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Lucky Edge Labeling of H Graph, N Copies of H-Graph, Theta Graph, Duplication of Theta Graphs and Path Union of Theta Graphs

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Abstract:

In this paper, it is proved that the H-graphs, n copies of H-graphs, Theta graph, Path union of Theta graph and Duplication of Theta graph are Lucky edge graphs.

Let G be a simple graph with vertex set $V(G)$ edge set $E(G)$ respectively. Vertex set $V(G)$ and edge set $E(G)$ by positive integer and $E(e)$ denote the edge label such that it is the sum of labels of vertices incident with edge. The labeling is said to be lucky edge labeling, if the edge set $E(G)$ is a proper coloring of G that is if we have $E(e_1) \neq E(e_2)$ wherever e_1 *and* e_2 are adjacent edges.

Keywords:

Lucky edge graphs, Lucky edge labeling, H-Graph and Theta Graphs.

Introduction:

In 1967, Rosa [6] introduced the concept of labeling.

Nellai Murugan [3,4], introduced the concept of Lucky edge labeling. A vertex labeling of a graph G is an assignment of labels to the vertices of G that includes for each edge uv a label depends on the vertex labels x and y.

In this paper we proved that the H graphs, n copies of H-graph, Path union of Helm, path union of closed helm, path union of Gear graph are lucky edge labeled graphs.

Preliminaries:

Definition 1.1

Lucky labeling is coloring the vertices such that the sum of labels of all adjacent vertices G vertex is not equal to the sum of labels of all adjacent vertices of vertex which is adjacent to it. [5,7]

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Definition 1.2

The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1,v_2, ...u_n$ by joining the vertices $V_{\frac{n+1}{2}}$ and $U_{\frac{n+1}{2}}$ by an edge if n is odd and the vertices $V_{\frac{n}{2}+1}$ and $U_{\frac{n}{2}}$ if n is even.[2].

Definition 1.3

A Theta graph $(T_\propto)$ is a block with two non-adjacent vertices of degree 3, and all other vertices of degree 2. [1,8]

Definition 1.4

A vertex V_i' is said to be a duplication of V_i if all the vertices which are adjacent to V_i are now adjacent to V_i'. [8]

Theorem:1

The Theta graph $(T_\propto)$ admits Lucky edge labeling whose Lucky number is 6.

Proof:

If $u_0, u_1,u_2, ...u_6$ are the vertices of the Theta Graph u_0 be the central vertex and rest of the vertices $u_1,u_2,u_3, u_4, u_5,u_6$ are the external vertices.

Edge set can be defined as,

$E(G) = \{u_0u_1,u_0u_4,u_iu_{i+1};1 \leq i \leq 5\}$

Let us define the vertex labeling $f: V(G) = \{1,2,3,4\}$ labeling has to be given by,

$i)\ f(u_0) = 4$

$ii)\ f(u_1) = 1$

$iii)\ f(u_2) = 2$

$iv)\ f(u_3) = 4$

$v)\ f(u_4) = 3$

v$i)\ f(u_5) = 3$

vi$i)\ f(u_6) = 1$

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Define the map f^* on E as follows,

Let $f^*E(G) \rightarrow \{1,2,3,4,5,6\}$ such that,

$i)\ f^*(u_0,u_1) = 5$

$ii)\ f^*(u_0,u_4) = 6$

$iii)\ f^*(u_i,u_{i+1}) = i + 1, where\ 1 \leq i \leq 5$

$vi)\ f^*(u_6,u_1) = 4$

Illustration:1

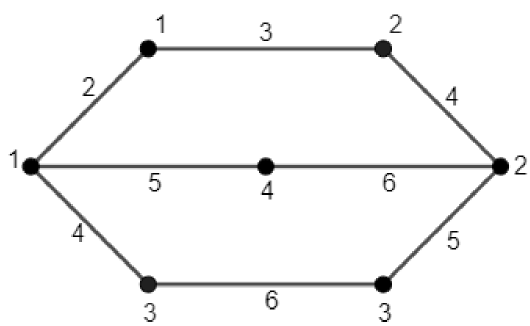

Figure: 1

Figure-1 shows Theta graph and its lucky number is 6.

Theorem:2

The duplication of any vertex of degree 3, in the Theta graph $T_\propto$ is a Lucky edge labelled graph and its Lucky edge number is 8.

