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Homotopy Analysis to MHD Visco-Elastic Fluid Flow and Heat Transfer over an Exponentially Stretching Sheet

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ABSTRACT: In this present study, we analyze the flow and heat transfer of Visco-elastic fluid over an exponentially stretching sheet of the MHD boundary layer. The equations governing the boundary layer are translated into ordinary differential equations by the use of sufficient similarity transformations. To solve such equations, the Homotopy Analysis Method (HAM) is applied. The effects of physical parameters such as Magnetic Parameter, Prandtl number for velocity and temperature are discussed in detail with graphical representation. The results of the solution are in strong agreement with the latest literature studies.
Key words: exponentially, HAM, MHD, stretching sheet, Visco-elastic fluid.

INTRODUCTION:

The flow of a compressible Visco-elastic fluid over a stretching surface in the polymer industry has a wide range of applications in the issue of extruded polymer sheet from a dye. Flow problems with magneto hydrodynamic (MHD) have many applications in the petroleum industry, crude oil purification, plastic sheet and foil production and plastic sheet cold drawing. In the presence of transverse magnetic field, H.I. Anderson [1] analyzes the movement of visco-elastic fluid through a stretching sheet showing that the current magnetic field has the same flow effect as the visco-elasticity.Keller and Magyari [2] described the transfer of heat and mass in the boundary layers over a continuous exponential stretching surface. K.V. Prasad et.al [3] studied the momentum and transfer of heat in visco-elastic fluid flow over an isothermal stretching surface in porous medium. S. Kumar Khan and E. Sanjayanand presented MHD flow of visco-elastic fluid in Porous medium over a quadratic stretching sheet [4]. Then A. Abdallah [7] analyzed heat transfer in MHD flow of visco-elastic fluid by using Homotopy Analysis Method. Besides that, M. Sajid, T. Hayat [8] applied the HAM for MHD viscous flow due to a shrinking sheet. B.Bidin and R.Nazar [9] numerically investigated the boundary flow and heat transfer of viscous fluid over an exponentially stretching sheet with thermal radiation. Hymavathi and Shankar [10] tested quasilinearization technique to MHD visco-elastic fluid flow over a non isothermal stretching sheet.Hymavathi [11] et.al studied the Soret and Dufour effects on a

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chemically reacting viscous fluid past a moving vertical plate. .Later, T. Hymavathi and B.Akkaya [12] present the analysis of heat transfer in visco-elastic fluid over exponentially stretching sheet in porous medium. Recently, T.Hymavathi and W. Sridhar [13] studied the diffusion of chemically reactive species in Casson fluid numerically by using Keller box method.

MATHEMATICAL FORMULATION:

Consider a steady two dimensional flow and heat transfer of electrically conducting visco-elastic fluid over an exponentially stretching sheet. The flow is confined to y>0. The x-axis is taken along the stretching surface in the direction of motion and y-axis is perpendicular to it. Two equal and opposite forces are applied along x-axis. A uniform magnetic field of strength $B = B_0 e^{x/2l}$ is applied normal to the stretching sheet. The governing equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right\} - \frac{\sigma B_0{}^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with the boundary conditions:

$$u = U_w(x) = U_0 e^{x/l}, \; v = 0, \; T = T_w = T_\infty + T_0 e^{x/2l} \qquad at \;\; y = 0$$
$$u = 0, \; u_y = 0. \; T = T_\infty \qquad as \;\; y \to \infty \qquad (4)$$

Where u and v are the components of velocity in the directions of x and y respectively, T is the temperature of the fluid, $\upsilon = \frac{\mu}{\rho}$ is the kinematic viscosity, ρ is the density of the fluid, μ is the dynamic viscosity, k_0 is the visco elastic parameter, σ is electrical conductivity of the fluid, C_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid. The continuity equation (1) is satisfied if we consider the stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x}, \tag{5}$$

by introducing the similarity transformations:

$$\eta = y\sqrt{\frac{U_0}{2\nu l}}\; e^{x/2l} \tag{6}$$

$$\psi(x, y) = \sqrt{2\nu l U_0}\, f(x.\eta)e^{x/2l}, \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\;, \tag{7}$$

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Where η is the similarity variable f is the dimensionless stream function, $\theta(\eta)$ be the dimensionless temperature.

