -Int(τ_γ -Cl((A, E))).
(6) γ-semiregular set if (A, E) is a t_γ -set and γ-soft semiopen.
(7) AB_γ -set if (A, E) = (U, E) ∩ (V, E), where (U, E) ∈ τ_γ and (V, E) is γ-semiregular.
(8) γ-dense if τ_γ -Cl((A, E)) = X .

The family of all γ-soft regular open (resp.γ-soft preopen, γ-soft semiopen, γ-soft β-open) sets of (X, τ, E, γ) is denoted by γ-SRO(X) (resp. γ-SPO(X), γ-SSO(X), γ-SβO(X)).

3. On γ-soft submaximal spaces

Definition 3.1. An operation-topological space (X, τ, E, γ) is said to be γ-soft submaximal if every γ-soft dense subset of X is γ-soft open.

Lemma 3.2. (A, E) ∈ γ-SP O(X) if and only if (A, E) = (U, E) ∩
(D, E) for some (U, E) ∈ τ_γ and τ_γ -dense set (D, E) ∈ S(X).

Proof. If (A, E) ∈ γ-SP O(X), then (A, E) ⊂ τ_γ -Int(τ_γ -Cl((A, E))). Let (U, E) ∈ τ_γ .

Let (D, E) = X \((U, E)\(A, E) = (X \(U, E)) ∪ (A, E). Then (D, E) is τ_γ -dense since X = τ_γ -Cl((A, E)) ∪ (X \τ_γ -Cl((A, E))) ⊂ τ_γ -Cl((A, E))∪(X \(U, E)) = τ_γ -Cl((D, E)). Also, (A, E) = (U, E)∩(D, E). Conversely, if (A, E) = (U, E)∩(D, E), where (U, E) ∈ τ_γ and (D, E) is τ_γ -dense, then (A, E) ⊂ (U, E), τ_γ -Int(τ_γ -Cl((A, E))) ⊂ τ_γ -Int(τ_γ -Cl((U, E)))

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

and (U, E) = (U, E)∩X = (U, E)∩τ_γ -Cl(D) ⊂ τ_γ - Cl((U, E) ∩ (D, E)) = τ_γ -Cl((A, E)), hence τ_γ -Int(τ_γ -Cl((U, E))) ⊂ τ_γ -Int(τ_γ -Cl((A, E))). Therefore, τ_γ -Int(τ_γ -Cl((A, E))) = τ_γ -Int(τ_γ - Cl((U, E))) so that (A, E) ∈ γ-SP O(X).

Lemma 3.3. If (X, τ, E, γ) is γ-soft submaximal, then τ_γ = γ-SPO(X).

Proof. Clearly τ_γ ⊂ γ-SP O(X). Now, (A, E) ∈ γ-SP O(X), then (A, E) = (U, E) ∩ (D, E) for some (U, E) ∈ τ_γ and τ_γ -dense set D ⊂ X . Therefore, if (X, τ, E, γ) is γ-soft submaximal, (D, E) ∈ τ_γ , then (A, E) ∈ τ_γ .

Theorem 3.4. For an operation-soft topological space (X, τ, E, γ), the following are equivalent:

(1) X is γ-soft submaximal.

(2) Every γ-soft preopen set is γ-soft open.

(3) Every γ-soft preopen set is γ-soft semiopen and every γ-soft α-open set is γ-soft open.

Proof. (1) ⇒ (2): It follows from Lemma 3.3.

(2) ⇒ (3): Suppose that every γ-soft preopen set is γ-soft open. Then every γ-soft preopen set is γ-soft semiopen.

(3) ⇒ (1): Let (A, E) be a τ_γ -dense subset of X . Since τ_γ -Cl((A, E)) = X , then (A, E) is γ-soft preopen. By (3), (A, E) is γ-soft semiopen.

Since a set is γ-soft α-open if and only if it is γ-soft preopen and γ-soft semiopen, then (A, E) is γ-soft α-open. Thus, by (3), (A, E) is γ-soft open; hence X is γ-soft submaximal.

Theorem 3.5. For an operation-soft topological space (X, τ, E, γ), the following are equivalent:

(1) X is γ-soft submaximal.

