Pythagorean Fuzzy Transportation Problem via Monalisha Technique.

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Abstract

Pythagorean set theory is renowned procedure to deal with ambiguity in the optimization problems. The Pythagorean fuzzy sets (PFS) are more comprehensive than IFS because it provides a novel technique for modeling uncertainty and vagueness. PFSs have meaningful applications in many different fields. In this work we have applied Monalisha approximation method to solve a Pythagorean fuzzy transportation problem. Apart from normal procedures for defuzzification, score function is used to convert the PFS, which gives a better optimal solution. An algorithm for solving the proposed method is explained with a numerical example. Currently existing methods like North West corner (NWC), Least Cost (LCM), and Heuristics method (HM) are compared with the proposed method and the results are reveled.

Keywords

Pythagorean Fuzzy, Pythagorean Fuzzy Transportation Problem, Score Function, Monalisha Approximation, Accuracy Function, IFS, Defuzzification.

1. Introduction

Transportation problem guarantee the proficient evolution and prudent accessibility of raw equipments and completed goods. A well-built accord to meet up the confront of how to provide the merchandise to the customers in more adept approaches is accomplished with the aid of transportation models. Transportation algorithm is one of the influential structures to provide the merchandise to the purchaser in proficient comportment. Transportation problems pact with the hauling of a single/multi artifact feigned at different plants (origins) to number of various storehouses (destinations). The core goals of Transportation problems gratify the claim at targets from the supply restraints at the least amount probable transportation cost. Hitchcock [8] in 1941 developed the essential transportation problem. Stepping stone technique which offered a different way of shaping the simplex method was suggested by Charnes et al [4] in 1953.

Dantzig [5] in 1963 urbanized the primal simplex transportation scheme. Ample researchers premeditated generally to decipher cost minimizing transportation problem in various schemes. Ringuset J.L.Rinks, D.B [18] in 1987 presented the interactive solutions for the linear multi objective transportation problem. In ongoing relevance's, all the constraints of the transportation problems may not be known exclusively owing to insurmountable features. This sort of blurred information is not constantly well symbolized by random variable preferred from a probability distribution. To designate this data, fuzzy numbers are initiated by Zadeh [24] in 1965. Zimmermann [27] in 1978 intended that the elucidations acquired by fuzzy linear programming are constantly proficient. If the sum of the member-ship function and non-membership function is greater than one then there arises a decisive situation, in order to trounce this case, Yager [22, 23] in 2013, 2014 established an additional category of non-standard fuzzy subset called Pythagorean fuzzy set (PFS).PFS are the sets

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wherethesquaresumofthemembership and the non-

membershipdegreessumisequaltoorlessthanone. The PFS are more comprehensive than IFS; which provide a new method for modeling uncertainty and vagueness. PFSs have meaningful applications in many different fields. For example, extension of TOPSIS to multiple criteria decision making. Recent literature suggests that Pythagorean fuzzy sets (PFS) can offer a better alternative, particularly when fuzzy sets have some extent of limitations in handling vagueness and uncertainty.

Chanas and Kuchta [2] in 1992 proposed the perception of optimal solution for the transportation problem with fuzzy coefficients which are expressed as fuzzy numbers. Narayanamoorthy, et al [14, 15] in 2013 and 2015 have recommended a new procedure for solving fuzzy transportation problems. Waiel F, Abd El.Wahed [21] in 2001 gave a detailed description of a Multi objective transportation problem under fuzziness. Bit A.K., Biswal M.P, Alam S.S [1] in 1992 have introduced the fuzzy programming approach to multi criteria decision making transportation problem. Charnas.S, et al, [3] in 1993 has explained in detail about the interval and fuzzy extension of classical transportation problems. Saad O.M. and Abbas S.A.[19] in 2003 had made a parametric study on transportation problem under fuzzy environment. Kadhirvel. K and Balamurugan. K [9] in 2012 suggested a new Method for solving transportation problems using trapezoidal fuzzy numbers. Pandian and Natrajan [16, 17] in 2010 proposed a new algorithm, namely fuzzy zero point method. Gani.A and Razak K.A [6] in 2006 have given the procedure to solve two stage fuzzy transportation problems. Monalisha [13] in 2014 and Vimala et al [20] in 2015 have proposed a new method to solve transportation and fuzzy transportation problem.