Proof:

Let $G_\propto$ be a graph obtained from $T_\propto$ after duplication vertex of the v_1 and duplication vertex of the vertex v_4.

Let us define vertex labeling $f : V(G) \rightarrow \{1,2,3,4\}$ as follows

Case(i)

Labeling of Duplication of v_1 we define $f(v_1') = 4$, v_1' is the Duplication vertex of v_1

Further

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$f(v_i) = 1, when\ i\ = 3,4$

$f(v_i) = 2, when\ i\ = 1,2$

$f(v_i) = 3, when\ i\ = 5,6$

Define the map f^* on E as follows,

Let $f^*E(G)$→$\{2,3,4,5,6,7,8\}$ such that,

$i) f^*(v_1v_2) = 4$

$ii) f^*(v_2v_3) = 3$

$iii) f^*(v_3v_4) = 2$

$vi) f^*(v_4v_5) = 4$

$v) f^*(v_5v_6) = 6$

$vi) f^*(v_6v_1) = 6$

$vii) f^*(v_6v_1) = 6$

$viii) f^*(v_1v_0) = 6$

$ix) f^*(v_4v_0) = 5$

$x) f^*(v_1{}'v_0) = 5$

$x) f^*(v_1{}'v_2) = 6$

$xi) f^*(v_1{}'v_6) = 7$

$xii) f^*(v_1{}'v_0) = 8$

$xii) f^*(v_4{}'v_3) = 6$

$xiv) f^*(v_4{}'v_5) = 7$

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Illustration:2

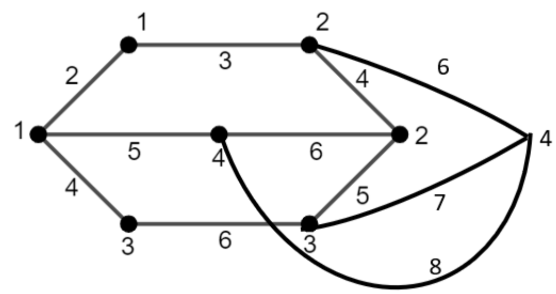

Figure: 2

Figure-2 shows the Duplication of Theta graph and its lucky number is 8.

Theorem:3

For every $m \geq 1$ there exists a path. Path union of "m" copies of Theta graph $T_{\propto}$ is a Lucky edge labeled graph whose Lucky number is 6.

Consider "m" copies of Theta graphs $T_{\propto}{}^{1}, T_{\propto}{}^{2},... T_{\propto}{}^{n}$

Then $V(T_{\propto}{}^{n}) = \{u_{i0},u_{i1},u_{i3},u_{i4},u_{i5},u_{i6},1 \leq i \leq 6\}$

$$E(T_{\propto}{}^{n}) = \{u_{i1}u_{i2},u_{i2}u_{i3},u_{i3}u_{i4},u_{i4}u_{i5},u_{i5}u_{i6}:1 \leq i \leq n\} \cup \{u_{i2}u_{i+1},u_{i2}u_{i3},u_{i0}u_{i6},1 \leq i \leq n\}$$

Proof:

Let the function $f : V(G)\rightarrow\{1,2,3,4\}$ there exists a Path union of Theta graph whose vertex labeling is defined by

$i)\ f(u_{i0}) = 4\ for\ all\ i$

$ii)\ f(u_{i1}) = 1\ for\ all\ i = 4,3,..n - 1$, $f(u_{i1}) = 3\ for\ all\ i = 2,4,...n,$

$iii)\ f(u_{i2}) = 2\ for\ all\ i = 1,3,...n - 1$

$f(u_{i2}) = 3\ for\ all\ i = 2,4,6,...n$

$f(u_{i3}) = 2\ for\ all\ i$

$f(u_{i4}) = 3\ for\ all\ i = 1,3,...n - 1$

$f(u_{i4}) = 2\ for\ all\ i = 2,4,...n$

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$f(u_{i5}) = 3\ for\ all\ i = 1,3,...n-1$