(2) For all (A, E) ⊂ X , if (A, E)\$\tau_\gamma$ -Int((A, E)) = ∅ , then (A, E)\$\tau_\gamma$ - Int(τ_γ -Cl((A, E))) = ∅ .

(3) τ_γ^* = {(U, E)\(A, E) : (U, E) ∈ τ_γ and τ_γ -Int((A, E)) = ∅ }.

Proof.(1) ⇒(2): Let (A, E) ⊂X and (A, E)\$\tau_\gamma$ -Int((A, E)) = ∅ . Suppose that (A, E)\$\tau_\gamma$ - Int(τ_γ -Cl((A, E))) = ∅ . Then (A, E) ⊂τ_γ - Int(τ_γ -Cl((A, E))). This implies that (A, E) is γ-soft preopen. Since x is γ-soft submaximal, by Theorem 3.4, (A, E) is γ-soft open. Thus, (A, E)\$\tau_\gamma$ -Int((A, E)) = ∅ . This is a contradiction.

(2) ⇒(1): Let (A, E) be γ-soft preopen. Then (A, E) ⊂τ_γ -Int(τ_γ - Cl((A, E))). Suppose that (A, E) is not γ-soft open. Then (A, E) * τ_γ - Int((A, E)) and (A, E)\$\tau_\gamma$ -Int((A, E)) = ∅ . By (2), (A, E)\$\tau_\gamma$ -Int(τ_γ - Cl((A, E))) = ∅ . Thus, (A, E) * τ_γ -Int(τ_γ -Cl((A, E))). This is a contradiction.

(1) ⇒(3): Suppose that τ_γ^* = {(U, E)\(A, E) : (U, E) ∈τ_γ and τ_γ -

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

Int((A, E)) = ∅ }. Let (G, E) ∈ τ_γ . Since (G, E) = (G, E)\∅ and

τ_γ -Int(∅) = ∅ , then $\tau_\gamma \subset \tau_\gamma^*$ .Let (G, E) ∈ τ_γ^* . Since (G, E) =

(G, E)\(A, E), where U ∈ τ_γ and τ_γ -Int((A, E)) = ∅ . We have (G, E) = $X \setminus \tau_\gamma - Int(A, E)$, then τ_γ -Cl(X \(A, E)) = X . Since X is γ-soft submaximal, X \(A, E) is γ- soft open. Thus, (G, E) is γ-soft open. Hence τ *(A, E) ⊂ τ_γ .
(3) ⇒ (1): Let (A, E) be γ-soft preopen. By Lemma 3.2, (A, E) = (G, E) ∩ (B, E), where (G, E) ∈ τ_γ and τ_γ -dense set B ⊂ X . We have τ_γ -Cl((B, E)) = X and hence τ_γ -Int(X \(A, E)) = ∅ . This implies that (A, E) = (G, E)\(X \(B, E)) = ∅ . Thus, by (3), (A, E) is γ-soft open.
Hence, by Theorem 3.4, X is γ-soft submaximal.

Theorem 3.6. For an operation-soft topological space (X, τ, E, γ), the following are equivalent:

(1) X is γ-soft submaximal.
(2) Every γ-soft preopen set is an AB_γ -set.
(3) Every τ_γ -dense set is an AB_γ -set.

Proof. (1) ⇒ (2): Let (A, E) ⊂ X be a γ-soft preopen set. Since X is γ-soft submaximal, by Theorem 3.4, (A, E) is γ-soft open. Since X is both τ_γ -open and γ-semiregular, every γ-soft open set AB_γ -set. It follows that (A, E) is an AB_γ -set.
(2) ⇒ (3): Let (A, E) ⊂ X be a τ_γ -dense set. Since every τ_γ -dense set
is γ-soft preopen, by (2) (A, E) is an AB_γ -set.
(3) ⇒ (1): Let (A, E) ⊂ X be a τ_γ -dense set. By (3), (A, E) is an AB_γ -set. Since every τ_γ -dense set is γ-soft preopen, (A, E) is γ- soft preopen. Then (A, E) ⊂ τ_γ -Int(τ_γ -Cl((A, E))). Furthermore, because (A, E) is an AB_γ -set, we have A = U ∩ V , where U ∈ τ_γ and V is γ-semiregular. Then (A, E) ⊂ τ_γ -Int(τ_γ -Cl(U ∩ V)) = τ_γ -Int(τ_γ -Cl((U, E))∩τ_γ -Cl(V)) = τ_γ -Int(τ_γ -Cl((U, E)))∩τ_γ -Int(τ_γ -Cl(V)). Since V is γ-semiregular, V is also t_γ -set. Then we have (A, E) ⊂ τ_γ - Int(τ_γ -Cl((U, E))) ∩ τ_γ -Int(V). Also A = U ∩ (A, E) ⊂ U ∩ (τ_γ -Int(τ_γ - Cl((U, E))) ∩ τ_γ -Int(V)) = U ∩ (τ_γ -Int(τ_γ -Cl((U, E)))) ∩ τ_γ -Int(V) = U ∩ τ_γ -Int(V) = τ_γ -Int(U ∩ V) = τ_γ -Int((A, E)). Hence, (A, E) ∈ τ_γ .
Therefore, X is γ-soft submaximal.

