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Inflationary Scenario in Bianchi Type II Space with Bulk Viscosity in General Relativity

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Abstract: This study is about bulk viscous inflationary model with flat potential under framework of LRS Bianchi type II metric. To drive relativistic solution of the field equations we choose theproportionally condition between coefficient of shear and expansion scalar which leads to asuitable relation $a = b^n$ between the metric functions where n is the constant other than one. Some dynamical features of the universe are also discussed.

Keywords: Bianchi Type II, Inflationary Model, Bulk Viscosity, Flat Potential, General Relativity

1.Introduction

In recent scenario, inflationary cosmology has become curious subject of study to explaining the evolution of universe and formation of galactic structures, it widely acceptable by many cosmologist that major cosmological problem like isotropy, homogeneity, flatness are successfully explain by the theory of inflation. The choice of anisotropic metric with system of field's equations allows us to construct mathematical model of cosmos to understanding the accelerated fate of current physical universe. The system of fields equations are basically set of non-linear differential equations we required solutions of its in various applications in astrophysics and cosmology.The analysis of microwave background has also provided some physical evidence regarding the cosmos inflation. Staronbinsky [1] explained the model of initial universe after epoch. Guth[2] studies the various aspects of cosmos inflation by proposing the fact that false vacuum energy are responsible for this phenomenon. The role of higg's fields with potential V are significantly are used in various research.Many cosmologists [3-11] have derived different model to understanding the theory of inflation and scalar field φ in various manner.
LRS Bianchi- II metric has significant role in developing models which helps to understanding the early evolution of cosmos and inflationary nature in more details. Inflationary model within framework of bulk viscosity is very helpful to illustrating many physical and structural features in dynamics of current universe. Mishner [12] studied inflationary model under effect of bulk viscosity in different manner. Heller and Klimek [13] has constructed viscous fluid model without initial singulaties. Gron [14-15] has discussed Bianchi - I space with bulk, nonlinear viscosity and in shearing mode.. The significant role of bulk viscosity in cosmos inflation is studied by many researchers [16-18] and Bali et al.[19-21] in different aspects. Agrawal [22] investigate LRS

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Bianchi II model inflationary model. Reddy [23] investigated Bianchi Type V space time for massless scalar field under flat potential. Sharma [24] derived Bianchi II inflationary model in general relativity. Bali and Poonia [25] constructed inflationary cosmological model under framework of Bianchi type III with bulk viscosity.

Motivated by this discussion, we have investigated Bulk viscous inflationary cosmological model with flat potential for LRS Bianchi type II metric. To get inflationary solution we suppose $\xi\theta = \alpha$ (constant) as proposed by Brevik et al.[26]. The paper work is classified as following sections given as: Section-2 concerned with metric and system of nonlinear fields equations. Section-3 contains solutions of fields equations in inflationary context.Section-4 are concerned with dynamical aspects of constructed model.Section-5 deals with conclusion and discussion.

2. Fields Equations with metric

LRS Bianchi-II space in orthonormal frame can be obtained by line element

$$ds^2 = g_{\mu\gamma}\theta^{\mu}\theta^{\gamma}\ , \quad g_{\mu\gamma} = \text{diag}\,(1, 1, 1, -1) \tag{1}$$

Here $a(t)$ and $b(t)$ are the metric function

where$\theta^1 = a(t)\omega^1, \theta^2 = b(t)\omega^2, \theta^3 = a(t)\omega^3\ , \theta^4 = dt$

$$\text{with}\ \omega^1 = dx\ , \omega^2 = dy - xdz\ ,\ \omega^3 = dz \tag{2}$$

from equation (1) and (2) metric can be written as

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)(dy - xdz)^2 + a^2(t)dz^2 \tag{3}$$

The field of gravity minimallyto scalar region with potential $V(\varphi)$ is given by

$$S = \int \left(R - \frac{1}{2}\varphi_{,i}\varphi_{,j}g^{ij} - V(\varphi)\right)\sqrt{-g}\ dx^4 \tag{4}$$