$f(u_{i5}) = 3\ for\ all\ i = 2,4,...n$

$f(u_{i6}) = 1\ for\ all\ i$

Define the amp f^* on E as follows,

Let $f^* E(G) \rightarrow \{2,3,...6\}$ such that,

$i) f(u_{i1}, u_{i2}) = 3, for\ i = 1,3,...n-1$

$ii) f(u_{i2}, u_{i3}) = 4, for\ i = 1,3,...n-1$

$iii) f(u_{i4}, u_{i5}) = 6, for\ i = 1,3,...n-1$

$iv) f(u_{i5}, u_{i6}) = 4, for\ i = 1,3,...n-1$

$v) f(u_{i6}, u_{i0}) = 5, for\ all\ i$

$vi) f(u_{i1}, u_{i2}) = 6, for\ all\ i$

$vii) f(u_{i1}, u_{i2}) = 6, for\ all\ i = 2,4,...n$

$viii) f(u_{i2}, u_{i3}) = 5, for\ all\ i = 2,4,...n$

$ix) f(u_{i3}, u_{i4}) = 4, for\ all\ i = 2,4,...n$

Illustration:3

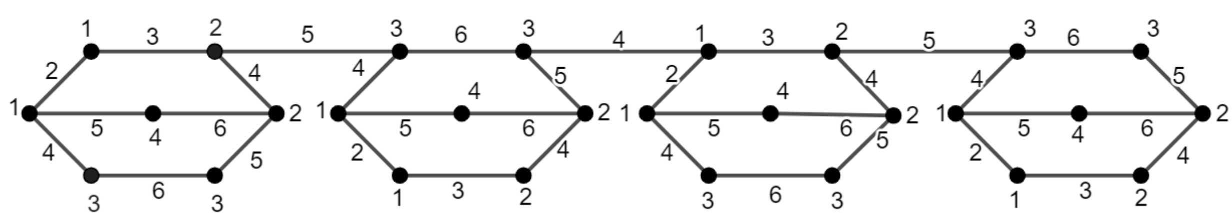

Figure: 3

Figure-3 shows the Path union of Theta graph and its lucky number is 6.

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Theorem:4

The graph H_n graph $(n \geq 3)$ is a lucky edge labelled graph having Lucky number is 4 and H_n graph $(n \geq 5)$, whose Lucky number is 5.

Proof:

Let $G = (V,E)$ be an H graph, with vertex partition $V = A \cup B$, vertices in A makes left arm of H and B makes right arm of it.

The component A contains the edges $\{a_i a_{i+1}$, where i $\geq$ 1, $b_i b_{i+1}$ $where\ \ i \geq 1$, $a_{\frac{n+1}{2}}\, b_{\frac{n+1}{2}}$, when n is odd $a_{\frac{n}{2}+1}$, $b_{\frac{n}{2}}$ when n is an even.

Then $|V(G)| = 2n$ vertices and $|E(G)| = 2n - 1$ edges.

Lucky edge labeling of H_n graph is divided into two cases.

Case(i)

When n is an odd

Sub case (ii)

When n=3, the vertex labeling 'f' is constructed as follows,

$(i) f(a_1) = 2$

$(ii) f(a_2) = 1$

$(iii) f(a_3) = 3$

$(iv) f(b_1) = 2$

$(v) f(b_2) = 1$

$(vi) f(b_3) = 3$

Edge labeling f_e must be given by

$f_e((b_1 b_2)) = 3$

$f_e((b_2 b_3)) = 4$

$f_e((a_2 b_2)) = 2$

$f_e((a_1 a_2)) = 3$

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$f_e((a_2 a_3)) = 4$

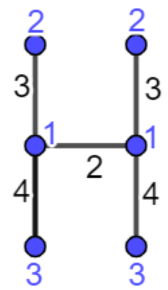

Figure: 4

Figure-4 shows H_3 the graph and its lucky number is 4.

Subcase (ii)

As explained the vertex labeling of H_3 above, it can be extended H_5, H_{11},H_{17},...H_{6k-1}, $\forall\ k \in Z^+$, inductively,

Pendant varices of H_5, H_{11},H_{17},...H_{6k-1} can be labelled as 2.

Edge labeling can easily be completed by adding the labels of the extreme verities of the given edge.

Illustration:5

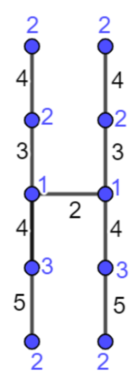

Figure: 5

Figure-5 shows H_5 the graph and its lucky number is 5. [when $n \geq 5$]

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Subcase (iii)

As Explained the vertex labeling of H_5 above, it can be extended, for $H_7, H_{13}, H_{19}, \ldots H_{6k+1}, \forall\ k \in\ Z^+$ inductively, pendant vertices of those H_n can be labelled as 1.