Definition 3.7. An operation-soft topological space (X, τ, E, γ) is said to be τ_γ -extremally disconnected if τ_γ -Cl((A, E)) ∈τ_γ for every (A, E) ∈τ_γ .

Theorem 3.8. For an operation-soft topological space (X, τ, E, γ), the following are equivalent:

(1) X is γ-soft extremally disconnected.
(2) τ_γ -Int((A, E)) is γ-closed for every γ-closed subset (A, E) of X .
(3) τ_γ -Cl(τ_γ -Int((A, E))) ⊂ τ_γ -Int(τ_γ -Cl((A, E))) for every subset (A, E) of X .
(4) Every γ-soft semiopen set is γ-soft preopen.
(5) The τ_γ -closure of every γ-soft β-open subset of X is γ-soft open.
(6) Every γ-soft β-open set is γ-soft preopen.
(7) For every subset (A, E) of X , (A, E) is γ-soft α-open if and only if it is γ-soft

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

semiopen.

Proof. (1) ⇒ (2): Let A ⊂ X be a γ-closed set. Then X \A is γ-soft open. By (i), τ_γ -Cl(X \A) = X \τ_γ -Int((A, E)) is γ-soft open. Thus, τ_γ -Int((A, E)) is γ-closed.
(2) ⇒ (3): Let (A, E) be any set of X . Then X \τ_γ -Int((A, E)) is
γ-closed in X and by (2) τ_γ -Int(X \τ_γ -Int((A, E))) is γ-closed in X .

Therefore, τ_γ -Cl(τ_γ -Int((A, E))) is γ-soft open in X and hence, τ_γ - Cl(τ_γ -Int((A, E))) ⊂ τ_γ -Int(τ_γ -Cl((A, E))).
(3) ⇒ (4): Let (A, E) be γ-soft semiopen. By (3), we have (A, E) ⊂ τ_γ - Cl((A, E)) ⊂ τ_γ - Int(τ_γ -Cl((A, E))). Thus, (A, E) is γ-soft preopen.
(4) ⇒ (5): Let (A, E) be a γ-soft β-open set. Then τ_γ -Cl((A, E)) is γ-soft semiopen. By (4), τ_γ -Cl((A, E)) is γ-soft preopen. Thus, τ_γ - Cl((A, E)) ⊂ τ_γ -Int(τ_γ -Cl((A, E))) and hence τ_γ -Cl((A, E)) is γ-soft open.
(5) ⇒ (6): Let (A, E) be γ-soft β-open. By (5), τ_γ -Cl((A, E)) = τ_γ - Int(τ_γ -Cl((A, E))) and hence (A, E) is γ-soft preopen.
(6) ⇒ (7): Let (A, E) be γ-soft semiopen set. Since a γ-soft semiopen
set is γ-soft β-open, then by (6), it is γ-soft preopen. Since (A, E) is γ-soft semiopen and γ-soft preopen, (A, E) is γ-soft α-open.
(7) ⇒ (1): Let (A, E) be a γ-soft open set of X . Then τ_γ -Cl((A, E)) is γ-soft semiopen and by (7) τ_γ -Cl((A, E)) is γ-soft α-open. There- fore, τ_γ -Cl((A, E)) ⊂ τ_γ -Int(τ_γ -Cl(τ_γ -Int(τ_γ -Cl((A, E))))) = τ_γ -Int(τ_γ - Cl((A, E))) and hence, τ_γ -Cl((A, E)) = τ_γ -Int(τ_γ -Cl((A, E))). Hence τ_γ -Cl((A, E)) is γ-soft open and X is γ-soft extremally disconnected.