The momentum and energy equations are converted into

$$f''' - 2f'^2 + ff'' - k_1\left\{3ff''' - \frac{1}{2}ff^{iv} - \frac{3}{2}f''^2\right\} - Mf' = 0 \qquad (8)$$

$$\theta'' = \Pr(f'\theta - f\theta') \qquad (9)$$

With the boundary conditions

$$\begin{array}{llll} f = 0, & f' = 1, & \theta = 1 & at\ \eta = 0 \\ f' = 0, & \theta = 0 & & as\ \eta \to \infty \end{array} \qquad (10)$$

Where $k_1 = \dfrac{k_0 U_w}{\upsilon l}$, $M = \dfrac{2\sigma B_0^2 l}{\rho U_0 e^{x/l}}$, $\Pr = \dfrac{\mu c_p}{k_\infty}$ are the dimensionlessvisco-elastic parameter, magnetic parameter, Prandtl number respectively.

HOMOTOPY ANALYSIS SOLUTION:

In this division, we apply HAM to interpret the solutions of the equations (8) and (9) under the boundary conditions (10). We have to take the initial guesses f_0, θ_0 as follows

$$f_0(\eta) = 1 - e^{-\eta} \qquad ` \ (11)$$

$$\theta_0(\eta) = e^{-\eta} \qquad (12)$$

Now we have to select the linear operators as

$$L(f) = f''' - f' \qquad (13)$$

$$L(\theta) = \theta'' - \theta \qquad (14)$$

This follows the property given below

$$L_f[C_1 + C_2 e^{\eta} + C_3 e^{-\eta}] = 0 \qquad (15)$$

$$L_\theta[C_4 e^{\eta} + C_5 e^{-\eta}] = 0 \qquad (16)$$

Where C_1, C_2, C_3, C_4, C_5 are the arbitrary constants.

By considering the embedding parameter $q \in [0,1]$, the non-zero auxiliary parameters $\hbar_1$ ***and*** $\hbar_2$ and the auxiliary functions $H_1(\eta)$ ***and*** $H_2(\eta)$, we have to construct the zeroth-order deformation equations as follows:

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$$(1-q)\,L(\tilde{\omega}_1(\eta;q)-f_0(\eta))=\hbar_1 q H_1(\eta)\,N_1[\tilde{\omega}_1(\eta\,;q)] \quad (17)$$

$$(1-q)\,L(\tilde{\omega}_2(\eta;q)-\theta_0(\eta))=\hbar_2 q H_2(\eta)\,N_2[\tilde{\omega}_1(\eta\,;q),\tilde{\omega}_2(\eta;q)] \quad (18)$$

with the following boundary conditions

$$\tilde{\omega}_1(0;q)=0 \;\; \tilde{\omega}_1'(0;q)=1 \;\; \tilde{\omega}_1'(\infty;q)=0$$

$$\tilde{\omega}_1(0;q)=1 \qquad \tilde{\omega}_2'(\infty;q)=0 \quad (19)$$

We define non linear operator as

$$N_1(\tilde{\omega}_1(\eta;q))=\frac{\partial^3\tilde{\omega}_1}{\partial\eta^3}-2\left(\frac{\partial\tilde{\omega}_1}{\partial\eta}\right)^2+\frac{\partial^2\tilde{\omega}_1}{\partial\eta^2}f-k_1\left(3\frac{\partial\tilde{\omega}_1}{\partial\eta}\frac{\partial^3\tilde{\omega}_1}{\partial\eta^3}-\frac{1}{2}f\frac{\partial^4\tilde{\omega}_1}{\partial\eta^4}-\frac{3}{2}\left(\frac{\partial^2\tilde{\omega}_1}{\partial\eta^2}\right)^2\right)-M\frac{\partial\tilde{\omega}_1}{\partial\eta} \quad (20)$$

$$N_2(\tilde{\omega}_1(\eta;q),\tilde{\omega}_2(\eta;q))=\frac{\partial^2\tilde{\omega}_2}{\partial\eta^2}+\Pr\left(\tilde{\omega}_1\frac{\partial\tilde{\omega}_2}{\partial\eta}-\frac{\partial\tilde{\omega}_1}{\partial\eta}\tilde{\omega}_2\right) \quad (21)$$

For $q=0 \;\; and \;\; q=1$, we have

$$\tilde{\omega}_1(\eta\,,0)=f_0(\eta), \qquad \tilde{\omega}_1(\eta,1)=f(\eta)$$

$$\tilde{\omega}_2(\eta\,,0)=\theta_0(\eta), \qquad \tilde{\omega}_2(\eta,1)=\theta(\eta) \quad (22)$$

