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λ-Ideally Statistical Convergence in n-Normed Spaces Over Non-Archimedean Field

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Abstract: This article extend the concept of Ideally Statistical Convergence in n-Normed space to λ-Ideally statistical convergence in n-Normed spaces over a complete, non-trivially valued, locally compact non-Archimedean field $\mathcal{K}$.

Key words: Statistical Convergence, Statistically Cauchy sequence, n-normed space

1. **Introduction:** This paper stems out from the concept of statistical convergence was introduced by Steinhaus in 1951. In the year of 1985 'Fridy .J.A', "On Statistical Convergence", [2] and in '2000' "λ-statistical convergence" by Mursaleen.M[6]. Later Reddy.B.S discussed "Statistical convergence in n-normed spaces" in 2010 [7]. In 2008 Sahiner A , G¨urdal M introduced "Ideal convergence in n -normed spaces and some new sequence spaces via n-norm" [8] also in 2014 "Yamanci U, Gurdal M. I-statistical convergence in 2-normed space"[10].

Throughout this article $\mathcal{K}$ denotes a complete, non-trivially valued, locally compact non-Archimedean field, ℕ denotes the set of all Natural numbers, $\mathfrak{I}$ be an admissible ideal.

Let$\lambda = \{\lambda_n\}$ be a positive integers tending to ∞, it is a non-decreasing sequence.
Such that $\lambda_{n+1} \leq \lambda_n + 1,\ \lambda_1 = 1$, Where $n \in \mathbb{N}$
Where $I_n = [\mathrm{n} - \lambda_n + 1, n]$

2. Preliminaries

Definition 2.1

Let a Sequence $x = \{x_k\}$ is Statistically Convergent to 'ℓ', if for any$\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n; n \in \mathbb{N}: \|x_k - \ell\| \geq \varepsilon\}| = 0$$

we write,

$stat. - lim_{k\to\infty}\, x_k = \ell$ (or) $x_k \xrightarrow{stat.} \ell$

Definition 2.2

Let a Sequence$x = \{x_k\}$ Statistically Cauchy Sequence if for any $\varepsilon > 0$, there exist an $n \in \mathbb{N}$,

Such that,

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$$\lim_{n\to\infty}\frac{1}{n}|\{k \le n; n \in \mathbb{N}: \left\|x_{k+1} - x_k\right\| \ge \varepsilon\}| = 0$$

Definition 2.3

A non-empty subset $\Im$ of a ring $\mathcal{R}$ of subsets of $\mathbb{N} \subset x$ is an ideal $\Im$ in $\mathcal{R}$ if and only if,

(i) $A, B \in \Im$ $which\ implies$ $A \cup B \in \Im$

(ii) $A \in \Im, B \in \mathcal{R}, B \subset A$ $which\ implies$ $B \in \Im$

$\Im$ is said to be non-trivial if $\Im \neq \emptyset$ and $\mathbb{N} \in \Im$, while a non-trivial ideal $\Im$ in $\mathbb{N}$ is said to be admissible if $\{x\} \in \Im$ for every $x \in \mathbb{N}$, where $\mathbb{N}$ denotes the set of all positive integers.

A non-trivial ideal $\Im$ in $\mathbb{N} \times \mathbb{N}$ is admissible if $\{\, n \,\} \times \mathbb{N}and\mathbb{N} \times \{\, n \,\}$ belongs to $\Im$ for every$n \in \mathbb{N}$.

Definition 2.4

'$\mathcal{F}$'is a non-empty class of subsets of x is said to be a filter in x.

provided:

(i) $\emptyset \epsilon \mathcal{F}$,

(ii) $A, B \ \in \mathcal{F}$ $which\ implies$ $A \cap B \ \in \mathcal{F}$

(iii)$A \ \in \ \mathcal{F}, A \subset B$ $which\ implies$ $B \ \in \mathcal{F}$

Definition 2.5

If $\Im \subset 2^x$ is a non-trivial ideal in X,

$X \neq \emptyset$.Then

$\mathcal{F}(\Im) = \{M \subset X: (M = X - A\ and\ A) \ \in \Im\,\}$ is a filter on X and conversely.