Computing of Edge labeling is mentioned in the subcase (ii)

Illustration: 6

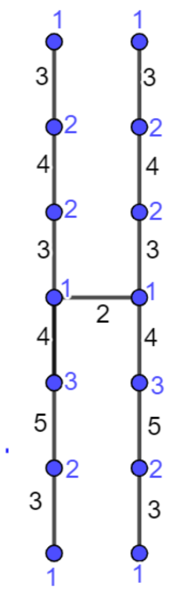

Figure: 6

Figure-6 shows H_7 the graph and its lucky number is 5.

Subcase (iv)

The vertex labeling of H_7 is explained in subcase (ii), from that it can be extended $H_9, H_{15}, H_{21}, \ldots H_{6k+3}, \forall\ k \in\ Z^+$ inductively.

The pendant vertices if $H_9, H_{15}, H_{17}, \ldots H_{6k+3}$ can be labelled as 3. Edge labeling can be computed by adding the labels of the extreme vertices of the given edge, but adjacent edge labels should not be in same.

Illustration:7

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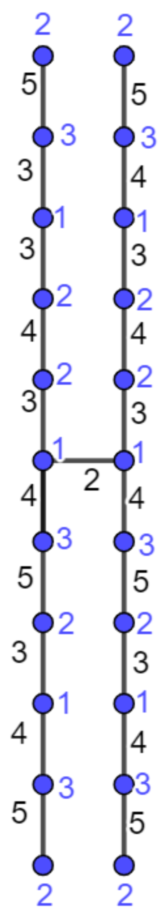

Figure: 7

Figure-7 shows H_{11} the graph and its lucky number is 5.

Case(ii)

When n is an even

Subcase(i)

When n =4, the vertex labeling f is constructed as follows

$(i)\ f(a_1)\ =\ 2$

$(ii)\ f(a_2)\ =\ 2$

$(iii)\ f(a_3)\ =\ 1$

$(iv)\ f(a_4)\ =\ 3$ and

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$f(b_1) = 3$

$f(b_2) = 1$

$f(b_3) = 2$

$f(b_4) = 2$

Edge labeling must be given by

$f_e((a_1a_2)) = 4$

$f_e((a_2a_3)) = 3$

$f_e((a_3a_4)) = 4$

$f_e((b_1b_2)) = 4$

$f_e((b_2b_3)) = 3$

$f_e((b_3b_4)) = 4$

$f_e((a_3b_2)) = 2$

Illustration:8

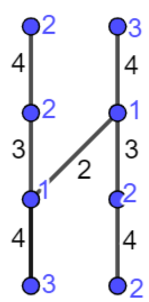

Figure: 8

Figure-8 shows H_4 the graph and its lucky number is 4.

Subcase(ii)

The vertex labeling of H_4 is explained in subcase (i) it can be extended, $H_6, H_{12}, H_{18}, \ldots H_{6k}$.

Pendant vertices of these H-graphs can be labelled by 3.

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Edge labeling can be computed by adding the labels of the extreme vertices of the given edge.

Illustration:9

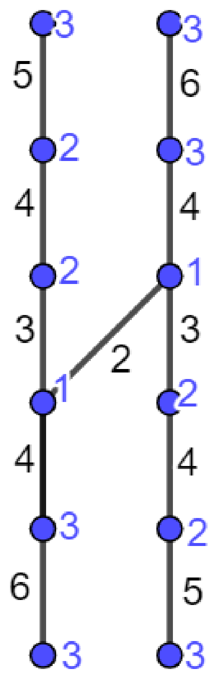

Figure: 9

Figure-11 shows H_6 the graph and its lucky number is 6.

Sub case (iii)

The vertex labeling of H_6 is explained above in the previous subcase, it can be extended for $H_8, H_{14}, H_{20},...H_{6k+2}, k \in Z^+$ inductively, pendant vertices of $H_8, H_{14}, H_{20},...H_{6k+2}, k \in Z^+$ can be labelled by 1.

Computing of Edge labeling is mentioned in the previous subcase.