Theorem 3.9. For an operation-soft topological space (X, τ, E, γ), the following are equivalent:

(1) X is γ-soft extremally disconnected.
(2) τ_γ -Cl((A, E)) ∩ τ_γ -Cl((B, E)) = ∅ for every disjoint γ-soft open sets (A, E) and B.
(3) τ_γ -Cl((A, E)) ∩τ_γ -Cl((B, E)) ⊂ τ_γ -Cl((A, E) ∩(B, E)) for every disjoint γ-soft open sets (A, E) and B.
(4) τ_γ -Cl(τ_γ -Int(τ_γ -Cl((A, E)))) ∩ τ_γ -Cl((B, E)) = ∅ for every disjoint γ-soft open sets (A, E) and (B, E).

Proof. (2) ⇒ (1): Suppose that τ_γ -Cl((A, E)) ∩ τ_γ -Cl((B, E)) = ∅ for every disjoint γ-soft open sets (A, E) and B. Let U be a γ-soft open subset of X . Since U and X \$\tau_\gamma$ -Cl((U, E)) are disjoint γ-soft open sets, τ_γ -Cl((U, E)) ∩ τ_γ -Cl(X \$\tau_\gamma$ -Cl((U, E))) = ∅. This implies that τ_γ - Cl((U, E)) ⊂ τ_γ -Int(τ_γ -Cl((U, E))). Thus, τ_γ -Cl((U, E)) is γ-soft open; hence X is γ-soft extremally disconnected.
(1) ⇒ (3): Let (A, E) and B be disjoint γ-soft open sets. Since τ_γ -Cl((U, E)) is γ-soft open and B is γ-soft open, τ_γ -Cl((A, E)) ∩ τ_γ - Cl((B, E)) ⊂ τ_γ -Cl(τ_γ -Cl((A, E)) ∩ τ_γ -Cl((B, E))) ⊂ τ_γ -Cl((A, E) ∩ B). (3) ⇒ (2): Let (A, E) and B be disjoint γ-soft open sets. By (3), τ_γ -Cl((A, E)) ∩ τ_γ -Cl((B, E)) ⊂ τ_γ -Cl((A, E) ∩ B) = τ_γ -Cl(∅) = ∅. Therefore, τ_γ -Cl((A, E)) ∩ τ_γ -Cl((B, E)) = ∅.
(2) ⇒ (4): Let (A, E) and B be disjoint γ-soft open sets. Since τ_γ - Int(τ_γ -Cl((A, E))) is γ-soft open and τ_γ -Int(τ_γ -Cl((A, E))) ∩ B = ∅. By (2), τ_γ -Cl(τ_γ -Int(τ_γ -Cl((A, E)))) ∩ τ_γ - Cl((B, E)) = ∅.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

(4) ⇒ (2): Let (A, E) and B be disjoint γ-soft open sets. By (4), τ_γ - Cl(τ_γ -Int(τ_γ -Cl((A, E)))) ∩ τ_γ -Cl((B, E)) = ∅ . Since τ_γ -Cl((A, E)) ⊂ τ_γ -Cl(τ_γ -Int(τ_γ -Cl((A, E)))), then τ_γ - Cl((A, E)) ∩ τ_γ -Cl((B, E)) = ∅ .

Lemma 3.10. If (A, E) is a γ-soft β-open set in an operation-soft topological space (X, τ, E, γ), then τ_γ -Cl((A, E)) = τ_γ -Cl_δ ((A, E)).

Theorem 3.11. For an operation-soft topological space (X, τ, E, γ), the following properties are equivalent:

(1) X is γ-soft extremally disconnected.
(2) τ_γ -Cl((A, E)) ∈ τ_γ for every (A, E) ∈ γ-SSO(X).
(3) τ_γ -Cl((A, E)) ∈ τ_γ for every (A, E) ∈ γ-SPO(X).
(4) τ_γ -Cl((A, E)) ∈ τ_γ for every (A, E) ∈ γ-SRO(X).

