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A Study about Forecasting Bangladesh by Using Verhulst Logistic Growth Model and Population Model

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ABSTRACT

Bangladesh is a densely populous country compared to other countries in the world. The population projection is expected by everyone and all countries from an initial age. Additionally, the population progress and the country's dimension are located foremost aspects that affect its economy and strategies. This article aims to present an overview of the forecasting of the forthcoming population progress of the country. We predicted the population growth of Bangladesh from 2020 to 2080 by using Verhulst Logistic Growth Model. Bangladesh will impact its carrying capacity of 276.33 million populations in the year 2080 and then it decreases as differently shaped curves. The article has delivered attention to the altering trends of the progress of the population of Bangladesh.

Keywords: Population Model, Verhulst Logistic Growth Model, Carrying Capacity, Future Population Projection.

1. INTRODUCTION

Population projection is quite possibly the most active concern to guarantee fast, compelling and feasible progression for humans. It is a valuable apparatus to exhibit the greatness of current issues and is liable to appraise the future size of the issue. In the quickly changing current world, populace projection has gotten perhaps the most pivotal issue. Population size and development in a nation baldly impact the circumstance of strategy, culture, training and climate and so forth of that country and cost of characteristic sources. Bangladesh is a densely populated united states than many different nations of the international. The population boom is named as alarming, however, understanding of boom in the future years might be beneficial in planning for the development of the country for the reason for population demonstrating and determining an assortment of fields, this model is generally utilized (Müller et al., 1989). To version growth of biological systems, several models have been introduced. These variously address population dynamics, both modeled discretely or for big populations, usually continuously. The easy exponential boom model can provide a good enough approximation to such an increase for the preliminary period. Predation and intraspecific competition are not covered for populations. (Verhulst & Pierre-François., 1838) took into consideration that, for the population model, a strong population could consequently have a saturation stage feature: this is normally known as the carrying capacity K, and paperwork a numerical top certain at the growth length. (Abayasekara, 2019) and (Abeykoon & A.T.P.L, 2011) have also studied how a country's population growth and demographic dimensions are important factors in its economic system and policies. Population forecasting is essential for all long-time period planning and the availability of a rural community's services. Moreover, mathematical models take much bureaucracy to predict the population increase, such as dynamical structures, statistical methods, methods with differential equations, etc. There are several issues related to the population growth along with the food elements, available land, era, delivery fee, loss of life rate and examples include emigration, winning conditions in a similar conflict and so on.(Fernando & N., 1991).Because of the aforementioned factors, the population will fluctuate, but it will rise enormously. This article is primarily based on the projection of the future population boom of the United States. Population dynamics makes use of the population model, essentially the main character form of a mathematical version. There are a lot of strategies that help to develop a sound populace version. Among these, the Malthusian exponential model through Thomas Robert Malthus (1766-1834) and the Verhulst (1838) logistic differential equation model is widely known by (Glass & Murray, 2003). Identification of Graph Thinking in Solving Mathematical Problems Naturally by (PRAYİTNO et al., 2022).The exponential model can't plan the environmental restrict factors because it provides a pressure-loose calculation for the future populace and can not trap the meticulous destiny population-wide variety. So the population will increase with the section of time to infinity which is unrealistic. The main dilemma of the Malthus model is that the populace is growing geometrically and once every 25 years or more (TR & Malthus, 1970). Several researchers in several fields have recently ended up inquisitive about finding out quantitative models of population increase (Shair et al., 2017). Five different perspectives on mathematical modeling in mathematics education have described by (Abassian et al., 2020) .Once it is modeled, it may be applied for planning populace applications, both for controlling increase or diffusing populace aimed at a balanced distribution. However, population predictions are an inevitable device for choice-makers and planners.(Tsoularis & Wallace, 2002) have investigated logistic growth models and their implications. The Verhulst–Pearl logistic equation, in which the population expansion stops at the carrying capacity, is the most common variant used to estimate the increase in populations with overlapping generations. (Ullah et al., 2019) have analyzed and Projection of Future Bangladesh Population Using Logistic Growth Model and (Turner et al., 1969) has successfully exploited validating a student assessment of mathematical modeling at the elementary school level.(Karim et al., 2020) have investigated population projection of Bangladesh with the help of Malthusian model, Sharpe-lotka model, and Gurtin Mac-Camy model and Malthusian model is related to caring capacity K of Population growth.We present the major models of this shape in this study. Finally, we introduce a generalized logistic equation that all of them are treated as special cases. In this paper, we determine forecasting Bangladesh by using Verhulst logistic growth model and population model.