In recent analysis allied with the application of PFS, various researchers have contributed many novel methods for unraveling many critical problems associated with PFS. Mohmed et al [12] in 2016 proposed a Pythagorean fuzzy analytic hier- archy process to multi-criteria decision making. Similaritymeasurefor Pythagorean fuzzy multiple criteria group decision making was presented by ZhangX [25] in 2016. Ma Z et al [11] in 2016 deliberated symmetric Pythagorean fuzzy weighted geo- metric/averaging operators and their application in multi criteria decision-making problems. Gou et al [7] in 2016 suggested an alternative queuing method for multiple criteria decision making with hybrid fuzzy and ranking information. Zhang et al [26] in 2014 introduced extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. Kumar et al [10]in 2019 presented a Pythagorean fuzzy approach to solve the transportation problem.

Compared with the other existing methods which have been correlated above, Monalisha's Approximation Method is executed with minimum steps. Based on the optimal solution, the decision can be taken interactively with the decision maker in decision space with the help of this proposed method. Additional information's like the violation of requirement factor, availability factor with respect to the constraints, compatibility of the cost of solution etc are provided.

In this work we have proposed a Pythagorean fuzzy Transportation problem and launched an algorithm named as Monalisha's approximation approach to unravel the specified problem. This paper is prearranged as follows; in section 2 prefaces of the Pythagorean fuzzy sets are provided. A model formulation for Pythagorean fuzzy Transportation problem is exhibited in section 3. The proposed algorithm is deliberated in section 4, a numerical exemplar is illustrated in section 5 and finally section 6 concludes the given work.

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2. Preliminaries

Definition 2.1 [10]

Let X is a fixed set, a pythagorean fuzzy set is an object having the form P = $\{(x,(\theta_P(x),\delta_P(x)))|x\in X\}$, where the function $\theta_P(x)$: $X\rightarrow [0, 1]$ and $\delta_P(x)$: $X \rightarrow [0,1]$ are the degree of membership and non-membership of the element $x \in X$ to P, respectively.

Also for every $x \in X$, it holds that $(\theta_P(x))^2 + (\delta_P(x))^2 \le 1$.

Definition 2.2 [10]

Let $\check{a}_1^p=(\;\theta_i^p\;,\delta_s^p\;)$ and $\check{b}_1^p=(\;\theta_0^p\;,\delta_f^p\;)$ betwoPythagoreanFuzzyNumbers(PFNs). Then the arithmetic operations are as follows:

(i) Additive property:
$$\check{a}_1^p \oplus \check{b}_1^p = \left(\sqrt{(\theta_i^p)^2 + (\theta_0^p)^2 - (\theta_i^p)^2(\theta_0^p)^2}, \delta_s^p, \delta_f^p\right)$$

(ii) Multiplicative property:
$$\check{a}_1^p \otimes \check{b}_1^p = \left(\theta_i^p \cdot \delta_s^p \sqrt{(\delta_s^p)^2 + (\delta_f^p)^2 - (\delta_s^p)^2(\delta_f^p)^2}\right)$$

(iii) Scalar product:
$$k \check{a}_1^p = \left(\sqrt{1 - (1 - \theta_i^p)^k}, (\delta_s^p)^k \right)$$

where k is nonnegative const..i.e.k > 0

Definition 2.3 [10] (*Comparison of two PFNs*) Let $\check{\alpha}_1^p = (\theta_i^p, \delta_s^p)$ and $\check{b}_1^p = (\theta_i^p, \delta_s^p)$ θ_0^p , δ_f^p) betwo Pythagorean Fuzzy Numbers such that the score and accuracy function are as follows:

- Score function: $S(\check{a}_1^p) = \frac{1}{2} (1 (\theta_i^p)^2 (\delta_s^p)^2)$
- Accuracy function: $(\check{a}_1^p) = (\theta_i^p)^2 + (\delta_s^p)^2$ (ii)

Then the following five cases arise:

Case 1: If
$$\check{a}_1^p > \check{b}_1^p$$
 iff $S(\check{a}_1^p) > S(\check{b}_1^p)$