Einstein's field equation for the model is given by

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j \tag{5}$$

(In geometrical unit 8πG = c = 1)

where energy momentum tensor for scalar field under consideration of bulk viscosity is given by

$$T_{ij} = \varphi_{,i}\varphi_{,j} - \left(\frac{1}{2}\varphi_{,l}\varphi^{,l} + V(\varphi)\right)g_{ij} - \xi\theta\left(g_{ij} + u_i u_j\right) \tag{6}$$

$$\text{with}\ \frac{1}{\sqrt{-g}}\partial_{,i}\left(\sqrt{-g}\varphi_{,i}\right) = -\frac{dV(\varphi)}{d\varphi} \tag{7}$$

whereξ and θ be the bulk viscosity coefficient and scalar expansion respectively

we assume comoving coordinate system as $u^i = (0,0,0,1)$

For LRS Bianchi type II metric (1), system of field equation (2) can be obtained as

$$2\frac{a_{44}}{a} + \frac{{a_4}^2}{a^2} - \frac{3}{4}\frac{b^2}{a^4} = -\left(\frac{1}{2}\varphi_4^2 - V(\varphi) - \xi\theta\right) \tag{8}$$

$$\frac{a_{44}}{a} + \frac{b_{44}}{b} + \frac{a_4 b_4}{ab} + \frac{1}{4}\frac{b^2}{a^4} = -\left(\frac{1}{2}\varphi_4^2 - V(\varphi) - \xi\theta\right) \tag{9}$$

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$$\frac{{a_4}^2}{a^2} + 2\frac{a_4 b_4}{ab} - \frac{1}{4}\frac{b^2}{a^4} = \left(\frac{1}{2}\varphi_4^2 + V(\varphi)\right) \tag{10}$$

where indices 4 in metric coefficient shows the ordinary differentiation with respect to cosmic time

Equation (7) provides

$$\varphi_{44} + \left(2\frac{a_4}{a} + \frac{b_4}{b}\right)\varphi_4 = -\frac{dV}{d\varphi} \tag{11}$$

The physical parameter of developed model which are significantly used to find solution of field equations and discussing geometrical features are given by

Proper volume of developed model (1) is given by

$$V = \sqrt{-g} = a^2 b \tag{12}$$

Expansion scalar (θ) for model is given by

$$\theta = u^i_{,i} = 2\frac{a_4}{a} + \frac{b_4}{b} \tag{13}$$

Shear scalar for model is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{14}$$

where $\sigma_{ij} = \frac{1}{2}\left(u_{i,k}D_j^k + u_{j,k}D_i^k\right) - \frac{1}{3}\theta\left(g_{ij} + u_i u_j\right)$ (15)

and $D_i^j = \delta_i^j - u^i u_j$

which leads to

$$\sigma^2 = \frac{1}{3}\left(\frac{a_4}{a} - \frac{b_4}{b}\right)^2 \tag{16}$$

Formula for Hubble parameter is given by

$$H = \frac{1}{3}\left(2\frac{a_4}{a} + \frac{b_4}{b}\right) \tag{17}$$

we obtained deceleration parameter by given relation

$$q = -\frac{R_{44}/R}{{R_4}^2/R^2} \tag{18}$$

3. Fields Equations with solutions

Equation (8-10) is independent equations with five unknown a, b, φ, θ and ξ . For this purpose we required extra condition

$\xi\theta = \alpha$ (Constant) (19)

i.e. coefficient of bulk viscosityis inversely proportional to expansion scalarand shear scalar (σ) is directly proportional to expansion (θ) which leads to

$$a = b^n n > 1 \tag{20}$$

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wehave also assumed constant potential V with flat region

i.e. $V(\varphi) = \varsigma$ (Constant)

Equation (11) leads to

$$\varphi_{44} + \left(2\frac{a_4}{a} + \frac{b_4}{b}\right)\varphi_4 = 0$$

on integrating we have

$$\varphi_4 = \frac{\varphi_0}{a^2 b} \quad (21)$$

From the equations (8) and (9), we obtained

$$\frac{a_{44}}{a} - \frac{b_{44}}{b} + \frac{{a_4}^2}{a^2} - \frac{a_4 b_4}{ab} - \frac{b^2}{a^4} = 0 \quad (22)$$

Equations (20) and (22) give

$$2b_{44} + 4n\frac{{b_4}^2}{b} = \frac{2}{n-1}b^{3-4n} \quad (23)$$

we consider $b_4 = f(b)$ (24)

which provide $b_{44} = ff'$ (25)

where$f' = \frac{df}{db}$

from equations (23),(24) and (25) we have

$$\frac{df^2}{db} + 4n\frac{f^2}{b} = \frac{2}{n-1}b^{3-4n} \quad (26)$$

which leads to

$$f^2 = \frac{1}{2(n-1)}b^{4-4n} + Db^{-4n} \quad (27)$$

where D is the integrating constant

From equation (17) and (18) we have

$$b_4 = \left[\frac{1}{2(n-1)}b^{4-4n} + Db^{-4n}\right]^{\frac{1}{2}} \quad (28)$$