2.1. SIMPLE MODELS

Let N_t and N_{t+1} denote the population at time t and $t+1$ respectively .This leads us to study the discrete time population model of the form

$$N_{t+1} = N_t F(N_t) = f(N_t) \qquad (1)$$

where $f(N_t)$ is a nonlinear function of N_t.

If we know the form of $f(N_t)$ then one can easily evaluate the value of N_{t+1} and subsequent generations by simply using (1) recursively. We shall always keep in mind that we are interested in non negative population whatever the form of $f(N_t)$.

Suppose that the population one step later is simply proportional to the current population, that is ,$(N_t) = r > 0$. Then from (1) we have

$N_{t+1} = rN_t \Longrightarrow N_t = r^t N_0$ (2)

Here r is the net reproductive rate and $N_o > 0$ is the starting value of the population. If $r > 1$ then population grows geometrically and if $r < 1$,then population decays geometrically. For most populations or for long times this simple model is not very realistic.

This is the discrete version of Malthus model. A slight modification of (2) could be

$N_{t+1} = rN_s, N_s = N_t^{1-b}$, b constantwhere N_s is the population that survives to breed.There must be restrictions so that $N_s \leq N_t$.

2. 2. FORMULATION OF LOGISTIC GROWTH MODEL

Let $N(t) = N$ be the population of a single species at time t. Then the rate of change

$\frac{dN}{dt} = births - deaths + migrations$ (1)

is a conservation equation for the population. The simplest model has no migration and the birth and death terms are proportional to N. Let b and d be the number of births and deaths per individual per unit time. Thus (1) takes the form

$\frac{dN}{dt} = bN - dN = (b - d)N = aN$ (2)

where b, d, a are constants and $a = b - d$.

From (2) we have

$$\int \frac{dN}{N} = \int adt$$

$$\Longrightarrow lnN = at + lnA , \qquad wherelnA = constant.$$

$$\Longrightarrow ln\frac{N}{A} = at$$

$$\Longrightarrow \frac{N}{A} = e^{at}$$

$\Longrightarrow N = N(t) = Ae^{at}$ (3)

At t=0, let the initial population $N(0) = N_o$.Then from (3) we have

$N(0) = Ae^0$

$\Longrightarrow N_o = A$

Putting this value in (3) we get,

$N(t) = N_o e^{at}$ (4)

Equation (4) shows that population grows exponentially if $a > 0$ $i.e. b > d$, decays exponentially if $a < 0$ $i.e. b < d$, and remains constant if $a = 0$ $i.e. b = d$. This approach, due to Malthus (1798), is fairly unrealistic.

Verhulst (1838 , 1845) proposed that a self-limiting process should operate when a population becomes too large . He suggested.

$$\frac{dN}{dt} = rN(1-\frac{N}{K}) \qquad (5)$$

where r and K are positive constants.

He called this logistic growth in a population. Here $r\ (1-\frac{N}{K})$ is the per capita birth rate and K is the carrying capacity of the environment, which is determined by the available sustaining resources.

$\frac{dN}{dt} = 0$ gives $N = 0$ and $N = K$. Thus there are two steady states or equilibrium states. $N = 0$is unstable since linearization about it gives

$$\frac{dN}{dt} \approx rN \qquad (6)$$

which is like as (2) and so N grows exponentially from any small initial value. Again, the equilibrium $N = K$ is stable since linearization about it , that is , $(N-K)^2$ is neglected compared with ($|N-K|$) gives

$$\frac{d\ (N-K)}{dt} \approx -r(N-K) \qquad (7)$$

and so $N \to K$ as $t \to \infty$.