Case 2: If
$$\check{a}_1^p < \check{b}_1^p$$
 iff $S(\check{a}_1^p) < S(\check{b}_1^p)$

Case 3: If
$$S(\check{a}_1^p) = S(\check{b}_1^p)$$
 and $H(\check{a}_1^p) < H(\check{b}_1^p)$, then $\check{a}_1^p < \check{b}_1^p$

Case 4: If
$$S(\check{a}_1^p) = S(\check{b}_1^p)$$
 and $H(\check{a}_1^p) > H(\check{b}_1^p)$, then $\check{a}_1^p > \check{b}_1^p$

Case 3: If
$$S(\check{\alpha}_1^p) = S(\check{b}_1^p)$$
 and $H(\check{\alpha}_1^p) < H(\check{b}_1^p)$, then $\check{\alpha}_1^p < \check{b}_1^p$
Case 4: If $S(\check{\alpha}_1^p) = S(\check{b}_1^p)$ and $H(\check{\alpha}_1^p) > H(\check{b}_1^p)$, then $\check{\alpha}_1^p > \check{b}_1^p$
Case 5: If $S(\check{\alpha}_1^p) = S(\check{b}_1^p)$ and $H(\check{\alpha}_1^p) = H(\check{b}_1^p)$, then $\check{\alpha}_1^p = \check{b}_1^p$

3. Model of Pythagorean fuzzy transportation problem

The balanced pythagorean fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

 c_{ij}^p =The pythagorean fuzzy transportation cost for unit quantity of the product from ith source

 a_{ii}^p = the Pythagorean fuzzy availability of the product at ith source

 b_{ij}^{p} = the Pythagorean fuzzy demand of the product at jth destination

 $\vec{x_{ij}}$ = the fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost.

Pythagorean fuzzy transportation problem is given by

$$minimize \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{p} * x_{ij}$$

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Subject to
$$\sum_{j=1}^{n} x_{ij} = a_i^p$$
, $i = 1,2,3,...,m$
 $\sum_{i=1}^{m} x_{ij} = b_j^p$, $j = 1,2,3,...,n$
 $\sum_{i=1}^{m} a_i^p = \sum_{j=1}^{n} b_j^p$

4. Algorithm for Solving Pythagorean Fuzzy Transportation Problem [8]

- **Step 1.** Frame the cost table from the givenPythagorean fuzzy transportation problem.
 - (i) Inspect whether total supply equals total demand. If yes, go to step 2.
 - (ii) If not, initiate a dummy row/column having all its cost elements as zero and supply/demand as the (+ve) difference of supply and demand.
- **Step 2**. Trace the smallest element in each row of the given cost matrix and then subtract the same from each element of that row.
- **Step 3**. In the reduced matrix obtained in step 2, trace the smallest element of each column and then subtract the same from each element of that column.
- **Step 4**. For each row of the transportation table recognize the smallest and the next to smallest costs. Determine the difference between them for each row. Exhibit them along the side of the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.
- **Step 5**. Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tiebreaking choice. Let the greatest difference correspond to i^{th} row and let 0 be in the i^{th} row. Allocate the maximum feasible amount $x_{ij} = \min(a_i,b_j)$ in the $(i,j)^{th}$ cell and cross off either the i^{th} row or the j^{th} column in the usual manner
- **Step 6**. Recompute the column and row differences for the reduced transportation table and go to step 5. Repeat the procedure until all the rim requirements (the various origin capacities and destination requirements are listed in the right most outer column and the bottom outer row respectively) are satisfied.

5. Numerical Example:

Consider a Pythagorean fuzzy transportation problem with three suppliers and four demand constraiants. The input data for Pythagorean fuzzy transportation problem is given bellow. The optimal aim of the process is to minimize the transportation cost and maximize the profit.

	\mathbf{D}_1	\mathbf{D}_2	\mathbf{D}_3	$\mathbf{D_4}$	Supply
O_1	(0.3,0.6)	(0.4,0.6)	(0.8,0.4)	(0.6,0.4)	25
O_2	(0.4,0.3)	(0.7,0.4)	(0.5,0.7)	(0.7,0.4)	26
O ₃	(0.6,0.2)	(0.8,0.2)	(0.7,0.3)	(0.9,0.1)	29
Demand	18	22	27	13	

Table.1. Pythagorean fuzzy transportation problem

In the given table the total supply is equal to the total demand which is equal to 80. Hence the transportation problem is balanced transportation problem.