3. n-Normed space over a non-Archimedean field $\mathcal{K}$

Definition 3.1

"Let X be a vector space with the dimension which is greater than $n-1$ over a non-Archimedean Valuation $|.|$with a valued field k.

A function $\|.,\ldots,.\| : X \times \ldots \times X \to [0,\infty)$ is called a non-Archimedean n-norm if

(i) $\| x_1, \ldots, x_n \| = 0$ $if\ and\ only\ if$ $x_1, \ldots, x_n$ are linearly dependent;

(ii) $\| x_1, \ldots, x_n \| = \| x_{j_1}, \ldots, x_{j_n} \|$ for every permutation $(j_1, \ldots, j_n)$ of $(1, \ldots, n)$;

(iii)$\| \alpha x_1, \ldots, x_n \| = |\alpha| \ \| x_1, \ldots, x_n \|$ for all $\alpha \in k$;

(iv)$\| x + x', x_2, \ldots, x_n \| \le max\{\| x, x_2, \ldots, x_n \|, \| x', x_2, \ldots, x_n \|\}$;

For all $x, x', x_1, \ldots, x_n \ \in X$

Then $(X, \|.,\ldots,.\|)$ is called a non-Archimedean n-normed space."

4. λ-$\Im$ Statistically Convergent and λ- $\Im$ Statistically Cauchy sequence in n-normed Space over non-Archimedean field $\mathcal{K}$

"Let $\lambda = \{\lambda_n\}$ be a non-decreasing sequence of positive integers tending to ∞.

Such that $\lambda_{n+1} \le \lambda_n + 1,\ \lambda_1 = 1$, Where $n \in \mathbb{N}$"

Definition 4.1

Let $\Im \subset 2^x$ is a non-trivial ideal in X, $X \neq \emptyset$, $X \in \mathbb{N}$.

A sequence $\{x_k\}$ of X is called λ- $\Im$-Statistically Convergent to 'ℓ' in n-Normed space over non-Archimedean field, if for every $\varepsilon > 0$,

Such that for all $Z_i \in X, i = 2,3, \ldots, n$,

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$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|x_k-\ell, z_2, z_3, \dots, z_n\right\|\geq\varepsilon\}\in\Im\right\|=0$$

Where $I_n=[\mathrm{n}-\lambda_n+1, n]$

We write,

$$\Im-stat_\lambda \lim_{k\to\infty}\left\|x_k-\ell, z_2, z_3, \dots, z_n\right\|=0$$

(or) $\Im-stat_\lambda\, lim_{k\to\infty}\left\|x_k, z_2, z_3, \dots, z_n\right\|-\left\|\ell, z_2, z_3, \dots, z_n\right\|=0$

(or) $\Im-stat_\lambda\, lim_{k\to\infty}\left\|x_k, z_2, z_3, \dots, z_n\right\|=\left\|\ell, z_2, z_3, \dots, z_n\right\|$

Where 'ℓ' is the $\Im$-limit of the Sequence $\{x_k\}$.

Definition 4.2

Let $\Im\subset 2^x$ is a non-trivial ideal in $X, X\neq\emptyset, X\in\mathbb{N}$.

A sequence $\{x_k\}$ of X is calledλ- $\Im$-Statistically Cauchy Sequence in n-Normed space over non-Archimedean field, if for every $\varepsilon>0$, then there exists an $n\in\mathbb{N}$ and all non-zero $Z_i\in X, i=2,3,\dots,n,$

Such that,

$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|x_{k+1}-x_k, z_2, z_3, \dots, z_n\right\|\geq\varepsilon\}\in\Im\right\|=0$$

5. Results

Theorem 5.1

Let $\{x_k\}$ be a sequence in n-normed space $(X, \|.,\dots,.\|)$, an admissible ideal$\Im$,