Now we give the solution of (5). From (5) we have

$$\frac{dN}{N\left(1-\frac{N}{K}\right)} = rdt$$

$$\Longrightarrow \frac{KdN}{N(K-N)} = rdt$$

$$\Longrightarrow \int \left(\frac{1}{N} + \frac{1}{K-N}\right) dN = \int rdt$$

$$\Longrightarrow \ln N - \ln(K-N) + \ln A = rt, \ln A = constant.$$

$$\Longrightarrow \ln\frac{AN}{K-N} = rt$$

$$\Longrightarrow \frac{AN}{K-N} = e^{rt}$$

$$\Longrightarrow AN = (K-N)e^{rt} \qquad (8)$$

By initial conditions we have

$$AN_o = (K-N_o)e^0$$

$$A = \frac{K-N_o}{N_o}$$

Putting the value of A in (8) we get,

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$$\frac{K-N_o}{N_o}N = (K-N)e^{rt}$$

$$\Rightarrow KN - NN_o = KN_oe^{rt} - NN_oe^{rt}$$

$$\Rightarrow N(K - N_o + N_oe^{rt}) = KN_oe^{rt}$$

$$\Rightarrow N = N(t) = \frac{N_o\,K\,e^{rt}}{K+N_o(e^{rt}-1)} \qquad (9)$$

$$\Rightarrow N = N(t) = \frac{N_o\,K}{K\,e^{-rt} + N_o(1-e^{-rt})}$$

When $t \to \infty$ then $N(t) \to \frac{N_o\,K}{0+N_o(1-0)}$

$$\Rightarrow N = N(t) \to K$$

which is the limiting behaviour of the Verhulst model as $t \to \infty$.

From (5) we see that , if $N_o < K,$ then $\frac{dN}{dt}$ is always positive and N(t) increases monotonically to a limiting population size K. If $N_o > K$ then $\frac{dN}{dt}$ is always negative and N(t) decreases monotonically to K.

Differentiating (5) , we get

$$\frac{d^2N}{dt^2} = r(1 - \frac{2N}{K}) \qquad (10)$$

If $N_o < \frac{K}{2}$, then $\frac{dN}{dt}$ increases as N varies from N_o to $\frac{K}{2}$ and decreases as N varies from $\frac{K}{2}$ to K.From (10), $\frac{d^2N}{dt^2}$ vanishes when $N = \frac{K}{2}$, so that there is a point of inflexion in the population growth curve when half the final population size is reached. Thus from (9) we have $\frac{K}{2} = \frac{N_o\,K\,e^{rt}}{K+N_o(e^{rt}-1)}$

$$\Rightarrow K + N_o(e^{rt} - 1) = 2N_oe^{rt}$$

$$\Rightarrow K - N_o = N_oe^{rt}$$

$$\Rightarrow \frac{K}{N_o} - 1 = e^{rt} \Rightarrow rt = \ln\,(\frac{K}{N_o} - 1)$$

$$\Rightarrow t = \frac{1}{r}\ln(\frac{K}{N_o} - 1) \qquad (11)$$

Thus, the point of inflexion occurs at time t given by (11)

2.3. MODELS FOR FORECASTING POPULATION

Under this phase, two boom fashions which are used to expect the populace of Bangladesh could be discussed. The very last model equations are obtained by way of solving the differential equations in order to expect the population of Bangladesh making use of predated population statistics from the census populace and the calculated mid-year populations, the beginning rate and the dying charge obtained by means of the Department of Registrar General of Bangladesh from 1991 to 2020. In order to check the accuracy and select the excellent model, three measures of the forecasted mistakes namely Root Mean Square Error (RMSE),

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Mean Absolute Percentage Deviation (MAPD) and Symmetric Mean Absolute Percentage Error (SAPE) are discussed.

Using reference curve fitting (Nelder, 1961)

Let us consider that equation

$$y = a + bx \qquad (1)$$

We know, Equations (normals) are

$$\sum y = ma + b\sum x \qquad (2)$$

$$\sum xy = a\sum x + b\sum x^2 \qquad (3)$$

Population of Bangladesh (2020 and Historical Table -1:

Year (x)	Population (million), $y = f(x)$
1950	37.895
1955	43.444
1960	50.102
1965	57.792
1970	66.881
1975	70.582
1980	80.624
1985	92.284
1990	105.256
1995	117.487
2000	127.658
2005	139.036
2010	147.575
2015	156.256
2020	164.689

Worldometer(www.Worldometers.info) Reference Elaboration of data by United nations , Department of Economic and Social Affairs, Population Division