Step 1. Determine the cost table from the given problem. Here total supply equals total demand, hence we can proceed to step 2. By the definition 2.2 the score function is given by, Scorefunction= $\frac{1}{2}\left(1-\left(\theta_i^\rho\right)^2-\left(\delta_i^\rho\right)^2\right)$

Here we use the score function for converting the Pythagoreanfuzzy numbers into crisp numbers.

$$S(C_{11}) = \frac{1}{2} \left(1 - \left(\theta_i^{\rho} \right)^2 - \left(\delta_i^{\rho} \right)^2 \right)$$
$$= \frac{1}{2} \left(1 - (0.3)^2 - (0.6)^2 \right) = \frac{1}{2} \left(1 - 0.09 - 0.36 \right) = \frac{0.55}{2} = 0.275$$

Applying the score function to all the values, we convert all the Pythagorean fuzzy numbers into crisp numbers. The defuzzified Pythagorean fuzzy transportation problem is given bellow.

	\mathbf{D}_1	\mathbf{D}_2	\mathbf{D}_3	\mathbf{D}_4	Supply
O ₁	0.275	0.24	0.1	0.24	25
O_2	0.375	0.175	0.13	0.175	26
O ₃	0.3	0.16	0.21	0.09	29
Demand	18	22	27	13	

Table.2. Defuzzified Pythagorean fuzzy transportation problem

Step 2. Locate the smallest cost in each row of the given cost matrix and then subtract the same from each cost of that row. In first row the minimum cost is 0.1, subtract 0.1 from the remaining costs in first row. Proceeding in the same process we can get the remaining values of the table.

Table 3 First	allocation	of Pythagorean	fuzzy transpo	ortation problem
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	$\mathbf{D_1}$	\mathbf{D}_2	\mathbf{D}_3	D ₄	Supply
O_1	0.175	0.14	0	0.14	25
O_2	0.245	0.045	0	0.045	26
O ₃	0.21	0.07	0.12	0	29
Demand	18	22	27	13	

Step 3. In the reduced matrix obtained in step 2, locate the smallest cost of each column and then subtracting the same from each cost of that column. In the first column the minimum cost is 0.175 subtract 0.175 from the remaining values in first column .Proceeding like this we can get the remaining costs.

Table.4. Second allocation of Pythagorean fuzzy transportation problem

	$\mathbf{D_1}$	\mathbf{D}_2	\mathbf{D}_3	\mathbf{D}_4	Supply
O_1	0	0.055	0	0.14	25
O_2	0.07	0	0	0.045	26
O ₃	0.035	0.025	0.12	0	29

18 22 27 13

Step 4. For each row of the transportation table after identifying the smallest and the next - to -smallest costs. Determine the difference between them for each row in the transportation table. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows of the transportation table. Similarly compute the differences for each column of the transportation table.

Table.5. Third allocation of Pythagorean fuzzy transportation problem

	$\mathbf{D_1}$	\mathbf{D}_2	\mathbf{D}_3	$\mathbf{D_4}$	Supply	Penalty
O ₁	0(18)	0.055	0	0.14	25(7)	0
O_2	0.07	0	0	0.045	26	0
O ₃	0.035	0.025	0.12	0	29	0.025
Demand	18(0)	22	27	13		
penalty	0.07(Max)	0.025	0	0.045		

Step 5. Identifying the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tiebreaking choice. Let the greatest difference correspond to i^{th} row and let 0 be in the i^{th} row. Allocating the maximum feasible amount $x_{ij} = \min(a_i,b_j)$ in the $(i,j)^{th}$ cell and cross off either the i^{th} row or the j^{th} column in the usual manner. Here the maximum penalty is in first column, and the minimum value is in the cell (1,1), allocate the minimum of (18,25), ie 18in (1,1) and subtract 18 from 25 and delete the first column.