$\ell, \ell'\in X$, for every$Z_i\in X$, if

$$\Im-stat_\lambda \lim_{k\to\infty}\left\|x_k-\ell, z_2, z_3, \dots, z_n\right\|=0$$

And $\Im-stat_\lambda\, lim_{k\to\infty}\left\|x_k-\ell', z_2, z_3, \dots, z_n\right\|=0$

Then $\ell-\ell'=0$

Proof:

Let $\ell=\ell'$(ie) $\ell-\ell'=0$

There exist a non-zero $z_2, z_3, \dots, z_n\in X,$

In such a way that$\ell-\ell'$ and $z_2, z_3, \dots, z_n$ are not linearly dependent.

(Soz_i exist as dimension of $X, d\leq n$)

Therefore every $\varepsilon>0,$

$lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|x_k-\ell, z_2, z_3, \dots, z_n\right\|\geq\varepsilon\}\in\Im\right\|=0$ ---------- (1)

$lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|x_k-\ell', z_2, z_3, \dots, z_n\right\|\geq\varepsilon\}\in\Im\right\|=0$ ---------- (2)

Now,

$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|\ell-\ell', z_2, z_3, \dots, z_n\right\|\geq\varepsilon\}\in\Im\right\|$$

$$=\lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|\ell-x_k+x_k-\ell', z_2, z_3, \dots, z_n\right\|\geq\varepsilon\}\in\Im\right\|$$

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$$= \lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|(\ell - x_k) + (x_k - \ell'), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\} \in \Im\right\|$$

$$\leq max\begin{Bmatrix}\lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|x_k - \ell, z_2, z_3, \dots, z_n\right\| \geq \varepsilon\} \in \Im\right\|, \\ \lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|x_k - \ell', z_2, z_3, \dots, z_n\right\| \geq \varepsilon\} \in \Im\right\|\end{Bmatrix}$$

$= 0$ (Using (1) & (2))

Thus $\ell - \ell' = 0$ (ie) $\ell = \ell'$

This completes the proof.

Theorem 5.2

Let an admissible ideal be$\Im$ for each $Z_i \in X$,for every $i = 2,3,\dots,n$,

(i) If$\Im - stat_\lambda\ lim_{k\to\infty}\left\|x_k, z_2, z_3, \dots, z_n\right\| = \left\|x, z_2, z_3, \dots, z_n\right\|$

and $\Im - stat_\lambda\ lim_{k\to\infty}\left\|y_k, z_2, z_3, \dots, z_n\right\| = \left\|y, z_2, z_3, \dots, z_n\right\|$

Then $\Im - stat_\lambda\ lim_{k\to\infty}\left\|x_k + y_k, z_2, z_3, \dots, z_n\right\| = \left\|x + y, z_2, z_3, \dots, z_n\right\|$

(ii) If $\Im - stat_\lambda\ lim_{k\to\infty}\left\|\alpha x_k, z_2, z_3, \dots, z_n\right\| = \left\|\alpha x, z_2, z_3, \dots, z_n\right\|$ for all $\alpha \in k$

Proof:

(i) Let $\Im - stat_\lambda\ lim_{k\to\infty}\left\|x_k, z_2, z_3, \dots, z_n\right\| = \left\|x, z_2, z_3, \dots, z_n\right\|$

and $\Im - stat_\lambda\ lim_{k\to\infty}\left\|y_k, z_2, z_3, \dots, z_n\right\| = \left\|y, z_2, z_3, \dots, z_n\right\|$

for every non-zero $Z_i \in X$,

Then $A_1 = 0$ and $A_2 = 0$

Where,

$$A_1 = A_1(\varepsilon) = \lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|x_k - x, z_2, z_3, \dots, z_n\right\| \geq \varepsilon\} \in \Im\right\|$$

$$A_2 = A_2(\varepsilon) = \lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|y_k - y, z_2, z_3, \dots, z_n\right\| \geq \varepsilon\} \in \Im\right\|$$

for every $Z_i \in X$,

Let,

$$A(\varepsilon) = \lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\{k\in I_n; n\in\mathbb{N}: \left\|(x_k + y_k) - (x + y), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\} \in \Im\right\|$$