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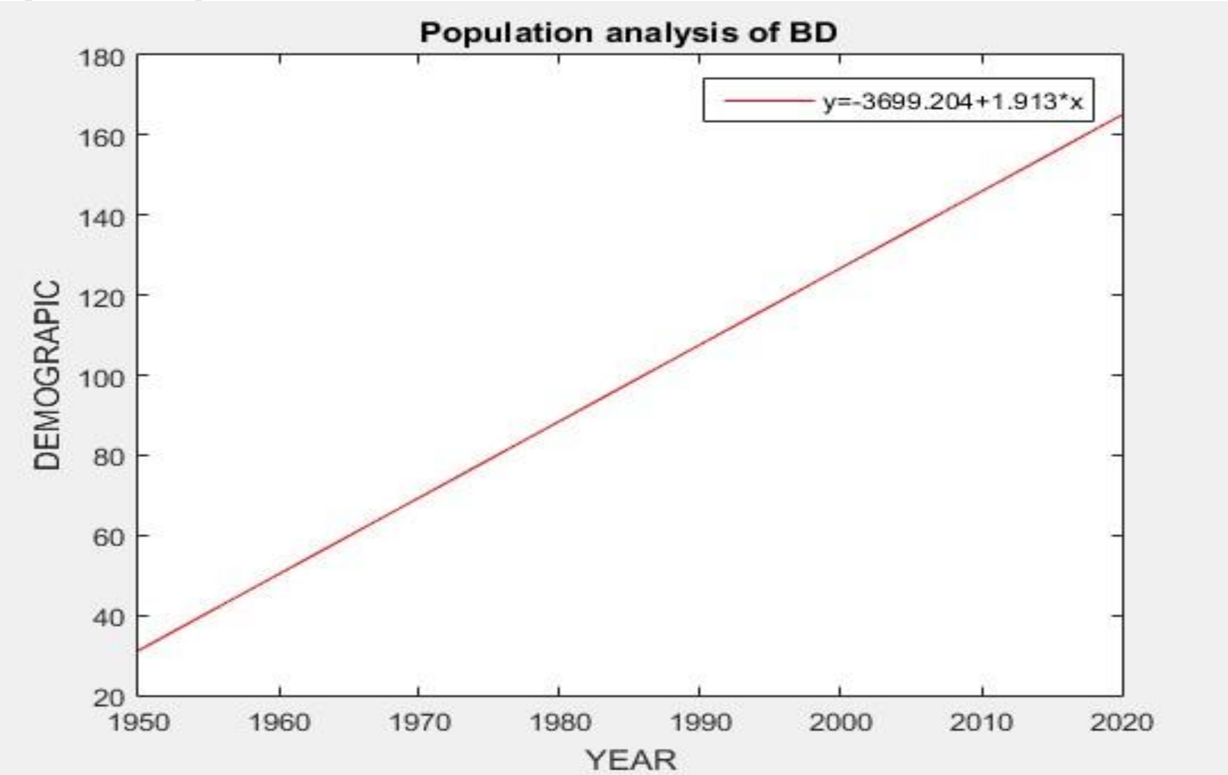
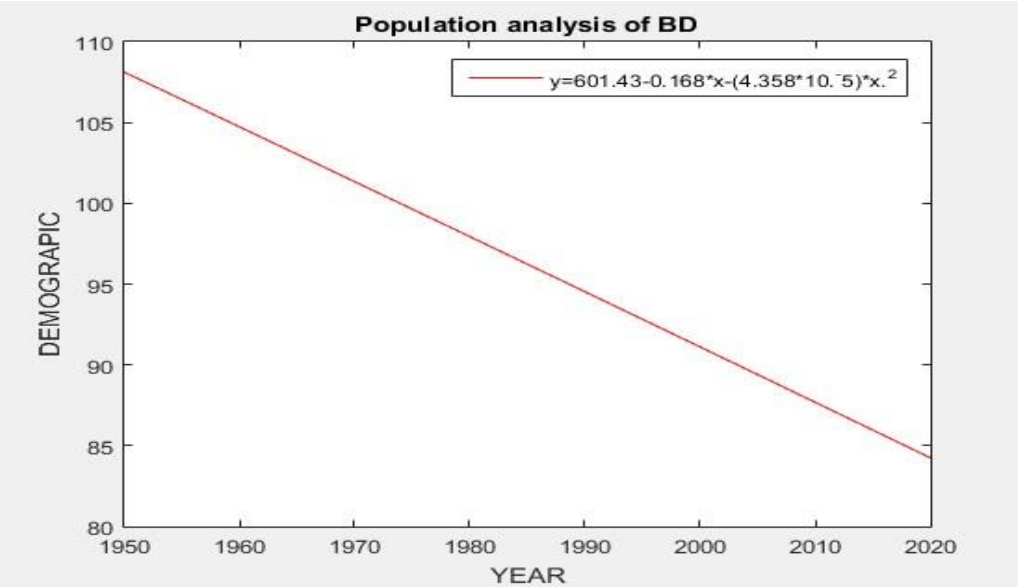
Solving by above table using [9],we get the equation $y = -3699.204 + 1.913\ x$which represent the equations of st^n (4)