Table.6. Fourth allocation of Pythagorean fuzzy transportation problem

	\mathbf{D}_2	\mathbf{D}_3	\mathbf{D}_4	Supply	Penalty
O_1	0.055	0(7)	0.14	7(0)	0.055(Max)
O_2	0	0	0.045	26	0
O ₃	0.025	0.12	0	29	0.025
Demand	22	27(20)	13		
penalty	0.025	0	0.045		

Step 6. Recomputing the column and row differences for the reduced transportation table and go to step 5. Repeating the procedure until all the rim requirements (the various origin capacities and destination requirements are listed in the right most outer column and the bottom outer row respectively) are satisfied.

Table.7. Fifth allocation of Pythagorean fuzzy transportation problem

\mathbf{D}_2	\mathbf{D}_3	$\mathbf{D_4}$	Supply	Penalty

O_2	0	0(20)	0.045	26(6)	0
O ₃	0.025	0.12	0	29	0.025
Demand	22	20(0)	13		
penalty	0.025	0.12(Max)	0.045		

Proceeding by the above steps, the final optimum solution is given bellow,

Table.8. Final allocation of Pythagorean fuzzy transportation problem

	$\mathbf{D_1}$	\mathbf{D}_2	\mathbf{D}_3	\mathbf{D}_4	Supply	Penalty
O ₁	0(18)	0.055	0(7)	0.14	25	0
O_2	0.07	0(6)	0(20)	0.045	26	0
O ₃	0.035	0.025(16)	0.12	0(13)	29	0.01
Demand	18	22	27	13		
penalty	0.07	0.025	0	0.045		

Table.9. Optimal solution of Pythagorean fuzzy transportation problem

	\mathbf{D}_1	\mathbf{D}_2	\mathbf{D}_3	\mathbf{D}_4	Supply
O ₁	0.275(18)	0.24	0.1(7)	0.24	25
\mathbf{O}_2	0.375	0.175(6)	0.13(20)	0.175	26
O ₃	0.3	0.16(16)	0.21	0.09(13)	29
Demand	18	22	27	13	

The above table satisfies the rim conditions with (m+n-1) non negative allocations at independent positions.

Thus the optimal allocation is: $X_{11}=18$, $X_{13}=7$, $X_{22}=6$, $X_{23}=20$, $X_{32}=16$, $X_{34}=13$.

The transportation cost according to the MAM's method is:

Total Cost

= (0.275x18) + (0.1x7) + (0.175x6) + (0.13x20) + (0.16x16) + (0.09x13)

=4.95+0.7+1.05+2.6+2.56+1.17=13.03

Total minimum cost will be Rs.13.03

In order to show the efficiency of the proposed method, the same problem is solved with various methods like North West corner (NWC), Least Cost (LCM) and Heuristics method (HM). We get the following results after solving the problem.

Table 9: Comparison Table

NWC	LCM	HEURISTIC	PROPOSED METHOD
15.21	14.29	14.71	13.03

The comparison of the proposed method with the other methods is provided in the chart bellow. Obviously the proposed method gives a better optimum solution. This method is more convenient than the other existing methods.



Fig.1: Comparison Chart

6. Conclusion

A pythagorean fuzzy transportation problem has been investigated in this work. Here we have introduced the monalisha's method to solve a pythagorean fuzzy transportation problem. By applying the score function the pythagorean fuzzy transportation problem is converted into crisp transportation problem. The proposed algorithm gives a best optimal solution when compared with other existing methods. In future, the proposed method can be applied to various applications of pythagorean fuzzy problems like job scheduling, neural network etc. The pronouncement maker can intrude in all the steps of the decision process. The proposed method can be utilized in an assortment of real-world problems where the information is uncertain with nonrandom, like environmental management, marketing, production area, cloud computing etc.

Reference:

- 1. Bit A.K., Biswal M.P., Alam S.S., "Fuzzy programming approach to multi criteria decision making transportation problem", *Fuzzy sets and system* 50 ,PP.135-142,1992.
- 2. Chanas.S and Kuchta.D, "A concept of the optimal solution of the transportation problem with fuzzy cost coefficients", *Fuzzy Sets and Systems* 82, pp 299-305,1992.
- 3. Charnas S.Delgado.M., Verdegy J.L., Vila M.A., "Interval and fuzzy extension of classical transportation problems", *Transporting planning technol*.17, PP.203-218.1993.
- 4. Charnes. A, Cooper W. W.and Henderson. A, "An introduction to Linear Programming", *Wiley*, New Work, 1953.
- 5. Dantzig G.B, "Linear programming and extensions", *Princeton University Press*, NJ, 1963.