Theorem 3.12. For aN operation-soft topological space (X, τ, E, γ), the following properties are equivalent:

(1) X is γ-soft extremally disconnected.
(2) τ_γ -Cl_δ ((A, E)) ∈ τ_γ for every (A, E) ∈ γ-SSO(X).
(3) τ_γ -Cl_δ ((A, E)) ∈ τ_γ for every (A, E) ∈ γ-SPO(X).
(4) τ_γ -Cl_δ ((A, E)) ∈ τ_γ for every (A, E) ∈ γ-SRO(X).

Proof. (1) ⇒ (2): Let X be γ-soft extremally disconnected.Let (A, E) ⊂ X be a γ-soft semiopen. By Lemma 3.10, we have τ_γ - Cl((A, E)) = τ_γ -Cl_δ ((A, E)). Since X is τ_γ - extremally disconnected, by Theorem 3.11 and Lemma 3.10, τ_γ -Cl((A, E)) = τ_γ -Cl_δ ((A, E)) is γ-soft open.
(2) ⇒ (1): Suppose that τ_γ -Cl_δ ((A, E)) ∈ τ_γ for every (A, E) ∈ γ- SO(X). Let (A, E) ⊂ X be γ-soft open set. By Lemma 3.10, τ_γ - Cl((A, E)) = τ_γ -Cl_δ ((A, E)). Thus, τ_γ -Cl((A, E)) is γ-soft open; hence X is γ-soft extremally disconnected.
(1) ⇒ (3): Let (A, E) be γ-soft β-open set. By Theorem 3.8, τ_γ - Cl((A, E)) is γ-soft open and hence by Lemma 3.10 τ_γ -Cl_δ ((A, E)) is γ-soft open.
(3) ⇒ (2): ((3) ⇒ (4):). Let (A, E) be a γ-soft semiopen subset (resp. γ-soft preopen) set of X . Since every γ-soft semiopen (resp. γ-soft preopen) set is γ-soft β-open, by (3) τ_γ - Cl_δ ((A, E)) is γ-soft open.
(2) ⇒ (1): ((4) ⇒ (1):). Let (A, E) be a γ-soft open set of X . Every γ-soft open set is γ-soft semiopen and γ-soft preopen. By (2) (resp (4)), τ_γ -Cl_δ ((A, E)) is γ-soft open and hence, by Lemma 3.10, τ_γ -Cl((A, E)) is γ-soft open. Therefore, X is τ_γ -extremally disconnected.

Lemma 3.13. A subset (A, E) of a operation-soft topological space (X, τ, E, γ) is γ-soft semiopen if and only if τ_γ -Cl((A, E)) = τ_γ -Cl(τ_γ - Int((A, E))).

Proof. Let (A, E) be a γ-soft semiopen set. We have (A, E) ⊂ τ_γ - Cl(τ_γ -Int((A, E))) and hence τ_γ -Cl((A, E)) ⊂ τ_γ -Cl(τ_γ -Int((A, E))). Since τ_γ -Cl(τ_γ -Int((A, E))) ⊂ τ_γ -Cl((A, E)), τ_γ -Cl((A, E)) = τ_γ -Cl(τ_γ - Int((A, E))). Conversely, since τ_γ -Cl((A, E)) = τ_γ -Cl(τ_γ -Int((A, E))), (A, E) ⊂ τ_γ -Cl((A, E)) = τ_γ -Cl(τ_γ -Int((A, E))). Thus, (A, E) is γ-soft semiopen.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

Theorem 3.14. For an operation-soft topological space (X, τ, E, γ), the following properties are equivalent:

(1) X is γ-soft extremally disconnected.
(2) If (A, E) is γ-soft β-open and (B, E) is γ-soft semiopen, then τ_γ -Cl((A, E)) ∩ τ_γ -Cl((B, E)) ⊂ τ_γ -Cl((A, E) ∩ (B, E)).
(3) If (A, E) and (B, E) are γ-soft semiopen sets, then τ_γ -Cl((A, E))∩ τ_γ -Cl((B, E)) ⊂ τ_γ -Cl((A, E) ∩ (B, E)).
(4) τ_γ -Cl((A, E))∩τ_γ -Cl((B, E)) = ∅ for every disjoint γ-soft semiopen sets (A, E) and (B, E).
(5) If (A, E) is γ-soft preopen and (B, E) is γ-soft semiopen, then τ_γ -Cl((A, E)) ∩ τ_γ -Cl((B, E)) ⊂ τ_γ -Cl((A, E) ∩ (B, E)).