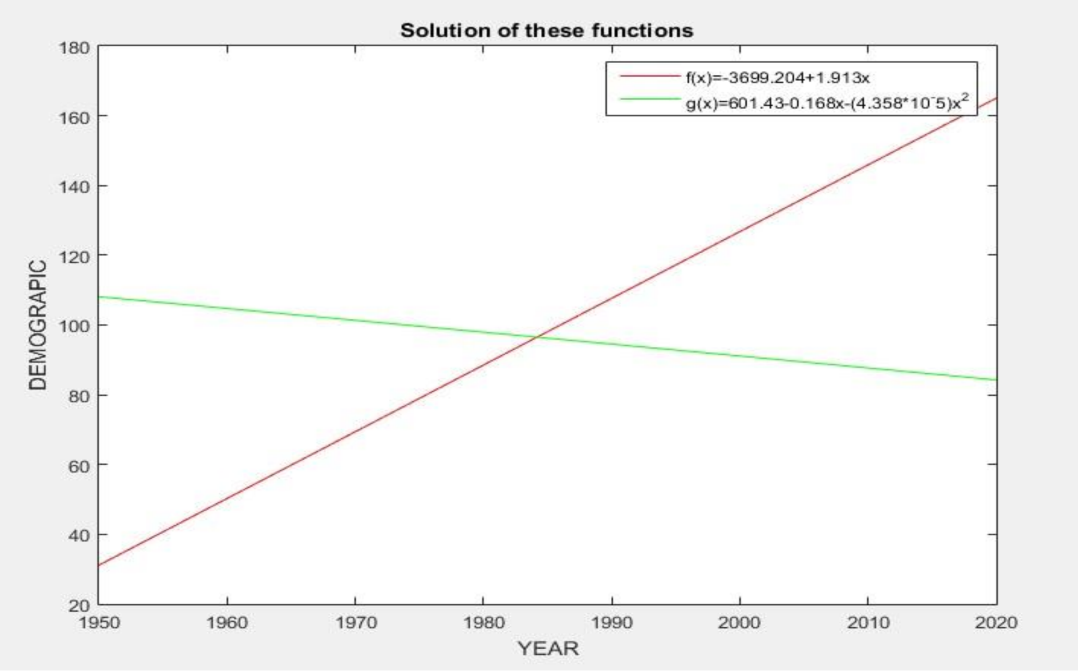
(Figure-1: Using Matlab)

And also considering for second degree parabola to the following data taking x as the independent variable where the curve of the second degree parabola is

$$y = 601.43 - 0.168x - (4.358\ \times 10^{-5})x^2 \qquad (5)$$



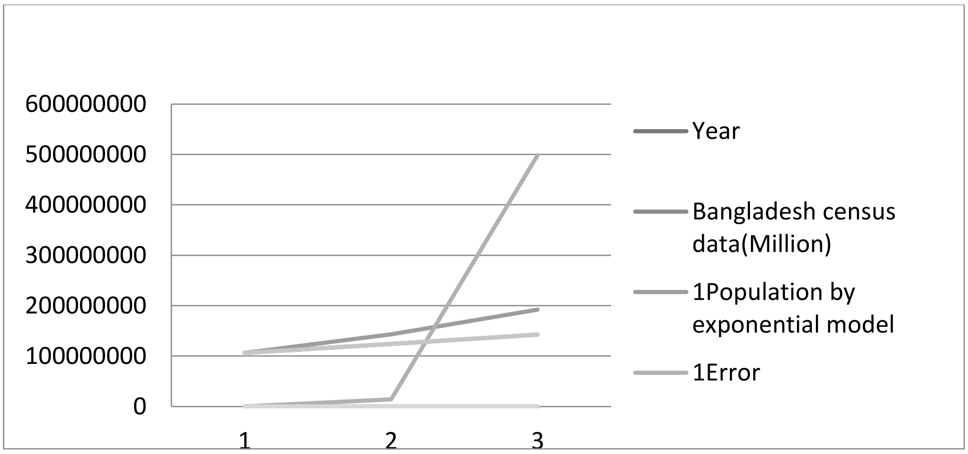
(Figure-2: Using Matlab)