- 6. Gani.A and Razak K.A., "Two stage fuzzy transportation problem" *Journal of physical Sciences*, pp 63-69,2006.
- 7. Gou X, Xu Z, Liao H, "Alternative queuing method for multiple criteria decision making with hybrid fuzzy and ranking information". *Inf Sci*357:144–160, 2016.
- 8. Hitchcock. F.L, "The distribution of a product from several sources to numerous localities", *Journal of mathematical physics*, pp 224-230,1941.
- 9. Kadhirvel. K, Balamurugan. K, "Method for solving transportation problems using trapezoidal fuzzy numbers", *International Journal of Engineering Research and Applications* (IJERA) ISSN: 2248-9622, Vol. 2, Issue 5, pp.2154-2158, 2012.
- 10. Kumar R,Edalatpanah.S.A,Jha.S,Singh.R, "A Pythagorean fuzzy approach to the transportation problem",*Complex and Intelleigent System*. 2019.
- 11. Ma Z, Xu Z, "Symmetric Pythagorean fuzzy weighted geo- metric/averaging operators and their application in multicriteria decision-making problems". *Int J Intell Syst31:1198–1219,2016*.
- 12. Mohd WRW, Lazim A, "Pythagorean fuzzy analytic hier- archy process to multi-criteria decision making". *AIP Conf Proc* 1905:040020, 2017.
- 13. Monalisha Pattnaik, "Transportation Problem By Monalisha's Approximation Method For Optimal Solution (Mamos)", *Scientific Journal of Logistics*, 11 (3), 267-273, 2015.
- 14. Narayanamoorthy.S and Kalyani.S, "Finding the initial basic feasible solution of a fuzzy transportation problem by a new method", *International Journal of Pure and Applied Mathematics*, Volume 101 No. 5, 687-692, 2015.
- 15. Narayanamoorthy.S,.Saranya.S &.Maheswari.S, "A Method for Solving Fuzzy Transportation Problem (FTP) using Fuzzy Russell's Method" I.J. *Intelligent Systems and Applications*, 02, 71-75, 2013.
- 16. Pandian .P, Natarajan. G, "A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem", *Applied Mathematical science*, vol. 4 pp 79-90,2010.
- 17. Pandian .P. Natarajan .G, "A new method for finding an optimal solution of fully interval integer transportation problems", *Applied Mathematical Sciences*, 4, 1819-1830,2010.
- 18. Ringuset J.L.Rinks, D.B., "Interactive solutions for the linear multi objective transportation problem". *European Journal of operational research*, 32, PP96-106.1987.
- 19. Saad O.M. and AbbasS.A. "A parametric study on transportation problem under fuzzy environment", *The Journal of Fuzzy Mathematics*, pp 115-124, 2003.
- 20. Vimala.S, Krishna prabha .S, "Fuzzy transportation problem through Monalisha's approximation method", *British Journal of Mathematics & Computer Science* 17(2): 1-11, Article no.BJMCS.26097,2016.
- 21. Waiel F., Abd El.Wahed, "A Multi objective transportation problem under fuzziness", *Fuzzy sets and systems* 117, PP.27-33,2001.
- 22. Yager RR, "Pythagorean fuzzy subsets. In: 2013 joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS)", pp 57–61, 2013.
- 23. YagerRR, "Pythagoreanmembershipgradesinmulticriteria decision making", *IEEE Trans Fuzzy Syst22:958–965, 2014*.
- 24. Zadeh, L. A, "Fuzzy sets", Information and control, vol 8, pp 338-353,1965.
- 25. ZhangX, "Anovelapproachbasedonsimilaritymeasurefor Pythagorean fuzzy multiple criteria group decision making". *Int J Intell* Syst31:593–611, 2016.
- 26. Zhang X, Xu Z, "Extension of TOPSIS to multiple crite- ria decision making with Pythagorean fuzzy sets". *Int J Intell Syst* 29:1061–1078, 2014.
- 27. Zimmermann H.J. "Fuzzy programming and linear programming with several objective functions", *Fuzzy Sets and Systems*, pp 45-55, 1978.