Proof. (1) ⇒ (2): Let (A, E) be γ-soft β-open and (B, E) be γ-soft semiopen. By Theorem 3.8, τ_γ -Cl((A, E)) is γ-soft open. We have, τ_γ - Cl((A, E)) ∩ τ_γ -Cl((B, E)) = τ_γ -Cl((A, E)) ∩ Cl(τ_γ -Int((A, E))) ⊂ τ_γ - Cl(τ_γ -Cl((A, E)) ∩τ_γ -Int(B, E)) ⊂ τ_γ -Cl(τ_γ -Cl((A, E)∩τ_γ -Int(B, E)) ⊂ τ_γ -Cl(τ_γ -Cl((A, E)) ∩ τ_γ -Int(B, E)) = Cl((A, E) ∩ τ_γ -Int(B, E)) ⊂ τ_γ -Cl((A, E)∩(B, E)). Thus, τ_γ -Cl((A, E))∩τ_γ -Cl((B, E)) ⊂ τ_γ -Cl((A, E)∩ (B, E)).
(2) ⇒ (3): It follows from the fact that every γ-soft semiopen set γ-soft β-open.
(3) ⇒ (4): Obvious.
(4) ⇒ (1): Let (A, E) be a γ-soft semiopen set. Since (A, E) and X \$\tau_\gamma$ - Cl((A, E)) are disjoint from γ-soft semiopen, respectively, by (4), we have τ_γ -Cl((A, E)) ∩ τ_γ -Cl(X \$\tau_\gamma$ -Cl((A, E))) = ∅ . This implies that τ_γ - Cl((A, E)) ⊂ τ_γ -Int(τ_γ -Cl((A, E))). Thus, τ_γ -Cl((A, E)) is γ-soft open.

Hence, by Theorem 3.11, X is τ_γ -extremally disconnected.
(2) ⇒ (5): It follows from the fact every γ-soft preopen set is γ-soft β-open.
(5) ⇒ (1): Let (A, E) and (B, E) be disjoint γ-soft open sets. Since (A, E) and (B, E) are γ-soft preopen and γ-soft semiopen, respectively, by (5) τ_γ -Cl((A, E)) ∩ τ_γ -Cl((B, E)) ⊂ τ_γ -Cl((A, E) ∩ (B, E)) = ∅ . Thus, τ_γ -Cl((A, E)) ∩ τ_γ -Cl((B, E)) = ∅ . By Theorem 3.9 X is γ-soft extremally disconnected.

Lemma 3.15. If (A, E) is a γ-soft semiopen set in an operation-soft topological space (X, τ, E, γ), then τ_γ -Cl((A, E)) = τ_γ -Cl_δ ((A, E)).

Corollary 3.16. For an operation-soft topological space (X, τ, E, γ), the following properties are equivalent:

(1) X is γ-soft extremally disconnected.
(2) If (A, E) is γ-soft β-open and (B, E) is γ-soft semiopen, then
τ_γ -Cl((A, E)) ∩ τ_γ -Cl_δ (B, E) ⊂ τ_γ -Cl((A, E) ∩ (B, E)).
(3) If (A, E) and (B, E) are γ-soft semiopen sets, then τ_γ -Cl((A, E))∩ τ_γ -Cl_δ (B, E) ⊂ τ_γ -Cl((A, E) ∩ (B, E)).
(4) τ_γ -Cl((A, E))∩τ_γ -Cl_δ (B, E) = ∅ for every disjoint γ-soft semiopen sets (A, E) and (B, E).
(5) If (A, E) and (B, E) are γ-soft preopen, then τ_γ -Cl((A, E)) ∩τ_γ - Cl_δ (B, E) ⊂ τ_γ -Cl((A, E) ∩ (B, E)).