Thus as q changes from 0 to 1, $\tilde{\omega}_1(\eta;q)$ varies from $\tilde{\omega}_1(\eta\,;0)$ ***to f*** (η) , $\tilde{\omega}_2(\eta;q)$ varies from $\tilde{\omega}_2(\eta\,;0)$ ***to*** $\theta(\eta)$

By using Taylor's theorem we have

$$\tilde{\omega}_1(\eta\,,q)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta)q^m \quad (23)$$

$$\tilde{\omega}_2(\eta\,,q)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta)q^m \quad (24)$$

Where $f_m(\eta)=\dfrac{1}{m!}\dfrac{\partial^m\tilde{\omega}_1(\eta,q)}{\partial q^m}\bigg|_{q=0}$ (25)

$$\theta_m(\eta)=\frac{1}{m!}\frac{\partial^m\tilde{\omega}_2(\eta,q)}{\partial q^m}\bigg|_{q=0} \quad (26)$$

Then the series (17) and (18) converges at q=1, and thus

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$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \tag{27}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \tag{28}$$

The deformation equations of the mth order are as follows:

$$L_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_1 H_1(\eta) R_m^f(\eta) \tag{29}$$

$$L_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = \hbar_2 H_2(\eta) R_m^\theta(\eta) \tag{30}$$

subject to the boundary conditions

$$f_m(0) = 0, \quad f_m'(0) = 0, \quad f_m'(\infty) = 0$$
$$\theta_m(0) = 0, \quad \theta_m(\infty) = 0 \tag{31}$$

$$R_m^f(\eta) = f'''_{m-1} + \sum_{i=0}^{m-1} f_i(f''_{m-1-i}) - 2\sum_{i=0}^{m-1} f_i'(f'_{m-1-i}) - K_1[3\sum_{i=0}^{m-1}(f'_{m-1-i})f'''_i$$
$$-\frac{1}{2}\sum_{i=0}^{m-1}(f_{m-1-i})f_i^{iv} - \frac{3}{2}\sum_{i=0}^{m-1}(f''_{m-1-i})f''_i] - M(f')_{m-1} \tag{32}$$

$$R_m^\theta(\eta) = \theta''_{m-1} + \Pr\sum_{i=0}^{m-1}\{f_i(\theta'_{m-1-i}) - \theta_i(f'_{m-1-i})\} \tag{33}$$

$$\chi_m = \begin{cases} 1, & m > 1, \\ 0, & m \le 1 \end{cases} \tag{34}$$

we choose the auxiliary function as

$$H_1(\eta) = 1 \qquad H_2(\eta) = 1 \tag{35}$$

If we let $f_m^*(\eta)$, $\theta_m^*(\eta)$ as the special functions of m-th order deformation equations, then the general solutions are given by

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^{-\eta} + C_3 e^{\eta} \tag{36}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_4 e^{-\eta} + C_5 e^{\eta} \tag{37}$$

Where the integral constants C_1, C_2, C_3, C_4, C_5 are determined using the boundary conditions (31). Now it is easy to solve the linear non-homogeneous equation (32)-(33) using MATHEMATICA software.

CONVERGENCE OF HAM:

The convergence of the HAM solution mainly depends on the non-zero auxiliary parameters

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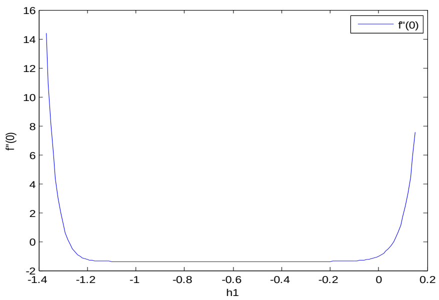
ђi(i=1, 2). For this $\hbar_i$ curves are plotted for 20th order approximations in Fig.1 and Fig.2 for different $-1.1 \le \hbar_1 \le 0.2$, $-2.1 \le \hbar_2 \le -0.18$ are the valid ranges of $\hbar_i$ respectively.