To Prove that, $A = 0$ it is sufficient to show that $A \subset A_1 \cup A_2$

Let $A_0 \in A$

Then

$lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\left\{k\in I_n; n\in\mathbb{N}: \left\|(x_{k_0} + y_{k_0}) - (x + y), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\right\} \in \Im\right\| = 0$----
(3)

Assume that $A_0 \in A_1 \cup A_2$

Then $A_0 \in A_1\ and\ A_0 \in A_2$

This implies,

$lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\left\{k\in I_n; n\in\mathbb{N}: \left\|(x_{k_0} - x), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\right\} \in \Im\right\| = 0$ ------------(4)

$lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\left\{k\in I_n; n\in\mathbb{N}: \left\|(y_{k_0} - y), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\right\} \in \Im\right\| = 0$------------(5)

Then we get,

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$$\lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\left\{k\in I_n; n\in\mathbb{N}: \left\|(x_{k_0}+y_{k_0})-(x+y), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\right\}\in\mathfrak{I}\right\|$$
$$\leq \max\begin{Bmatrix}\lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\left\{k\in I_n; n\in\mathbb{N}: \left\|(x_{k_0}-x), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\right\}\in\mathfrak{I}\right\|, \\ \lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\left\{k\in I_n; n\in\mathbb{N}: \left\|(y_{k_0}-y), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\right\}\in\mathfrak{I}\right\|\end{Bmatrix}$$
$= 0$ (Using (4) & (5))

Hence (3) is True.

Therefore, $A_0 \in A_1 \cup A_2$ that is $A \subset A_1 \cup A_2$

This completes the proof.

(ii) $\mathfrak{I} - stat_\lambda\, lim_{k\to\infty} \left\|x_k, z_2, z_3, \dots, z_n\right\| = \left\|x, z_2, z_3, \dots, z_n\right\|$ $\alpha \in k$ and $\alpha \neq 0$

Then

$lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\left\{k\in I_n; n\in\mathbb{N}: \left\|(x_k-x), z_2, z_3, \dots, z_n\right\| \geq \frac{\varepsilon}{|\alpha|}\right\}\in\mathfrak{I}\right\|$ ------------- (6)

Now, we shall show that

$$\mathfrak{I} - stat_\lambda \lim_{k\to\infty}\left\|\alpha x_k, z_2, z_3, \dots, z_n\right\| = \left\|\alpha x, z_2, z_3, \dots, z_n\right\| \ \alpha \in k$$

This is to prove that,

$$\mathfrak{I} - stat_\lambda \lim_{k\to\infty}\left\|\alpha x_k - \alpha x, z_2, z_3, \dots, z_n\right\| = 0$$

This implies,

$$\lim_{n\to\infty}\frac{1}{\lambda_n}\|\{k\in I_n; n\in\mathbb{N}: \left\|\alpha x_k-\alpha x, z_2, z_3, \dots, z_n\right\| \geq \varepsilon\}\in\mathfrak{I}\| = 0$$
$$\lim_{n\to\infty}\frac{1}{\lambda_n}\|\{k\in I_n; n\in\mathbb{N}: \left\|\alpha(x_k-x), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\}\in\mathfrak{I}\|$$
$$= \lim_{n\to\infty}\frac{1}{\lambda_n}\|\{k\in I_n; n\in\mathbb{N}: \left\||\alpha|(x_k-x), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\}\in\mathfrak{I}\|$$
$$= \lim_{n\to\infty}\frac{1}{\lambda_n}\left\|\left\{k\in I_n; n\in\mathbb{N}: \left\|(x_k-x), z_2, z_3, \dots, z_n\right\| \geq \frac{\varepsilon}{|\alpha|}\right\}\in\mathfrak{I}\right\|$$

Now, Using (4)

$\mathfrak{I} - stat_\lambda\, lim_{k\to\infty}\left\|\alpha x_k - \alpha x, z_2, z_3, \dots, z_n\right\| = 0$ for every $\alpha \in k$

This completes the proof.