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

Proof. The proof follows from Theorem 3.14 and Lemma 3.15.

Theorem 3.17. For an operation-soft topological space (X, τ, E, γ), the following properties are equivalent:

(1) X is γ-soft submaximal and γ-soft extremally disconnected.

(2) Any subset of X is γ-soft β-open if and only if γ-soft open.

Proof. (1) ⇒ (2): Let X be γ-soft submaximal and γ-soft extremally disconnected. By Theorem 3.8, every γ-soft β-open set is γ-soft pre- open. By Theorem 3.4, every γ-soft preopen set is γ-soft open. Thus, every γ-soft β-open set is γ-soft open. The converse follows from the fact that every γ-soft open set is γ-soft β-open.
(2) ⇒ (1): Suppose that any subset of X is γ-soft β-open if and only if it is γ-soft open. Since every γ-soft β-open set is γ-soft open and so γ-soft preopen, by Theorem 3.8, X is γ-soft extremally disconnected. Since every γ-soft preopen set is γ-soft open, by Theorem 3.4 X is τ_γ -submaximal.

Corollary 3.18. For a γ-soft submaximal and γ-soft extremally dis- connected space (X, τ, E, γ), the following properties are equivalent:

(1) (A, E) is γ-soft β-open,

(2) (A, E) is γ-soft semiopen,

(3) (A, E) is γ-soft preopen,

(4) (A, E) is γ-soft α-open,

(5) (A, E) is γ-soft open.

Proof. The proof follows from Theorem 3.17.

Theorem 3.19. For a operation-soft topological space (X, τ, E, γ), the following properties are equivalent:

(1) X is γ-soft extremally disconnected.

(2) γ-soft regular open sets coincide with γ-soft regular closed sets.

Proof. (1) ⇒ (2): Suppose (A, E) is a γ-soft regular open subset of X . Since γ-soft regular open sets are γ-soft open, by (1), A = τ_γ -Cl((A, E)) = τ_γ -Cl(τ_γ -Int((A, E))) and so (A, E) is γ-soft regu- lar closed. If (A, E) is γ-soft regular closed, then A = τ_γ -Cl(τ_γ - Int((A, E))) = τ_γ -Int(τ_γ -Cl(τ_γ -Int((A, E)))) = τ_γ -Int((A, E)) so (A, E) is γ-soft open. Also, A = τ_γ -Cl(τ_γ -Int((A, E))) = τ_γ -Int(τ_γ -Cl(τ_γ - Int((A, E)))) = τ_γ -Int(τ_γ -Cl((A, E))). Hence (A, E) is γ-soft regular open.
(2) ⇒ (1): Let (A, E) be a γ-soft open subset of X . Then τ_γ -Int(τ_γ - Cl((A, E))) is γ-soft regular open and so it is γ-soft regular closed, by (ii). Hence τ_γ -Int(τ_γ -Cl(τ_γ -Int(τ_γ -Cl(((A, E)))))) = τ_γ -Int(τ_γ -Cl((A, E))) which implies that τ_γ -Cl(τ_γ -Int(τ_γ -Cl((A, E)))) = τ_γ -Int(τ_γ -Cl((A, E))). Therefore, τ_γ -Cl((A, E)) = τ_γ -Cl(τ_γ -Int((A, E))) ⊂ τ_γ -Cl(τ_γ -Int(τ_γ -Cl((A, E)))) = τ_γ -Int(τ_γ -Cl((A, E))) and so τ_γ -Cl((A, E)) = τ_γ -Cl(τ_γ - Int((A, E))). Hence τ_γ -Cl((A, E)) is γ-soft open. This shows that X is γ-soft extremally disconnected.

Theorem 3.20. For an operation-soft topological space (X, τ, E, γ), the following properties are equivalent:

(1) X is γ-soft extremally disconnected.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

(2) Every γ-soft regular closed sets is γ-soft preopen.

Proof. (1) ⇒ (2): The proof follows from the fact that every γ-soft regular open set is γ-soft preopen set.