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Illustration:10

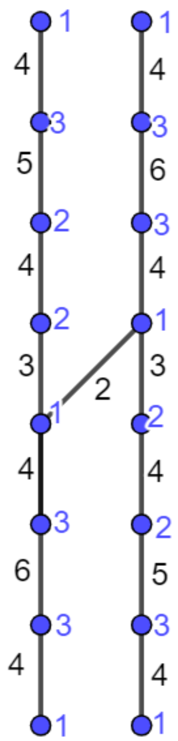

Figure: 10

Figure-10 shows H_8 the graph and its lucky number is 6.

Subcase (iv)

The vertex labeling of H_8 has explained in the previous case, In addition to that pendant vertices of $H_{10}, H_{16}, H_{22}, \ldots H_{6k+4}$ can be labelled by 2.

Edge labeling can be computed by adding the labels of the extreme vertices of the given edge.Labeling of the adjacent edges should not be in same.

Theorem:5

The path union of n copies of H_3 graph and H_4 graph is lucky edge labeled graph.[8].

Proof:

N copies of H_3- graph.

Consider an H-graph with 2n vertices and 2n-1 edges.

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Let $U_1^k, U_2^k, U_3^k, \ldots V_1^k, V_2^k, V_3^k$, $k = 1,2,3,\ldots n$ are the vertices of H-graph. V_i^k and V_1^{k+1} are connected by an edge, where $k = 1,2,\ldots n-1$.

Define $f: V(G) \rightarrow \{1,2,3\}$ then the vertex labeling as

$i)\ f(U_1^k) = 2\ when\ k = 1,2,5,6,9,10,\ldots$

$ii)\ f(U_2^k) = 1\ when\ k = 1,2,5,6,9,10,\ldots$

$iii)\ f(U_3^k) = 3\ when\ k = 1,2,5,6,9,10,\ldots$

$iv)\ f(V_1^k) = 3\ when\ k = 3,4,7,8,\ldots$

$v)\ f(V_2^k) = 1\ when\ k = 3,4,7,8,\ldots$

$vi)\ f(V_3^k) = 2\ when\ k = 3,4,7,8,\ldots$

$vii)\ f(U_1^k) = 3\ when\ k = 3,4,7,8,\ldots$

$viii)\ f(U_2^k) = 1\ when\ k = 3,4,7,8,\ldots$

$ix)\ f(U_3^k) = 2\ when\ k = 3,4,7,8,\ldots$

$v)\ f(V_1^k) = 2\ when\ k = 1,2,5,6,\ldots$

$f(V_1^k) = 1\ when\ k = 1,2,5,6,\ldots$

$f(V_1^k) = 3\ when\ k = 1,2,5,6,..$

Edge labeling must be given by

$i) f((U_1^k, U_2^k)) = 3\ when\ k = 1,2,5,6,\ldots$

$ii) f((U_1^k, U_2^k)) = 4\ when\ k = 3,4,7,8,\ldots$

$iii) f((U_2^k, U_3^k)) = 4\ when\ k = 1,2,5,6,\ldots$

$vi) f((U_2^k, U_3^k)) = 3\ when\ k = 3,4,7,8,\ldots$

$v) f((V_1^k, V_2^k)) = 3\ when\ k = 1,2,5,6,\ldots$

$vi) f((V_1^k, V_2^k)) = 4\ when\ k = 3,4,7,8,\ldots$

$vii) f((V_2^k, V_3^k)) = 4\ when\ k = 1,2,5,6,\ldots$

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$viii) f((V_2{}^k, V_3{}^k)) = 3 \ when \ k = 3,4,7,8,...$

$xi) f((U_2{}^k, V_2{}^k)) = 2 \ when \ k = 1,2,3,4,...$

$x) f((V_1{}^{k+1}, V_1{}^{k+1})) = 4 \ when \ k = 1,5,...$

$f((V_2{}^k, V_1{}^{k+1})) = 5 \ when \ k = 2,4,6..$

$f((V_1{}^k, V_1{}^{k+1})) = 6 \ when \ k = 3,7..$

Illustration:11

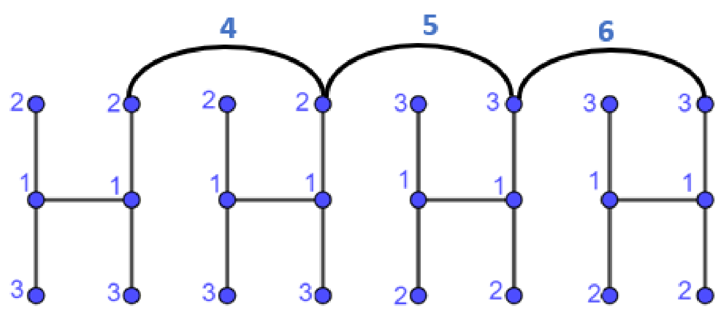

Figure: 11

Figure-11 represents s the Path union 4-Copies of H_3 graph and its lucky number is 6