We suppose$X$ to give dimension d,

where $2 \leq n \leq d < \infty$.

Let, $u = \{u_1, u_2, \dots, u_{(n-1)}\}$ to be a basis for X.

Theorem 5.3

Let an admissible ideal be $\mathfrak{I}$. The sequence $\{x_k\} \in X\lambda$-$\mathfrak{I}$-Statistically Convergent to $x \in X$ iff$\mathfrak{I} - stat_\lambda\, lim_{k\to\infty}\left\|x_k - x, \underbrace{u_i, u_i, \dots, u_i}_{(n-1)times}\right\| = 0$ for every $i = 1,2,\dots,(n-1)$.

Proof:

Let $\{x_k\} \in X$ is λ-$\mathfrak{I}$-Statistically Convergent to $x \in X$,

Then by the definition of λ-$\mathfrak{I}$-Statistically Convergent,

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We have,

$$\mathfrak{I} - stat_\lambda \lim_{k\to\infty} \left\| x_k - x, z_2, z_3, \dots, z_n \right\| = 0$$

Then

$$\mathfrak{I} - stat_\lambda \lim_{k\to\infty} \left\| x_k - x, \underbrace{u_i, u_i, \dots, u_i}_{(n-1)times} \right\| = 0$$

For every $i = 1,2, \dots, (n-1)$, is trivial for all Z_i can be expressed as a linear combination of u_i for $i = 1,2, \dots, (n-1)$

Assume,

$\mathfrak{I} - stat_\lambda\, lim_{k\to\infty} \left\| x_k - x, u_i, u_i, \dots, u_i \right\| = 0$ ----------------(7)

for $i = 1,2, \dots, (n-1)$

To show that

$lim_{n\to\infty} \frac{1}{\lambda_n} \|\{k \in I_n; n \in \mathbb{N}: \left\|(x_k - x), z_2, z_3, \dots, z_n\right\| \geq \varepsilon\} \in \mathfrak{I}\| = 0$ ----------------(8)

Now, Let us consider the n-norm

$$\left\|(x_k - x), z_2, z_3, \dots, z_n\right\|$$

Also $u = \{u_1, u_2, \dots, u_{(n-1)}\}$ is a basis of X,

Then we have,

$$Z_2 = \sum_{i=1}^{n-1} \alpha_i^2\, u_i, Z_3 = \sum_{i=1}^{n-1} \alpha_i^3\, u_i, \dots, Z_n = \sum_{i=1}^{n-1} \alpha_i^n\, u_i$$

This implies,

$$\|(x_k - x), z_2, z_3, \dots, z_n\| = \left\| x_k - x, \sum_{i=1}^{n-1} \alpha_i^2\, u_i, \sum_{i=1}^{n-1} \alpha_i^3\, u_i, \dots, \sum_{i=1}^{n-1} \alpha_i^n\, u_i \right\|$$

Where 'n' is any non-negative integer,

Then we have,

$$\|(x_k - x), z_2, z_3, \dots, z_n\| = \left\| n(x_k - x), \sum_{i=1}^{n-1} \alpha_i^2\, u_i, \sum_{i=1}^{n-1} \alpha_i^3\, u_i, \dots, \sum_{i=1}^{n-1} \alpha_i^n\, u_i \right\|$$

By using ultrametric inequality and distribution components for each Z_i over n-norm we get,

$$\|(x_k - x), z_2, z_3, \dots, z_n\|$$
$$\leq max \left\{ \begin{array}{c} \left\|(x_k - x), \underbrace{\alpha_1^2 u_1, \alpha_1^3 u_1, \dots, \alpha_1^n u_1}_{(n-1)times}\right\| \\ + \left\|(x_k - x), \underbrace{\alpha_2^2 u_2, \alpha_2^3 u_2, \dots, \alpha_2^n u_2}_{(n-1)times}\right\| \\ + \cdots + \left\|(x_k - x), \underbrace{\alpha_{(n-1)}^2 u_{(n-1)}, \alpha_{(n-1)}^3 u_{(n-1)}, \dots, \alpha_{(n-1)}^n u_{(n-1)}}_{(n-1)times}\right\| \end{array} \right\}$$