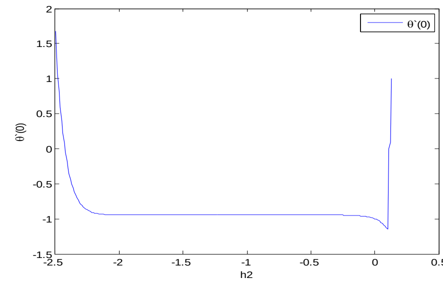


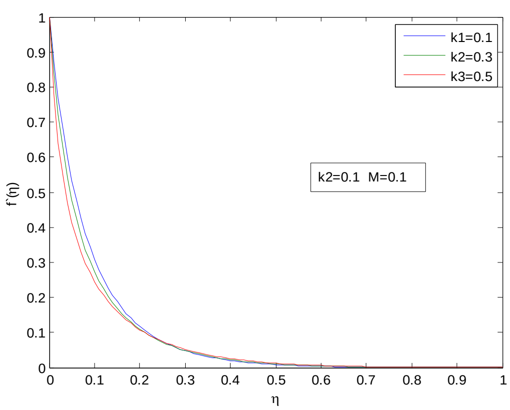
Fig.1: ħ1- curve for f ``(η)**Fig.2:** ħ2- curve for θ´(η)

TABLE1: Convergence of HAM solution for various orders of Approximations when $\boldsymbol{k}_1 = \boldsymbol{M} = 0.1 \ \& \ \Pr = 1$

order	$-(f''(0))$	$-\theta'(0)$
5	1.37478	0.945019
10	1.37486	0.945114
15	1.37493	0.945145
20	1.37498	0.945146
25	1.37498	0.945147
30	1.37498	0.945147
35	1.37498	0.945147

RESULTS AND DISCUSSION:

Graphically the results are illustrated by plotting the Figures.



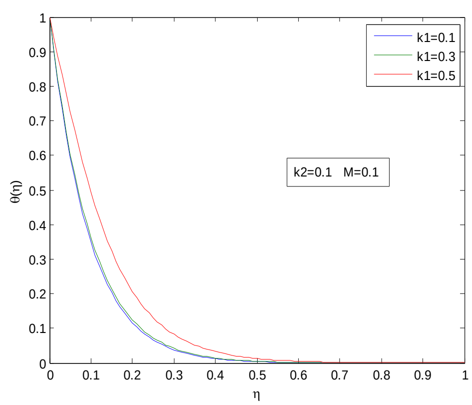


Fig.3: f´ (η) for various k_1 values **Fig.4:** θ (η) for various k_1 values

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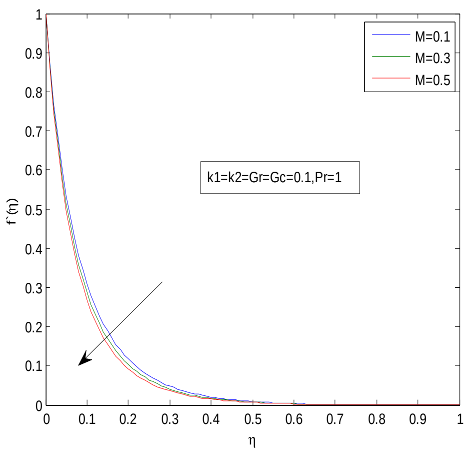
Fig.3-Fig.4 illustrates the effect of visco-elastic parameter k_1 on velocity & temperature. From the figure, it is obvious that the velocity decreases and the temperature increases by increasing the visco-elastic parameter (k1).

Fig.5 shows the effect of the magnetic parameter M on velocity. It is observed that with an increase in magnetic parameter there is a decrease in velocity. The effect of the Prandtl number Pr on temperature is shown in Fig.6. Temperature decreases by increasing Prandtl number Pr.



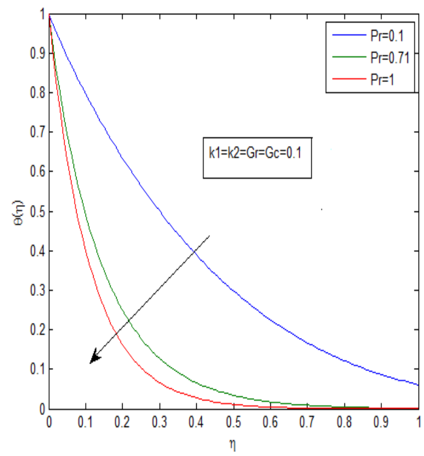


Fig.5: f′ (η) for various M values **Fig.6:** θ (η) for different values of Pr

CONCLUSIONS:

In this paper, we use Homotopy Analysis Method (HAM) to analyze the MHD flow of Visco-elastic fluid and heat transfer over an exponentially stretching surface. The findings of this paper are

* The velocity of the fluid decreases by increasing the magnetic parameter.
* The effect of increasing the number of Prandtl number is to lower the fluid temperature.
* The visco-elastic parameter has the effect of increasing the rate of heat transfer

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