(2) ⇒ (1): If (A, E) is γ-soft open, then τ_γ-Cl(τ_γ-Int((A, E))) is γ-soft regular closed and so it is γ-soft preopen. Therefore, τ_γ-Cl((A, E)) = τ_γ-Cl(τ_γ-Int((A, E))) ⊂ τ_γ-Int(τ_γ-Cl(τ_γ-Cl(τ_γ-Int((A, E))))) = τ_γ-Int(τ_γ- Cl(τ_γ-Int((A, E)))) = τ_γ-Int(τ_γ-Cl((A, E))). Thus, τ_γ-Cl((A, E)) = τ_γ-Int(τ_γ-Cl((A, E))) which implies that τ_γ-Cl((A, E)) is γ-soft open. Hence X is γ-soft extremally disconnected.

References

[1] Abdulkadir Aygunoglu, Halis Aygun: Some notes on soft topological spaces. Neural Comput and Applic. DOI: 10.1007/s00521-011-0722-3.

[2] J. Biswas and A.R. Prasannan, An introduction to weaker and stronger form of soft open sets by γ-operation, Inter. J. Pure and Appl. Math., 116 (2) 2017, 285-298.

[3] Bozena Kostek: Soft set approach to the subjective assessment of sound qual- ity. in: IEEE Conferences. 1 (1998), 669-674.

[4] D. Chen, E.C.C. Tsang, D.S. Yeung: Some notes on the parameterization reduction of soft sets, in: International Conference on Machine Learning and Cybernetics, vol. 3, 2003,pp. 1442-1445.

[5] D. Chen, E.C.C. Tsang, D.S. Yeung, X. Wang: The parameterization reduction of soft sets and its applications. Computers and Mathematics with Applica- tions. 49 (2005), 757-763.

[6] Feng Feng, Young Bae Jun, Xianzhong Zhao: soft semirings. Computers and Mathematics with Applications. 56 (2008), 2621-2628.

[7] P. Gomathi sundari, N. Rajesh and B. Jaya Bharathi, Soft semiopen sets via operations (submitted).

[8] P. Gomathi sundari, N. Rajesh and B. Jaya Bharathi, Some weak forms of soft open sets via operations (under preparation).

[9] Z. Kong, L. Gao, L. Wang, S. Li: The normal parameter reduction of soft sets and its algorithm, Computers and Mathematics with Applications 56 (2008), 3029-3037.

[10] E. F. Lashin, A. M. Kozae, A. A. Abo Khadra and T. Medhat: Rough set for topological spaces, Internat. J. Approx. Reason. 40 (2005), 35-43.

[11] D. Molodtsov: Soft set theory-first results. Computers and Mathematics with Applications. 37 (1999), 19-31.

Annals of R.S.C.B., ISSN:1583-6258, Vol. 25, Issue 4, 2021, Pages. 1405 - 1416
Received 05 March 2021; Accepted 01 April 2021.

[12] P.K. Maji, R. Biswas, A.R. Roy: Soft set theorys. Computers and Mathematics with Applications. 45 (2003), 555-562.

[13] P.K. Maji, A.R. Roy: An application of soft sets in a decision making problem.
Computers and Mathematics with Applications. 44 (2002), 1077-1083.

[14] Milind M. Mushrif, S. Sengupta, A.K. Ray: Texture Classification Using a Novel, Soft Set Theory Based Classification Algorithm, Springer, Berlin, Hei- delberg. (2006), 246-254.

[15] D. Pei, D. Miao: From soft sets to information systems, in: X. Hu, Q. Liu, A. Skowron, T.Y. Lin, R.R. Yager, B. Zhang (Eds.), Proceedings of Granular Computing, vol. 2, IEEE, 2005, pp. 617-621.

[16] S. Hussain and B. Ahmad: Some properties of soft topological spaces. Com- puters and Mathematics with Applications. 62 (2011), 4058-4067.

[17] M. Shabir, M. Naz: On soft topological spaces. Computers and Mathematics with Applications. 61 (2011), 1786-1799.

[18] Z. Xiao, L. Chen, B. Zhong, S. Ye: Recognition for soft information based on the theory of soft sets, in: J. Chen (Ed.), Proceedings of ICSSSM-05, vol. 2, IEEE, 2005, pp. 1104-1106.

[19] I. Zorlutuna, M. Akdag, W. K. Min, S. Atmaca: Remarks on soft topological spaces. To appear in Annals of Fuzzy Mathematics and Informatics.