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Let $max\ \alpha_1^i = \alpha_1$, for all $i = 2,3, \dots, n$
Similarly,
Let $max\ \alpha_{(n-1)}^i = \alpha_{(n-1)}$ for all $i = 2,3, \dots, n$
Substituting these values in above equation, we get

$$\|(x_k - x), z_2, z_3, \dots, z_n\| \le max \left\{ \begin{array}{c} \|(x_k - x), \alpha_1 u_1, \alpha_1 u_1, \dots, \alpha_1 u_1\| \\ +\|(x_k - x), \alpha_2 u_2, \alpha_2 u_2, \dots, \alpha_2 u_2\| \\ + \dots + \left\|(x_k - x), \alpha_{(n-1)} u_{(n-1)}, \alpha_{(n-1)} u_{(n-1)}, \dots, \alpha_{(n-1)} u_{(n-1)}\right\| \end{array} \right\}$$

$$\le max \left\{ \begin{array}{c} |\alpha_1|^{n-1} \|(x_k - x), u_1, u_1, \dots, u_1\| \\ +|\alpha_2|^{n-1} \|(x_k - x), u_2, u_2, \dots, u_2\| \\ + \dots + \left|\alpha_{(n-1)}\right|^{n-1} \left\|(x_k - x), u_{(n-1)}, u_{(n-1)}, \dots, u_{(n-1)}\right\| \end{array} \right\}$$

By our assumption (1), we have

$$\Im - stat_\lambda \lim_{k\to\infty} \left\|x_k - x, u_i, u_i, \dots, u_i\right\| = 0$$

For every $i = 2,3, \dots, (n-1)$
Which implies,

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \left\| \left\{ k \in I_n; n \in \mathbb{N}: \left\{ \begin{array}{c} |\alpha_1|^{n-1} \|(x_k - x), u_1, u_1, \dots, u_1\| + \\ |\alpha_2|^{n-1} \|(x_k - x), u_2, u_2, \dots, u_2\| + \dots + \\ \left|\alpha_{(n-1)}\right|^{n-1} \left\|(x_k - x), u_{(n-1)}, u_{(n-1)}, \dots, u_{(n-1)}\right\| \end{array} \right\} \ge \varepsilon^{(n-1)} \right\} \in \Im \right\|$$

Therefore we have,

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \|\{k \in I_n; n \in \mathbb{N}: \|(x_k - x), z_2, z_3, \dots, z_n\| \ge \varepsilon\} \in \Im\|$$

$$\subset \lim_{n\to\infty} \frac{1}{\lambda_n} \left\| \left\{ k \in I_n; n \in \mathbb{N}: \{(x_k - x), u_1, u_1, \dots, u_1\} \ge \frac{\varepsilon^{(n-1)}}{|\alpha_1|^{(n-1)}} \right\} \in \Im \right\|$$

$$\cup \lim_{n\to\infty} \frac{1}{\lambda_n} \left\| \left\{ k \in I_n; n \in \mathbb{N}: \{(x_k - x), u_2, u_2, \dots, u_2\} \ge \frac{\varepsilon^{(n-1)}}{|\alpha_2|^{(n-1)}} \right\} \in \Im \right\|$$

$$\cup \dots \cup \lim_{n\to\infty} \frac{1}{\lambda_n} \left\| \left\{ k \in I_n; n \in \mathbb{N}: \left\{(x_k - x), u_{(n-1)}, u_{(n-1)}, \dots, u_{(n-1)}\right\} \ge \frac{\varepsilon^{(n-1)}}{|\alpha_2|^{(n-1)}} \right\} \in \Im \right\|$$