(Figure-3: Using Matlab)

Table- 2: Census data of Bangladesh (Ullah et al,2019) reference use table

Year	Bangladesh census data(Million)	[1]Population by exponential model	[1]Error	[2]Population by logistic model	[2]Error
1991	106313000	106313000	0.0	106313000	0.0
2001	124355000	142936945.14	14169339.51	124355034	34
2011	142319000	192177535.08	498588535.08	142319066.3	66.3
2021	169101000	243078454.03	73977454.03 1	169109045.6	98.5



(Figure-4: Census data of Bangladesh and population by exponential model)

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Table-3:population of yearly change

Year (x)	Population (million) $y = f(x)$	Yearly % Change	Yearly change
1950	37.895	1.88%	185.675
1955	43.444	2.12%	838.324
1960	50.102	2.67%	118.544
1965	57.792	2.90%	147.432
1970	66.881	3.01%	176.947
1975	70.582	1.75%	116.676
1980	80.624	2.59%	191.463
1985	92.284	2.65%	222.493
1990	105.256	2.60%	248.155
1995	117.487	2.22%	239.959
2000	127.658	2.08%	249.758
2005	139.036	1.72%	227.553
2010	147.575	1.20%	170.798
2015	156.256	1.15%	173.616
2020	164.689	1.01%	164.322

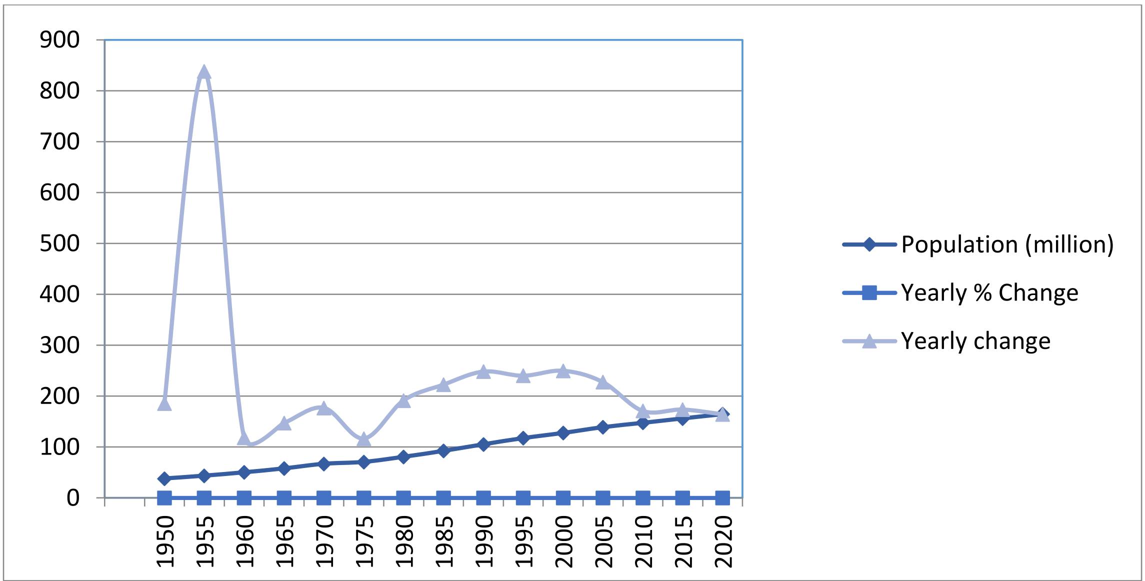
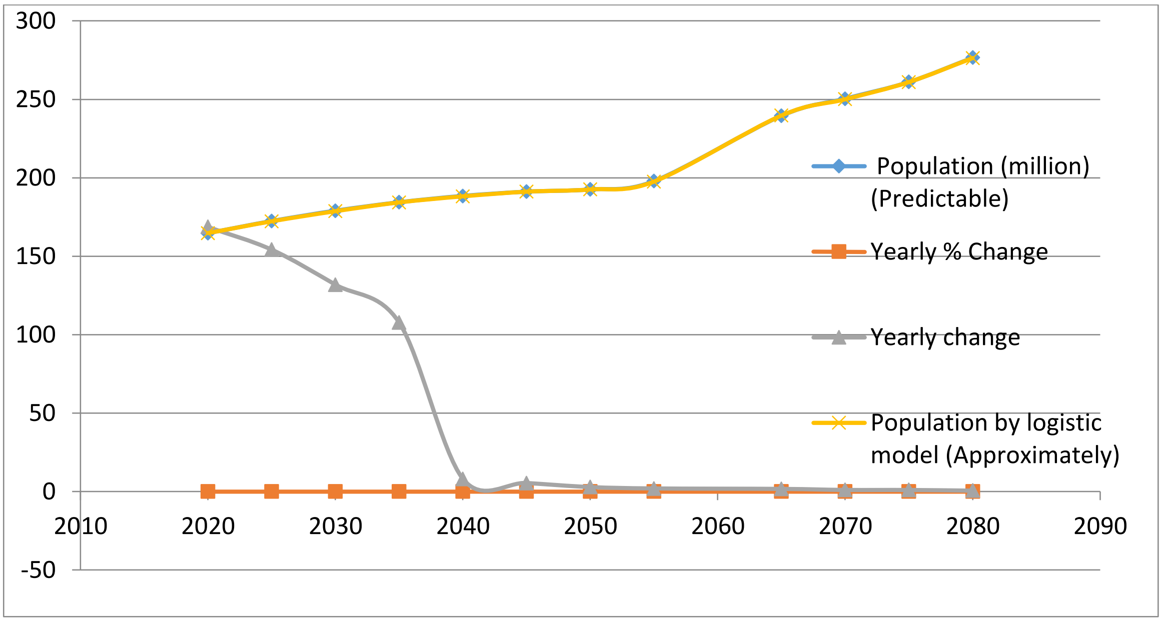