Equation (28) leads to

$$\int \left[\frac{1}{2(n-1)}b^{4-4n} + Db^{-4n}\right]^{-\frac{1}{2}} db = \pm(t - t_0)$$

wheret_0 is the integration constant

The metric (1) can be reduces in to form

$$ds^2 = -\left[\frac{1}{2(n-1)}T^{4(1-n)} + DT^{-4n}\right]^{-1} dT^2 + T^{2n}[dX^2 + dZ^2] + T^2[dY - XdZ^2]^2 \quad (29)$$

using the transformation $b = T,\ x = X,\ y = Y,\ and\ z = Z$

4. Structural and dynamical features of the model

The proper volume is given by

$$V = T^{2n+1} \quad (30)$$

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The shear scalar (σ^2) of model is obtained by

$$\sigma^2 = \frac{1}{3}(n-1)^2\left[\frac{1}{2(n-1)T^{2(2n-1)}} + D\,T^{-2(2n+1)}\right] \tag{31}$$

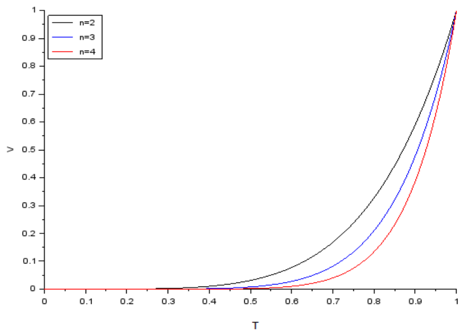

Fig. 1 volume (V) versus time (T)

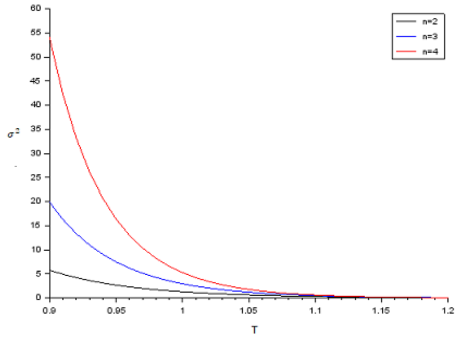

Fig. 2 coefficient of shear (σ)verses time(T)

The expansion (θ) for constructed model is given by

$$\theta = (2n+1)\left[\frac{1}{2(n-1)T^{2(2n-1)}} + D\,T^{-2(2n+1)}\right]^{\frac{1}{2}} \tag{32}$$

The result for hubble parameter (H) is given as

$$H = \frac{1}{3}(2n+1)\left[\frac{1}{2(n-1)T^{2(2n-1)}} + D\,T^{-2(2n+1)}\right]^{\frac{1}{2}} \tag{33}$$

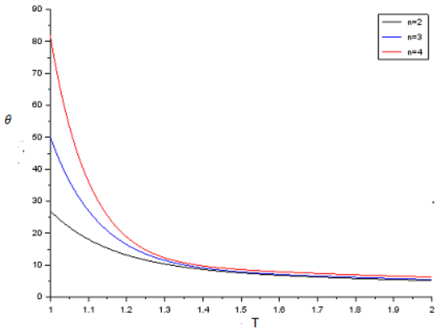

Fig. 3 coefficient of expansion (θ) versus time(T)

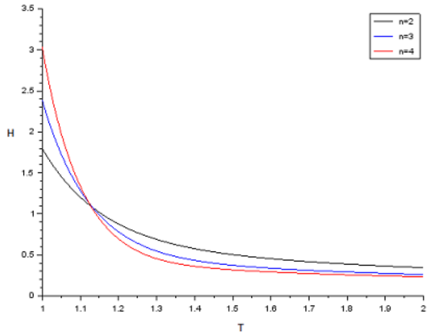

Fig. 4 Hubble parameter (H)verses time (T)

The deceleration parameter (q) for model is obtained by

$$q = -1 + \frac{3}{2n+1} \tag{33}$$

Coefficient of bulk viscosity (ξ) is given by

$$\xi = \frac{\alpha}{(2n+1)\left[\frac{1}{2(n-1)T^{2(2n-1)}} + D\,T^{-2(2n+1)}\right]^{\frac{1}{2}}} \tag{34}$$

Scalar field is given by

$$\varphi = \int \frac{\varphi_0}{T^{2n+1}}\,dT + C$$

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5. Conclusion and discussion

The proper volume growths with time and become infinite at late time which indicates that cosmic inflation is possible in developed model. Since $\frac{\sigma}{\theta} =$ constant i.e. model maintained the anisotropy at late time, but the model become shear free and isotropic at n=1. The negative deceleration parameter shows accelerated phase of universe. The scalar of expansion and Hubble parameter become divergent at initial epoch, t=0 and tends to zero for infinite T and at $n = -\frac{1}{2}$ i.e. universe start with infinite expansion. The shear scalar is decreasing function of time and tends to zero for infinite large T. Higgs field decrease slowly and become finite for large time. The bulk viscosity coefficient leads to cosmic inflation in present scenario.

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