Which gives,

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \|\{k \in I_n; n \in \mathbb{N}: \|(x_k - x), z_2, z_3, \dots, z_n\| \ge \varepsilon\} \in \Im\|$$

$$\subset \lim_{n\to\infty} \frac{1}{\lambda_n} \left\| \left\{ k \in I_n; n \in \mathbb{N}: \{(x_k - x), u_1, u_1, \dots, u_1\} \ge \frac{\varepsilon}{|\alpha_1|} \right\} \in \Im \right\|$$

$$\cup \lim_{n\to\infty} \frac{1}{\lambda_n} \left\| \left\{ k \in I_n; n \in \mathbb{N}: \{(x_k - x), u_2, u_2, \dots, u_2\} \ge \frac{\varepsilon}{|\alpha_2|} \right\} \in \Im \right\|$$

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$$\cup \ldots \cup \lim_{n\to\infty} \frac{1}{\lambda_n} \left\| \left\{ k \in I_n; n \in \mathbb{N}: \left\{ (x_k - x), u_{(n-1)}, u_{(n-1)}, \ldots, u_{(n-1)} \right\} \geq \frac{\varepsilon}{\left| \alpha_{(n-1)} \right|} \right\} \in \Im \right\|$$

Hence the right hand side be a member of ideal, Thus left hand side.
Then$\Im - stat_\lambda \, lim_{k\to\infty} \left\| x_k - x, z_2, z_3, \ldots, z_n \right\| = 0$, for every non-zero $Z_i \in X$.
This Completes the proof.

Let us now determine a norm on X, Denoted by $\|x\|_\infty$, with respect to basis
$u = u_1, \ldots, u_d$,

$$\|x\|_\infty = max \left\{ \| \, x, \underbrace{u_i, u_i, \ldots, u_i}_{(n-1)} \, \| : i = 1,2, \ldots, d \right\}$$

Theorem 5.4

Anyλ-$\Im$-Statistically Cauchy sequence $\{x_k\}$ in n-normed space $(X, \|., \ldots, .\|)$ is λ-$\Im$-Statistical Convergence iff if any λ-$\Im$-Statistical Cauchy sequence is λ-$\Im$-Statistical Convergence with respect to $\|.\|_\infty$.

Proof:

Clearly λ-$\Im$-Statistical convergence in n-norm is equivalent to that in $\|.\|_\infty$.
For all $z_i \in X, i = 2,3, \ldots, n$

$$\Im - stat_\lambda \lim_{k\to\infty} \left\| x_k - x, z_2, z_3, \ldots, z_n \right\| = 0$$

If and only if $\Im - stat_\lambda \, lim_{k\to\infty} \left\| x_k - x \right\|_\infty = 0$

It suffices to prove that $\{x_k\}$ is λ-$\Im$-Statistical Cauchy sequence, with respect to n-Norm iff it is λ-$\Im$-Statistically Cauchy sequence with respect to $\|.\|_\infty$.
Let $\{x_k\}$ is λ-$\Im$-Statistical Cauchy sequence with respect to n-Norm.
Then there exists $N \in \mathbb{N}$,
such that for every $k, m \geq N$
Now we have,

$$\lim_{n\to\infty} \frac{1}{\lambda_n} \|\{k \in I_n; n \in \mathbb{N}: \left\| x_k - x_m, z_2, z_3, \ldots, z_n \right\| \geq \varepsilon\} \in \Im\|$$

Consider,
$\left\| x_k - x_m, z_2, z_3, \ldots, z_n \right\| \geq \varepsilon$,
Now, we have $\left\| x_k - x_m, u_i, u_i, \ldots, u_i \right\| \geq \varepsilon$ for all $i = 1,2, \ldots, n$.
Hence, $max \left\| x_k - x_m, u_i, u_i, \ldots, u_i \right\| \geq \varepsilon$ for all $i = 1,2, \ldots, n$.
By definition,

$$\|x_k - x_m\|_\infty \geq \varepsilon$$

Therefore, $\{x_k\}\lambda$-$\Im$-Statistically Cauchy, with respect to $\|.\|_\infty$.
This completes the proof.

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