Case (ii)

n copies of H graph

Let $U_1{}^k$, $U_2{}^k$,$U_3{}^k$,$U_4{}^k$ and $V_1{}^k$,$V_2{}^k$,$V_3{}^k$,$V_4{}^k$ are the vertices of H_4 graph.

$U_3{}^k$ and $V_2{}^k$ are connected by an edges for all k.

Also, $V_1{}^k$ and $V_2{}^k$ are connected by a path for all k.

Define $f:V(G)\rightarrow\{1,2,3\}$ then the vertex labeling as

$i) \ f(U_1{}^k) = 2 \ for \ all \ k$

$ii) \ f(U_2{}^k) = 2 \ for \ all \ k$

$iii) \ f(U_3{}^k) = 1 \ for \ all \ k$

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$iv)\ f({V_1}^k) = 3\ when\ k\ =\ 1,2,5,6...$

$v)\ f({V_2}^k) = 1\ for\ all\ k.$

$vi)\ f({V_3}^k) = 2\ when\ k\ =\ 1,2,5,6,...$

$vii)\ f({V_4}^k) = 1\ for\ all\ k.$

$viii)\ f({V_1}^k) = 2\ when\ k\ =\ 3,4,7,8,...$

$viii)\ f({V_3}^k) = 3\ when\ k\ =\ 3,4,7,8,...$

Edge labeling must be given as

$i)\ f^*(({U_1}^k,{U_2}^k)) = 4\ for\ all\ k.$

$ii) f^*(({U_2}^k,{U_3}^k)) = 3\ for\ all\ k.$

$iii)\ f^*(({U_3}^k,{U_4}^k)) = 4\ \ for\ all\ k.$

$iv)\ f^*(({V_1}^k,{V_2}^k)) = 4\ when\ k\ =\ 1,2,5,6,...$

$v)\ f^*(({V_1}^k,{V_2}^k)) = 3\ when\ k\ =\ 3,4,7,8,...$

$vi)\ f^*(({V_2}^k,{V_3}^k)) = 3\ when\ k\ =\ 1,2,5,6,...$

$vii)\ f^*(({V_3}^k,{V_4}^k)) = 4\ when\ k\ =\ 1,2,5,6,...$

$viii)\ f^*(({V_3}^k,{V_4}^k)) = 5\ when\ k\ =\ 3,4,7,8...$

$xi) f^*(({U_3}^k,{V_2}^k)) = 2\ for\ all\ k.$

$x) f^*(({V_1}^k,{V_1}^{k+1})) = 6\ when\ k\ =\ 1,5,...$

$xi) f^*(({V_1}^k,{V_1}^k)) = 5\ when\ k\ =\ 2,4,6,...$

$xii) f^*(({V_1}^k,{V_1}^k)) = 4\ when\ k\ = 3,7,...$

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Illustration:12

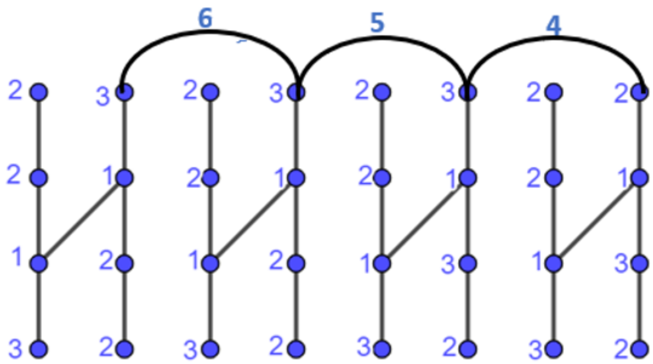

Figure: 12

Figure-12 indicates s the Path union 4-Copies of H_4 graph and its lucky number is 6.

Conclusion:

In this paper, we shown that the H-graphs, n copies of H-graphs, Theta graph, Path union of Theta graph and Duplication of Theta graph are Lucky edge graphs.

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