Fig-5 :population of yearly change

Table-4: Population of Bangladesh (2020 and Historical)

Year (x)	Population (million) $y = f(x)$ (Predictable)	Yearly % Change	Yearly change	Population by logistic model (Approximately)
2020	164.689	1.01%	168.662	164.689
2025	172.399	0.92%	154.193	172.23
2030	178.993	0.75%	131.895	178.678
2035	184.374	0.59%	107.694	184.374
2040	188.416	0.43%	8.085	188.416
2045	191.142	0.29%	5.456	191.142
2050	192.556	0.15%	2.851	192.56
2055	197.85	0.11%	1.975	197.57
2065	239.57	0.87%	1.675	239.7
2070	250.43	0.05%	1.008	250.03
2075	261.05	0.03%	0.976	261.01
2080	276.55	0.01%	0.564	276.33



(Figure-6: Population yearly change and using Logistic growth model)

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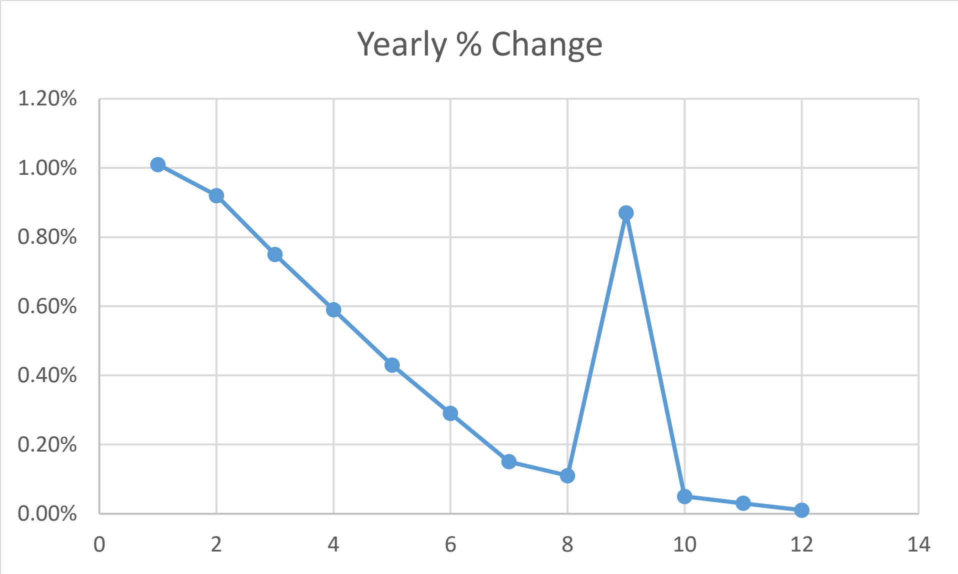
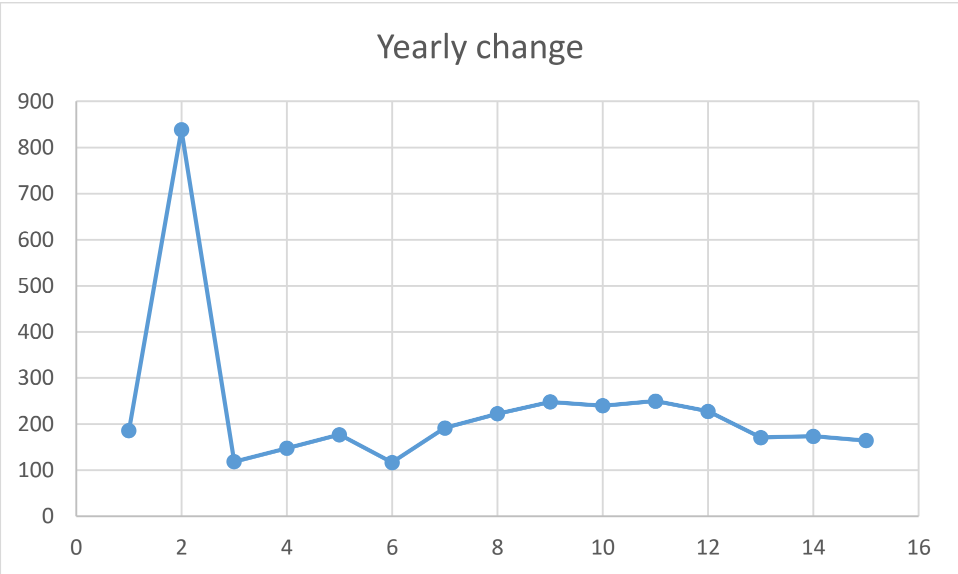


(**Figure-7: Population yearly change and using Logistic growth model**)



(Figure-8:Population yearly change and using Logistic growth model)

2.4. RESULT AND DISCUSSION

In this study, we predicted the population of Bangladesh applying Verhulst Logistic Growth Model. Firstly we take the population growth data. Then using table-1 with the help of curve fitting, we draw the figure-1 and figure-2. Figure-1 and figure -2 have shown the graphical representation of the projected population from 1950 to 2020. We solve the equation by using Newton's forward formula. From the solution, we get the figure-3. In table-2, we have analyzed the census data of the population of Bangladesh during 1991-2021 compared with the logistic model that approach as congenial harmony with the census data of the population of Bangladesh and graphically representation shown by respectively Figures 4-Figures 8. From the above table, we see that the population rate will gradually decrease. At a time population growth rate will be near zero.

2.5. CONCLUSION

In conclusion, this study provides a methodology for studying the behavior of the population projection. In this study, Verhulst Logistic Growth Model was introduced for predicting the population growth of Bangladesh and to investigate the pattern of the country's population growth in the long run. Predictions for the future population of Bangladesh were calculated from 2020 to 2080 using the Logistic Growth Models. The Logistic Growth Model is appropriate for predicting the human population. The model provides a good fit to the population data that was obtained from the Department of Census and Statistics, Bangladesh. The projected population of the country using the Logistic Growth Model is about 276.33 million by the year 2080. Verhulst Model is a model that recognizes the constrained nature of resources and how resources should match the population. Bangladesh and other developing countries, need to try and address the problem of high population growth since such countries have the lowest access to useful resources such as medical care, housing, and food.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interest related to the publication of this article

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